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AUTHOR Beatty, Leslie; And Others
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ABSTRACT

Detailed lesson plans are provided in this teacher's guide for the SMSG text materials for grade 2. Included are chapters on sets and numbers (review), addition and subtraction (review), sets of points, addition and subtraction (extension), linear measurement, computing sums and differences, congruence of angles and triangles, arrays and multiplication, and division and rational numbers. Mathematical background, objectives, vocabulary, and materials are presented, followed by suggested discussion questions and activities. Answers for worksheets are also included. (MS)

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School Mathematics Study Group

# Mathematics for the Elementary School

## Book 2

Unit 55

# Mathematics for the Elementary School

## Book 2

*Teacher's Commentary*

REVISED EDITION

Prepared under the supervision of the
Panel on Elementary School Mathematics
of the School Mathematics Study Group:

| | |
|---|---|
| Leslie Beatty | Chula Vista City School District,<br>Chula Vista, California |
| E. Glenadine Gibb | State College of Iowa |
| William T. Guy | University of Texas |
| Stanley B. Jackson | University of Maryland |
| Irene Sauble | Detroit Public Schools |
| Marshall H. Stone | University of Chicago |
| J. Fred Weaver | Boston University |
| Raymond L. Wilder | University of Michigan |

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# PREFACE

The increasing contribution of mathematics to the culture of the modern world, as well as its importance as a vital part of scientific and humanistic education, has made it essential that the mathematics in our schools be both well selected and well taught--at all levels, from the kindergarten through the graduate school.

With this in mind, mathematical organizations in the United States cooperated in the formation of the School Mathematics Study Group (SMSG). The general objective of SMSG is the improvement of the teaching of mathematics in grades K-12 in the schools of this country. The National Science Foundation has provided substantial funds for the support of this endeavor.

One of the prerequisites for the improvement of the teaching of mathematics in our schools is an improved curriculum--one which takes account of the increasing use of mathematics in science and technology and in other areas of knowledge, and at the same time one which reflects recent advances in mathematics itself. Among the projects undertaken by SMSG has been that of enlisting a group of outstanding mathematicians, educators, and mathematics teachers to prepare a series of sample textbooks which would illustrate such an improved curriculum. This is one of the publications in that series.

The development of mathematical ideas among young children must be grounded in appropriate experiences with things from the physical world and the immediate environment. The text materials for grades K-3 provide for young children an introduction to the study of mathematics that reflects clearly this point of view, in which growth is from the concrete to the abstract, from the specific to the general. Major emphasis is given to the exploration and progressive refinement of ideas associated with both number and space.

These texts for grades K-3 were developed following the completion of texts for grades 4-6. The dynamic nature of SMSG permitted serious reconsideration of several crucial issues and resulted in some modification of earlier points of view. The texts for grades K-3 include approaches to mathematics which appear to be promising as well as approaches whose efficacy has been demonstrated through classroom use.

It is not intended that this book be regarded as the only definitive way of introducing good mathematics to children at this level. Instead, it should be thought of as a sample of the kind of improved curriculum that we need and as a source of suggestions for the authors of commercial textbooks. It is sincerely hoped that this and other texts prepared by SMSG will lead the way toward inspiring a more meaningful teaching of Mathematics, the Queen and Servant of the Sciences.

Based on the teaching experience of elementary teachers in all parts of the country and the estimates of the authors of the revisions, it is suggested that teaching time be approximately as follows:

BOOK 2

| Chapter | Time |
|---|---|
| 1 | 4-5 weeks |
| 2 | 4-5 weeks |
| 3 | 2-3 weeks |
| 4 | 4-5 weeks |
| 5 | 3-4 weeks |
| 6 | 4-5 weeks |
| 7 | 3-4 weeks |
| 8 | 2-4 weeks |
| 9 | 2-4 weeks |

Teachers are urged to try not to exceed these approximate time allotments so that pupils will not miss the important ideas contained in later chapters.

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## Chapter I

## SETS AND NUMBERS: REVIEW

### Background

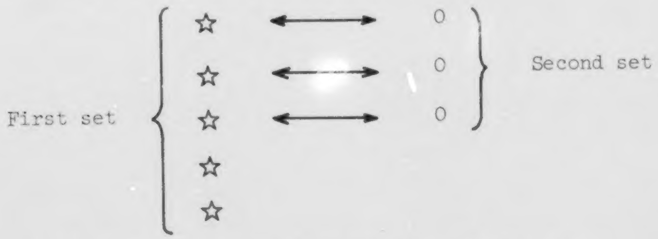
Section I-1 reviews ideas and terminology associated with sets. A set is just a collection of things. The things belonging to a set are called its members. A set may be defined by some property common to its members (the set of all children in this room; the set of all bicycles). Sets may also consist of quite unrelated objects. For instance, it would be quite legitimate (though perhaps of dubious interest) to discuss the set whose members are the number 7, George Washington, and the North Pole.

It is possible for a set to have only one member, as in the case of the set whose only member is the number 7, or the set whose only member is the number 0. There is also a special set, called the empty set, which by definition has no members at all.

One set is called a subset of another if every member of the first set is also a member of the second. Thus, the set of all children in this school room is a subset of the set of all children in this school. We need to notice especially two tricky facts about subsets. First, according to the definition of subset, every set is a subset of itself. For instance, it is undeniably a fact (though not a very exciting one) that the set of all bicycles is a subset of the set of all bicycles. Second, according to the definition of subset, the empty set is a subset of every set. For example, the empty set is a subset of the set of all bicycles, because every member of the empty set (there aren't any!) is a member of the set of all bicycles--or of any other set you want to name.

Section I-2 is concerned with the idea of comparing two sets to see which, if either, has more members than the other. A way of doing this which is probably more primitive than counting consists in *pairing off* the members of one set with those of the other. When such a *pairing-off* process "comes out even", we say that the two sets have been placed in *one-to-one correspondence*. In this case we may also say that the two sets are *equivalent* and that they have the *same number of members*. Notice that this is *not* the same thing as saying that these sets are equal. We say that a set is *equal* to another set only when they are actually *the same set*, i.e., when every member of the first set is also a member of the second and vice versa. In general, when sets are merely equivalent, there are many different ways of pairing off their members to place them in one-to-one correspondence.

Sometimes no matter how we carry out the pairing process we find that it does *not* "come out even". For instance, in the example illustrated below



there would always be members of the first set left unpaired with members of the second. In this case we say that the first set has *more* members than the second (and hence of course that the second has *fewer* members than the first). The case where the first set has just *one more member*, or *one fewer member*, than the second prepares the way for the idea that each whole number has a next-larger whole number and a next-smaller whole number (except that of course 0 has no next-smaller whole number).

Section I-3 is devoted to the operation of joining sets and the related operation of adding numbers. Taking a set and *joining* to it a second set results in forming a new set whose members are all those of the first set together with all those of the second. This new set is called the *union* of these two sets. Thus, if we take the set of all boys in our room and join to it the set of all girls in our room we get as the union the set of all children in our room. In this example the two sets joined are disjoint, that is, they have no members in common. When this happens, adding the number of members in the second set to the number of members in the first set gives the number of members in their union. It is obvious that joining the first of two sets to the second gives the same union as we get by joining the second set to the first. This clarifies the commutative property of addition: adding the first of two numbers to the second gives the same sum as we get by adding the second number to the first.

Just as joining sets serves as an approach to adding numbers, so removing from a set one of its subsets serves as an approach to subtracting numbers (Section I-4). Section I-5 points out the inverse relationship ("doing and undoing") between first joining and then removing a given set. Corresponding to this, we have the inverse relationship between first adding and then subtracting a given number.

Section I-6 concerns partitioning a set into equivalent (disjoint) subsets. For instance, we may partition a set of 43 members into 4 sets of 10 members each and a remaining set of 3 members. This will lead smoothly, in Section I-7, to the decimal place value system. But in Section I-6 we also partition sets into equivalent subsets of other sizes: 2 members each, 3 members each, 5 members each, etc. This prepares the way for later introducing, in Chapter VIII, the idea of dividing a number by 2, by 3, by 5, etc. Partitioning a set into sets of 2, or 3, or 5 members each, and counting the resulting number of sets, also clearly leads to the idea of counting by 2's, 3's and 5's.

As noted above, partitions into equivalent subsets are used in Section I-7 to introduce place value. At this time we do not go beyond the idea of "tens and ones".

A main objective is simply to lead children to realize that, for example,

34

3 tens and 4 ones

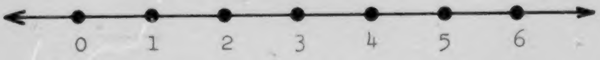
30 + 4

are three names--three ways of naming--the same number. Of these three, the last

30 + 4

is often the least familiar and the most useful. You may wish to use an abacus for class demonstrations. Also, pictures of an abacus are employed on one of the pages in the pupil's book (page 29).

Section I-8 reviews the idea of representing the whole numbers 0, 1, 2, 3, ... by points equally spaced in order from left to right along the so-called number line, as shown below.



We recall specifically that in pictures of the number line arrow-heads are used to suggest that the line extends indefinitely far in both directions. Later on, the number line will be used in a variety of ways: in visualizing number operations and their properties (e.g., in Chapter II), in introducing linear measurement (Chapter III), etc. The immediate use of the number line comes in Section I-9: greater than (>) for numbers corresponds to to the right of on the number line; and similarly less than (<) corresponds to to the left of. Finally, Section I-10 introduces the idea and terminology of the even numbers (0, 2, 4, 6 8, ...) and the odd numbers (1, 3, 5, 7, 9, ...).

## I-1. Sets

**Objective:** To review the ideas of set, member of a set, subset of a set, and the empty set.

**Vocabulary:** (Review) Set, member, subset, empty set.

**Materials:** Objects in the room, materials for the flannel board, counting blocks, books of various sizes and colors, etc.

### Suggested Procedure:

The time spent on this lesson will depend on the extent of the children's experience with the idea and vocabulary of sets.

### Sets

Use sets of objects found in various locations about the room. Have them described in different ways: the set of books on the book shelf, the set of things on the science table, the set of coats and jackets on the rack, the set of pencils in the children's desks.

Move toward more detailed descriptions: the set of blue, yellow and red books on the top shelf of the bookcase; the set of yellow pencils in the desks in the second row, etc. Also explain to children that a set may be designated by simply listing its members, whether or not they have any particular property in common. For instance, a set might have as its members Billy Jones, an eraser, the school building, and the letter A.

Begin to use the word member, as in the set which has as its members the girls wearing red dresses or the set whose members are the stapler, this book, and that eraser. If the word seems unfamiliar to children, talk about the members of the class, the members of a family, the members of a baseball team, etc. Sometimes the idea of member can be reinforced by asking such questions as, "Is this book a member of the set of pencils? Is the flag a member of the set of children? Is Billy a member of the set of boys named Joe, Tom, and Larry?"

Bring out the fact that a set may have only one member; e.g., the set of American flags in the classroom; the set of pencil sharpeners; the set of boys in our room named _____.

- Subset

This term may be less readily recalled than the term set and member.

> Let's think of the people in this school. Who are members of this set? (Pupils, teachers, principal, secretary, nurse, custodians, cafeteria workers, etc.) This set has many subsets. What are some of the subsets you can name? (See above.)
> Of which of these sets are you a member?

Be sure to mention many such subsets: the subset of children in grade one, the subset of boys, the subset of children 9 years old, the subset of adults, etc. For several of these sets ask whether each of its members is a member of the set of people in this school. Also, emphasize the idea of subset negatively:

> Is the set of boys in our school a subset of the set of boys in our room? (No, because not every boy in our school is a member of the set of boys in our room.)

- The empty set

Put a variety of objects on a desk or table. Ask a child to point out the set of books, pencils, etc., on the table. Then ask him to point out the set of apples. If there are no apples on the table, this will be the empty set. Use several examples: the set of readers with pink and purple covers, the set of children more than 11 years old in this class, etc.

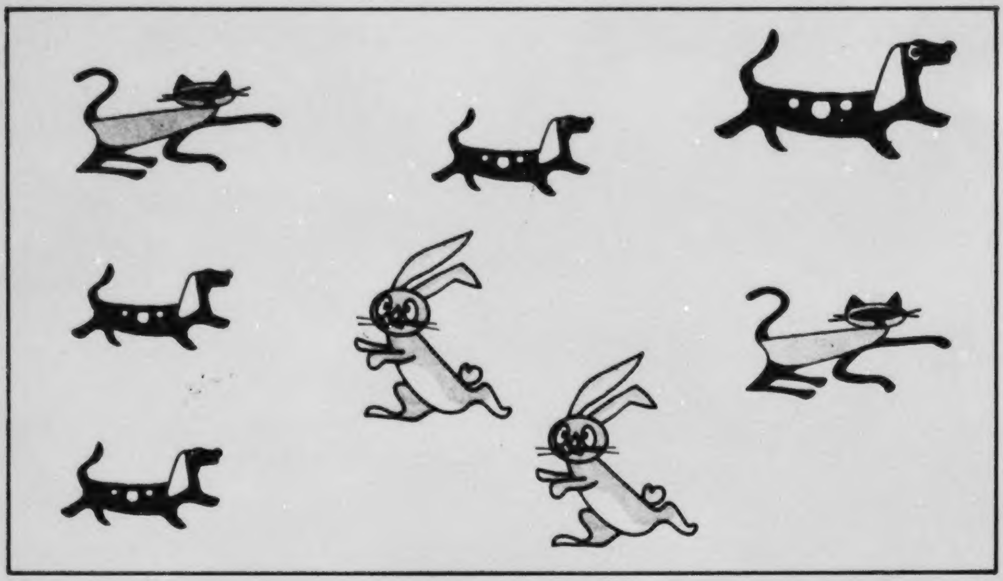
Pupil's book, page 1: Read the page with the children. Note that the pictured set is described in more than one way. Ask if there are other ways to describe each set. (Sets of pets;

---

* The dot is used throughout the commentary to note a change in activity or development within a section.

Set of animals with 4 legs. Set of things with fur. Etc.) Have children count to fill the blanks. Notice that the set of houses in the picture is the empty set, and that the number of the subset of animals is the same as the number of the set of animals.

Pupil's book, page 2: Read page with the children. Describe pictured set and subsets. Notice that some of the rings will overlap; the children should realize that things may be members of more than one subset of a set.



SETS AND SUBSETS

This is a set of animals.

This is a set of pets.

This is a set of cats, dogs, and rabbits.

This set of animals has 8 members.

This set has many subsets.

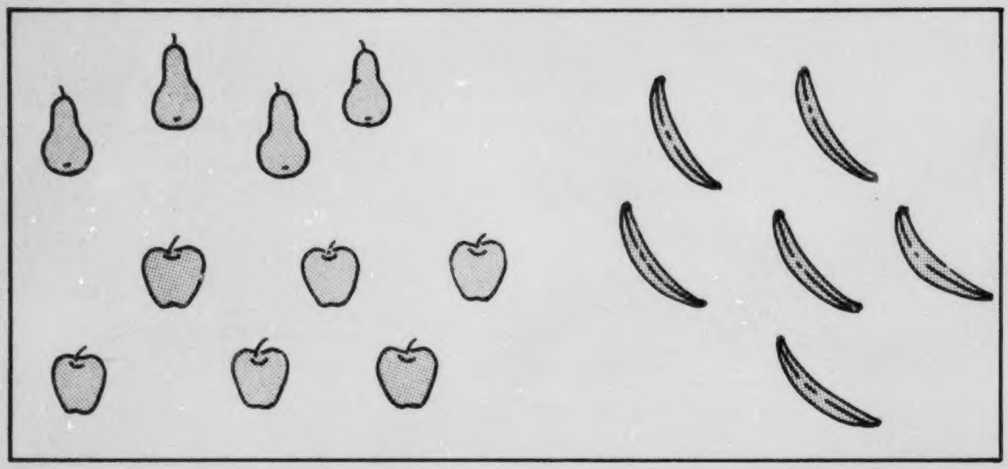
The subset of dogs has 4 members.

The subset of cats has 2 members.

The subset of rabbits has 2 members.

The subset of houses has 0 members.

The subset of animals has 8 members.



This is a set of fruit. It has many subsets.

Draw a red ring around the subset of 🍎's.

The subset of 🍎's has 6 members.

Draw a blue ring around the subset of 🍐's.

The subset of 🍐's has 4 members.

Draw a yellow ring around the subset of 🍌's.

The subset of 🍌's has 6 members.

Draw a green ring around a subset of two 🍎's and three 🍐's.

This subset has 5 members. Answers will vary.

Draw a brown ring around a subset of one 🍐, one 🍎 and two 🍌's.

This subset has 4 members. Answers will vary.

## I-2. Comparing sets

Objective: To review pairing and one-to-one correspondence using the expressions: equivalent, as many as, more than, fewer than, one more than, one fewer than.

Vocabulary: (Review) Equivalent, as many as, more than, fewer than, pairing, order.

Materials: Flannel board objects, children's set materials (beans, pegs, cardboard disks, toothpicks, bottle caps, etc.).

Suggested Procedure:

Place a set of 5 objects on the left side of the flannel board. Display other felt objects on a desk or table near by. Ask a child to come up and show a set with just as many members as the set you placed on the flannel board.

> How do you know it has just as many members? (Child will probably say he counted them; both have 5 members.)
>
> If you couldn't count, how could you still tell that this set has just as many members as that?

Review the idea of pairing: for each member of the set on the left there is just one member of the set on the right, and for each member of the set on the right there is just one member of the set on the left. (Suppose the first set is a set of trees and the second set is a set of disks. Then, for every tree there is a disk, and for every disk there is a tree.) Ask what we call sets which can be paired in this way. (Equivalent sets.)

Discuss various sets in the classroom. For instance:

> Is the set of books on the table equivalent to the set of pencils in this box? Find out without counting.

Let children pair books and pencils to determine whether the sets are equivalent. Repeat with other sets. If sets are not equivalent, ask:

> Which set has more members? Which set has fewer members? Let's say "The set of books has *more* members *than* the set of pencils. The set of pencils has *fewer* members *than* the set of books."

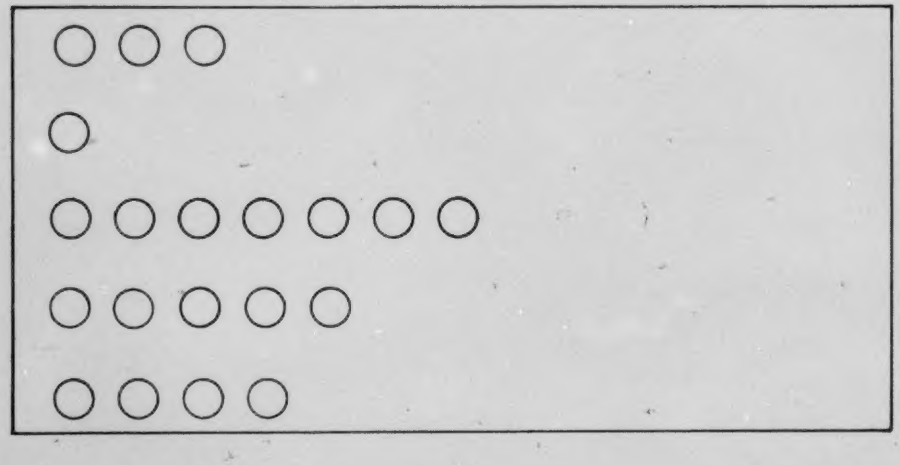
- Display a set of 4 objects on the flannel board. Have a child show a set with more members than your set. Have a child show a set with fewer members. Encourage use of the sentences: "This set has more members than that set. This set has fewer members than that set."

If children have difficulty with the term "fewer than", use manipulative materials. Have each child show on his desk a set equivalent to one you place on the flannel board. Then have him show a set with fewer members than the one you place on the flannel board.

*Pupil's book, page 3:* Read instructions with the children. The word equivalent is not used on the page, but it should be used orally.

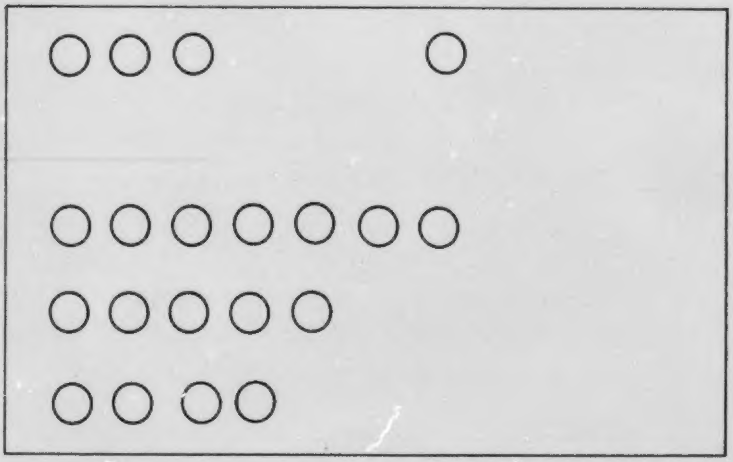
*Pupil's book, pages 4 and 5:*
Read instructions with the children. (Answers will vary.)

- *Order*
  Arrange sets on flannel board as shown below.



Can you put these sets in order so that each has more members than the one above it? Which set has fewer members than any of the others? (Set of 1.)

Let's put it over here.



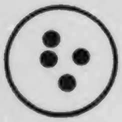
Which set will come next?

Have children finish ordering the sets on the right side of the flannel board. Point out that the set of 3 has more members than the set of 1, the set of 4 more than the set of 3, etc. Suggest that it is possible to put other sets into the display and still have the sets ordered.

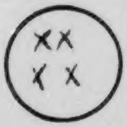
Where could we put another set? (Between the set of 1 and the set of 3; between the set of 5 and the set of 7.)

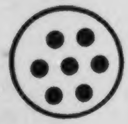
## Comparing Sets

Show a set with as many members. Use X's.



has just as many members as





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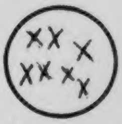

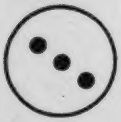

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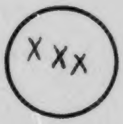

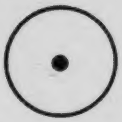

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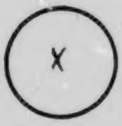

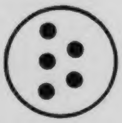

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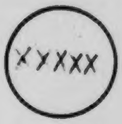

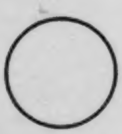

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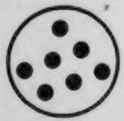

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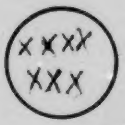
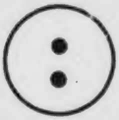
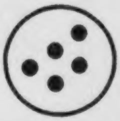
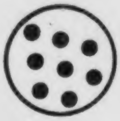


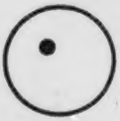
## Comparing Sets

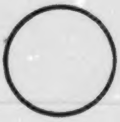
Show a set with more members. Use X's.

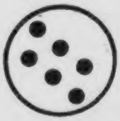
has more members than 

has more members than 

has more members than 

has more members than 

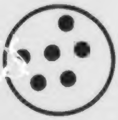
has more members than 

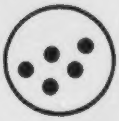
has more members than 

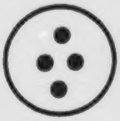
Answers will vary.

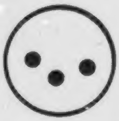
## Comparing Sets

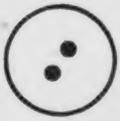
Show a set with fewer members. Use X's.

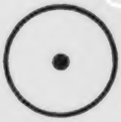
○ has fewer members than 

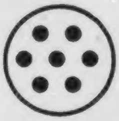
○ has fewer members than 

○ has fewer members than 

○ has fewer members than 

○ has fewer members than 

○ has fewer members than 

○ has fewer members than 

Answers will vary for all but next to last.
It must be empty.

How *many* more members has the set of 5 than the set above it? (One.)

How many fewer members has the set of 3 than the set below it? (One.)

More practice can be given with *one more than* and *one fewer than* using flannel board or manipulative materials as needed. Call attention to the relationship between the order of sets with one more member and the order of the counting numbers.

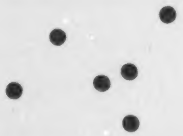
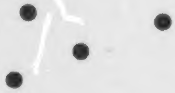
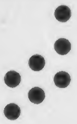
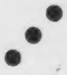
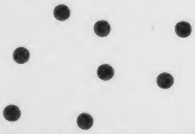
★ *Pupil's book, page 6*: (Optional). Read instructions with the children and have them notice that the set on the left must have only *one* fewer member than the set on the right. Therefore, the set they make must have one *more* member than the set on the left.

★ *Pupil's book, page 7*: (Optional). Children should notice that the set on the left must have only one more member than the set on the right. Therefore, the set they make must have one fewer member than the set on the left.

★ *Pupil's book, page 8*: (Optional). Children should use X's in the ring on the right, as indicated by the sentence to be completed.

## ★Comparing Sets

Show a set with one more member. Use X's.

| | | |
|---|---|---|
|  | has one fewer member than | X X X X X X X |
|  | has one fewer member than | X X X X X |
|  | has one fewer member than | X X X X X X X X |
|  | has one fewer member than | X X X X |
|  | has one fewer member than | X X X X X X X X X |
| | has one fewer member than | X |

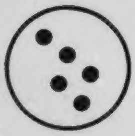
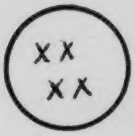
## ★ Comparing Sets

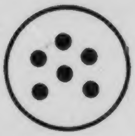
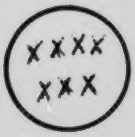
Use X's. Show a set with one fewer member.

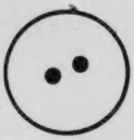
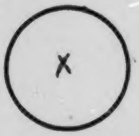
| | | |
|---|---|---|
| ● ● ● ● ● | has one more member than | X XXX |
| ● ● ● ● | has one more member than | XXX |
| ● ● ● ● ● ● | has one more member than | X X XXX |
| ● ● ● ● ● ● ● ● | has one more member than | XXX XXX X |
| ● ● | has one more member than | X |
| ● | has one more member than | |

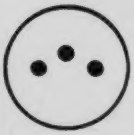
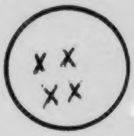
## ★Comparing Sets

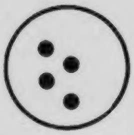
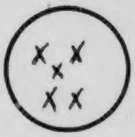
Use X's to show sets.

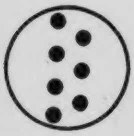
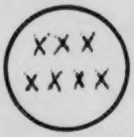
 has one more member than 

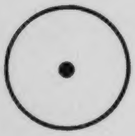
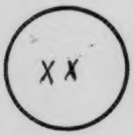
 has one fewer member than 

 has one more member than 

 has one fewer member than 

 has one fewer member than 

 has as many members as 

 has one fewer member than 

## I-3. Joining sets and adding numbers

Objective: To use the joining of sets to review addition facts through sums of 6 and the commutative property of addition.

Vocabulary: Union, addend; (Review) joining, adding, sum, equation, plus (+), equals (=), commutative.

Materials: Flannel board objects, 4 red books and 2 blue books, manipulative materials for children as before.

Suggested Procedure:

Place a set of 3 pears on the left side of the flannel board and a set of 2 oranges on the right side. Have each set described. Ask a child to join the oranges to the pears.

> Describe the new set. (A set of pears and oranges.) We have a special name for the new set--the set you get when you join two sets. We call it the union of the two sets.

Write union on the chalkboard.

> Is each member of the set of pears a member of the union? (Yes.)
>
> Is each member of the set of oranges a member of the union? (Yes.)

Return the set of oranges to its original position. Have someone join the pears to the oranges. Discuss the resulting set.

> Is the union we formed by joining the pears to the oranges the same set as the union formed when we joined the oranges to the pears? (Yes.)

- Direct 3 boys to stand at the left of the room, and 4 girls to stand at the right of the room. Have them described as a set of boys and a set of girls. Direct the boys to join the girls. Discuss the union of the two sets. (It is a set of boys and girls, a set of children, a set with 7 members.)

> Let's make a record of what has happened.
> How many girls did we have? (4.)

Write on the chalkboard.

> How many boys were in the set of boys that joined the set of girls? (3.)

Leave a space after the 4 and write 3.

> How many members are in the union of the two sets? (7.)

Leave a space after the 3 and write 7.

> We have 4, 3, and 7 as the numbers we need. How can we use these numbers to write an equation suggested by what has happened? ($4 + 3 = 7$.)

Some children may not remember the names of the plus and equals signs, although they may be able to write them. Take time to make sure that children understand the relationship between the terms and the operations. A set is joined to a set and the result is their union. A number is added to a number and the result is their sum. Leave the equation on the chalkboard: $4 + 3 = 7$. (It is not necessary to use the word equation yet.)

Have the boys return to their places.

> This time let's have the set of girls join the set of boys.

Continue, recording as you go, until you arrive at $3 + 4 = 7$.

Read the two equations aloud.

> If $4 + 3 = 7$ and $3 + 4 = 7$, could we say that $4 + 3 = 3 + 4$? (Yes.)

- Display 4 red books on one desk and 2 blue books on another. Have sets joined in two ways and write the equation suggested by each.

> If $4 + 2 = 6$ and $2 + 4 = 6$, can we say that $4 + 2 = 2 + 4$? (Yes.)

[*Note*. The above experiences have illustrated intuitively the idea of the commutative property of addition. While it is not necessary to use the term "commutative property" with the class unless you wish to do so, it is important that the children realize that the order in which the addends appear does not affect the sum.]

- Have children use their set materials to "act out" situations associated with equations you write on the chalkboard, supplying the sum for you.

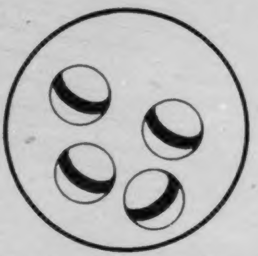
*Pupil's book, page 9*: Read the page with the children. They should think of joining the pictured sets in two ways and fill in the blanks.

*Pupil's book, page 10*: Children look at the two sets pictured and complete the equations beside them as a record of the two ways of joining the sets. In the last two rows they have to supply the number of members in the second set as well as the number of members in the union.

- On the chalkboard write:

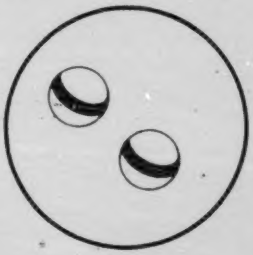
6 1 7

## Joining Sets and Adding Numbers



This set has

4 members.



This set has

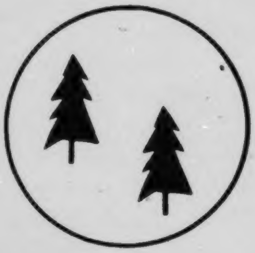
2 members

Think about the union of these sets.

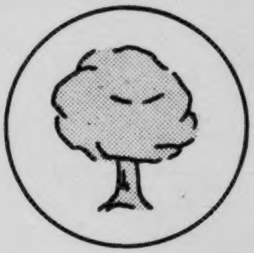
It would have 6 members.

4 + 2 = 6

2 + 4 = 6



2 members



1 member

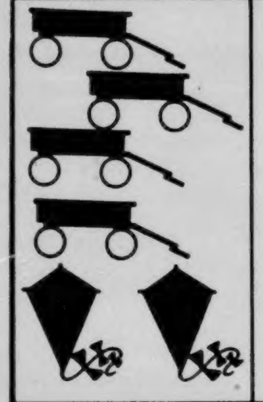
Think about the union of these sets.

It would have 3 members.

2 + 1 = 3

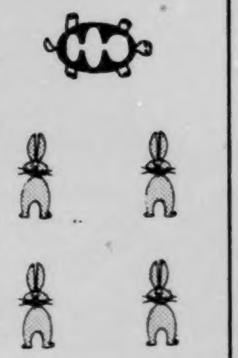
1 + 2 = 3

## Joining Sets and Adding Numbers



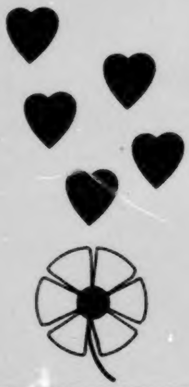
2 + 4 = 6

4 + 2 = 6



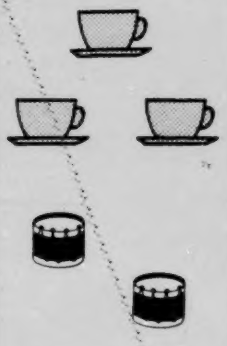
1 + 4 = 5

4 + 1 = 5



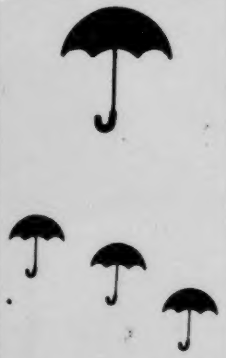
5 + 1 = 6

1 + 5 = 6



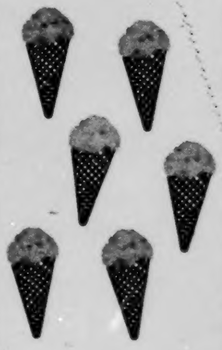
3 + 2 = 5

2 + 3 = 5



1 + 3 = 4

3 + 1 = 4



6 + 0 = 6

0 + 6 = 6

Have children use flannel board or their set materials to arrange a set of 6 and a set of 1.

> What is the number of members in the union of these two sets? (7)
>
> Do you think that joining a set of 1 to to a set of 6 will always mean that the union has 7 members or is it possible that sometimes you will find 6 members in the union, or maybe 8)? (There will always be 7 members in the union.)
>
> Jim, come finish the record on the chalkboard. Put in what is needed.

After Jim inserts the plus and equals symbols, ask what the equals sign means. Bring out the fact that one can say 6 + 1 is another name for 7 or that 6 + 1 is the same as 7.

> When we use the equals sign in a sentence like this (point to 6 + 1 = 7) we call the sentence an equation.

Write equals and equation on the chalkboard and underline equa in each.

> Since we are thinking of adding two numbers, we call each of the numbers we add an addend.

Write add and addend on the chalkboard.

Review the word "sum", and write several equations on the chalkboard asking children to point out the first addend, the second addend and the sum. Write some equations, leaving out an addend or the sum and ask which part of the equation is missing. Occasionally leave out the plus sign and ask whether children can tell that the numbers must be added. (Don't do this if zero is one of the addends.)

- Zero as an addend

Write:

6 + 0 = ___

Have children use objects to "act out" a situation associated with the equation.

> What is the number of the set you join to 6? Does it change the set at all? Is the union the set you started with? (Yes.)

Provide other illustrations in which the empty set is joined to a set having at least one member. Record equations such as $4 + 0 = 4$, $9 + 0 = 9$, etc. Lead to the generalization that the sum is the same as the first addend.

Write:

$$0 + 6 = \_\_\_$$

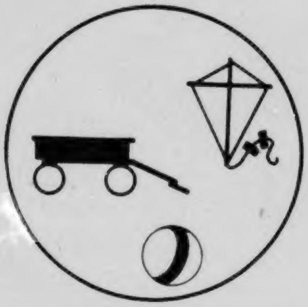
Ask children to "act out" a situation associated with the equation. Using other equations such as $0 + 5 = 5$, $0 + 8 = 8$, etc. Lead to the generalization that the sum is the same as the second addend.

| | |
|---|---|
| Pupil's book, page 11: | Children are to imagine joining the pictured sets in two ways and write equations to record both ways. |
| Pupil's book, page 12: | This is a practice page for sums through 6. |

## Joining Sets and Adding Numbers

Write equations for each of these.

Join



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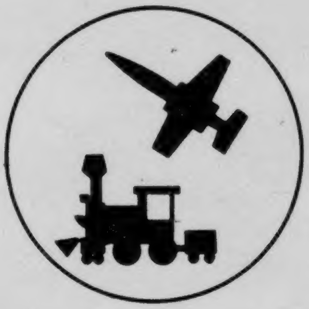
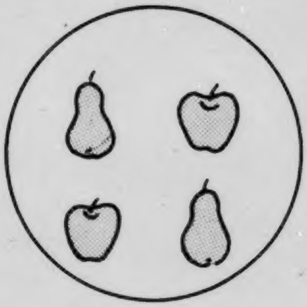


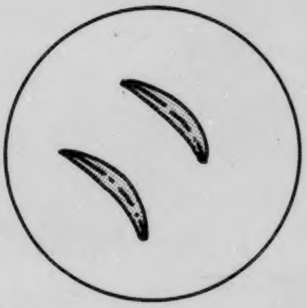
Equations: 3 + 2 = 5

2 + 3 = 5

Join



and



Equations: 4 + 2 = 6

2 + 4 = 6

## Adding Numbers

Fill the blanks.

5 + 1 = 6

1 + 4 = 5

6 + 0 = 6

3 + 2 = 5

1 + 2 = 3

4 + 1 = 5

0 + 2 = 2

5 + 0 = 5

1 + 5 = 6

2 + 3 = 5

1 + 1 = 2

2 + 2 = 4

3 + 3 = 6

4 + 0 = 4

0 + 3 = 3

2 + 4 = 6

3 + 1 = 4

2 + 1 = 3

0 + 6 = 6

3 + 0 = 3

## I-4. Removing sets and subtracting numbers

**Objective:** To review the idea of removal of a subset from a set, and to relate this to the operation of subtracting one number from another.

**Vocabulary:** (Review) remove, subtract, minus, difference.

**Materials:** A set of red blocks and green blocks, a flannel board and felt figures such as rabbits and birds, small objects (disks, sticks, etc.) to be used by children.

**Suggested Procedure:**

Place a set of red blocks and green blocks on the table, desk, or other convenient display place.

> We will start with this set. We can call it our starting set.
> Who can describe it?

Have children identify subsets of this set. (Subset of red blocks and subset of green blocks.)

Ask a child to remove the subset of red blocks.

Direct another child to show and describe the set that remains.

Ask a child to describe what the first child did with the set. (He took it away; he removed it, etc.) Direct the discussion so that the children see that the subset of red blocks was removed from the set of red blocks and green blocks. (Note that it was not removed from the set of green blocks, as children very often visualize it. One way to help children see this is for the teacher to start with a set of green blocks and then direct a child to try to remove a subset of red blocks from it.)

- Using a set of rabbits and birds on the flannel board ask a child to remove the subset of rabbits and describe the set that remains.

Then ask another child what took place. Be sure he responds that the first child removed the subset of rabbits from the set of rabbits and birds.

Return the rabbits to their original places on the flannel board. Direct a child to remove the subset of birds and describe the set remaining. Again ask another child to describe this action.

> (He removed the subset of birds from the set of rabbits and birds.)

- Continue with similar activities until you are sure the children understand that the subset is removed from the starting set, not from the remaining set.

> Now let's record what we are doing, using numbers and writing an equation.

Make a chart on the board as shown below.

> For instance, what was the number of blocks in the starting set?
>
> How many blocks were in the set removed? How many blocks were in the remaining set?

Suppose the numbers were 5, 3, and 2. Write them on the chart and help children to recall that the equation recording the removal of a set of 3 from a set of 5 is 5 - 3 = 2.

| Number of starting set | Number of set removed | Number of set remaining | Equation |
|---|---|---|---|
| 5 | 3 | 2 | 5 - 3 = 2 |
| | | | |

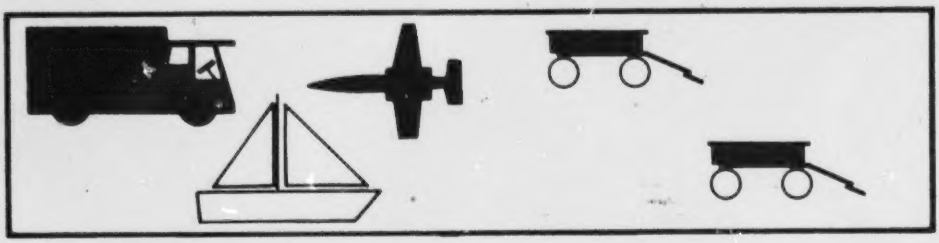
Review the term subtraction and use of the minus symbol, and note that 2 is the difference of 5 and 3. Record on the chart for each of the previous experiences.

- You may also wish to let children "act out" equations with their materials on their desks either from equations written on the chalkboard or from oral instructions.

Pupil's book, pages 13, 14, 15: Read instructions with children. Pages 14 and 15 may be completed independently.

Pupil's book, page 16: Have children use manipulative materials to complete page, selecting the number of objects for the starting set, removing a set as directed, identifying the number of objects in the remaining set and then writing the equation. Notice that in some sections of the chart they will have to find the number of the starting set or of the set removed.

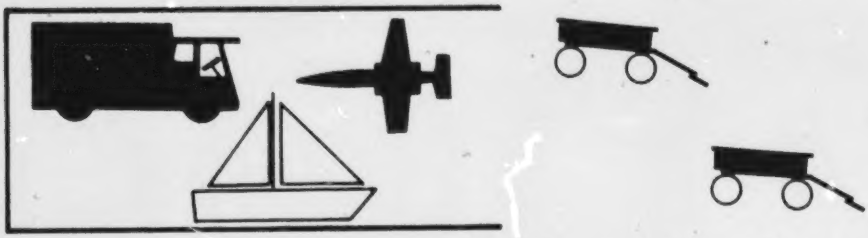
## Removing Sets and Subtracting Numbers



Here is a picture of a set of toys.

It has 5 members.

Think of removing a set of 2 wagons.



These pictures will help us think about the remaining set.

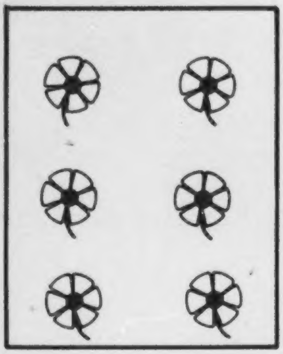
The remaining set has 3 members.

We may write this equation: 5 - 2 = 3.

## Removing Sets and Subtracting Numbers

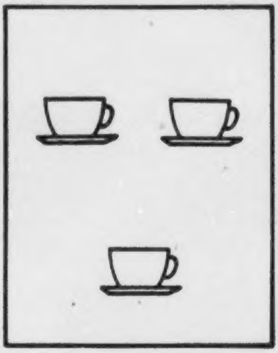
**Draw a ring around the set you think of removing.**

**Write an equation for each picture.**



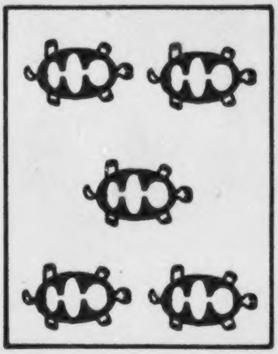
**Remove 2 ✿ 's.**

6 - 2 = 4



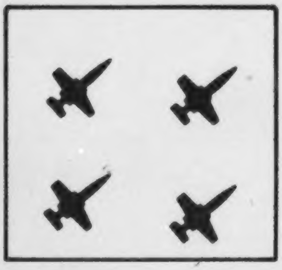
**Remove 3 ☕ 's.**

3 - 3 = 0



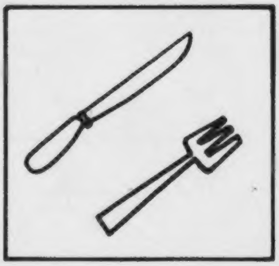
**Remove 2 🐢 's.**

5 - 2 = 3



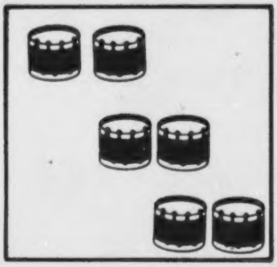
**Remove 0 ✈ 's.**

4 - 0 = 4



**Remove 1 🍴.**

2 - 1 = 1



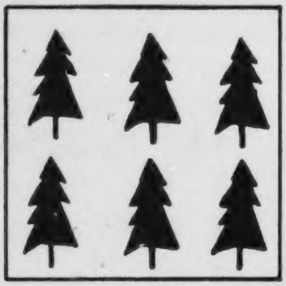
**Remove 5 🥁 's.**

6 - 5 = 1

## Removing Sets and Subtracting Numbers

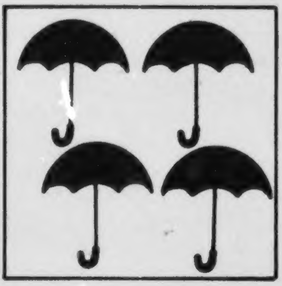
Draw a ring around the set you think of removing.

Write an equation for each picture.



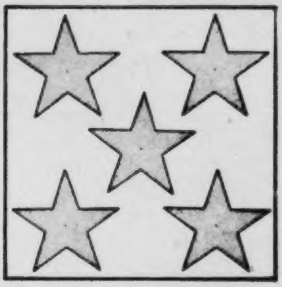
Remove 3 🌲's.

6 - 3 = 3



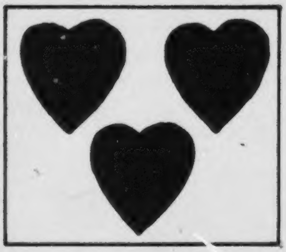
Remove 2 ☂'s.

4 - 2 = 2



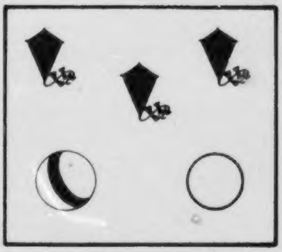
Remove 1 ☆.

5 - 1 = 4



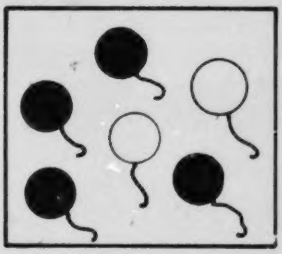
Remove 2 ♥'s.

3 - 2 = 1



Remove 3 ◆'s.

5 - 3 = 2



Remove 4 ●'s.

6 - 4 = 2

## Removing Sets and Subtracting Numbers

Complete the chart.

| Number of starting set | Number of set removed | Number of remaining set | Equation |
|---|---|---|---|
| 6 | 4 | 2 | 6 - 4 = 2 |
| 5 | 1 | 4 | 5-1 =4 |
| 3 | 3 | 0 | 3-3 =0 |
| 3 | 0 | 3 | 3 -0 =3 |
| 4 | 1 | 3 | 4-1 = 3 |
| 5 | 3 | 2 | 5 - 3 = 2 |
| 5 | 4 | 1 | 5-4 =1 |
| 2 | 2 | 0 | 2-2=0 |

Fill in the blanks:

6 - 6 = 0

4 + 2 = 6

3 - 2 = 1

5 - 1 = 4

1 + 5 = 6

2 + 0 = 2

2 + 4 = 6

6 + 0 = 6

2 - 0 = 2

## I-5. Doing and undoing

Objective: To review the idea that adding a number is "undone" by subtracting that number, and vice versa.

Vocabulary: (Review) doing and undoing.

Materials: Materials for flannel board, manipulative materials.

Suggested Procedure:

Adding and undoing

To introduce doing and undoing, you might tell the class the following story, recording on the chalkboard as indicated:

> Jimmy went to a store with his mother. He asked if he might be allowed to buy some candy and she said, "Yes."
>
> He picked up two candy bars and asked
> "Is this all right?"
> His mother said, "Yes."

Write 2 on the chalkboard to record his first purchase.

> As he was walking along, he saw some other candy bars he liked so he picked up three more.

Now write 3 on the chalkboard to the right of 2.

> He put the 3 candy bars with the 2 candy bars. Then, how many did he have? (2 + 3, or 5.)

Write 5 to the right of 3.

> How can we use these 3 numbers to write an equation about the number of objects in the union of the two sets? (Hopefully, children will suggest using "+" and "=" and you can complete the equation. (2 + 3 = 5.)
>
> When he showed the five candy bars to his mother, she said, "No, Jimmy, not five bars! You may have only two."

What did Jimmy have to do? (Put three candy bars back.) How many candy bars did Jimmy show to his mother? (5.)

Write 5 on the chalkboard.

How many was he allowed to have? (2.)

Write 2 to the right of 5 leaving space for three symbols.

What do you think we should write to show that he put three candy bars back? If we start with 5 and we want to end with 2, what equation can we write to name the number of candy bars that Jimmy could have? (5 - 3 = 2.)

Complete the equation, writing "- 3 =" between 5 and 2.

What sign did we use here (point) and here (point)? (Minus and equals.)

- Put a set of 3 rabbits on the left side of the flannel board and a rabbit on the right. Have a child join the set of 1 to the set of 3. Write the equation. (4 + 1 = 5.)

Suppose we don't want a set of 4 rabbits. We want a set of 3 rabbits.

We have to undo what we have done. We want only 3 rabbits. What can we do? (Remove 1 rabbit.) What set are we going to remove one from? (The set of 4.)

Write 4 on the chalkboard

How many will we remove? (1.)
What can we write on the chalkboard to show that we have removed a set of 1 from a set of 4? (4 - 1.)

Write - 1 to the right of 4.

How many will be in the remaining set? (3.)

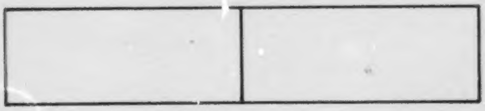
What equation can we write using 4 - 1 and 3? (4 - 1 = 3.)

Write this equation on the chalkboard.

- Give several examples of joining and removing. Use individual set materials if needed, and record on the chalkboard. Encourage children to generalize: If you start with a set, join another set to it, then remove the second set from the union, the remaining set has the same number of members as the starting set. If you start with a number, add a number to it, then subtract the second number from the sum, you get back the starting number.

Pupil's book, page 17: At least the first few problems should be done as a class activity. Tell children that the second equation in each box must show how to undo the operation shown in the first. They should notice that the first number in the second equation is the sum in the first.

- Give several other examples in the following form. First make a chart such as this on the chalkboard:



Put two sets of objects on the flannel board--3 members in one set and 5 members in the other set. Have each child display two sets at his desk--3 things in one set, 5 things in the other.

**Doing and Undoing**

**Fill the blanks.**

| | | |
|---|---|---|
| 4 + 2 = 6<br>6 - 2 = 4 | 3 + 1 = 4<br>4 - 1 = 3 | 6 + 0 = 6<br>6 - 0 = 6 |
| 1 + 5 = 6<br>6 - 5 = 1 | 2 + 2 = 4<br>4 - 2 = 2 | 1 + 3 = 4<br>4 - 3 = 1 |
| 3 + 2 = 5<br>5 - 2 = 3 | 4 + 1 = 5<br>5 - 1 = 4 | 3 + 3 = 6<br>6 - 3 = 3 |
| 0 + 5 = 5<br>5 - 5 = 0 | 1 + 2 = 3<br>3 - 2 = 1 | 2 + 4 = 6<br>6 - 4 = 2 |
| 5 + 1 = 6<br>6 - 1 = 5 | 2 + 1 = 3<br>3 - 1 = 2 | 4 + 0 = 4<br>4 - 0 = 4 |

On the flannel board join the set of 5 members to the set of 3 members. Have children make a similar joining with their sets. Ask children to give the equation for this action and record the equation in the chart on the chalkboard.

| | |
|---|---|
| 3 + 5 = 8 | |

Ask children how they could undo the joining they have done (by removing the set of 5 from the set of 8). Ask the children to give the equation for this "undoing" and record the equation in the chart on the chalkboard.

| | |
|---|---|
| 3 + 5 = 8<br>8 - 5 = 3 | |

Now consider the commuted situation in which the set of 3 is joined with the set of 5, and then the set of 3 is removed. Perform the manipulations and record the equations on the chalkboard in the chart.

| | |
|---|---|
| 3 + 5 = 8<br>8 - 5 = 3 | 5 + 3 = 8<br>8 - 3 = 5 |

Use several similar examples, before using page 18 of the pupil's book.

Pupil's book, page 18:

Here the chart is to be completed with a set of related equations. Some children may have to draw pictures or use manipulative materials.

**Adding and Undoing**

Finish each set of equations.

| | |
|---|---|
| 6 + 3 = 9 | 3 + 6 = 9 |
| 9 - 3 = 6 | 9 - 6 = 3 |

| | |
|---|---|
| 2 + 8 = 10 | 8 + 2 = 10 |
| 10 - 8 = 2 | 10 - 2 = 8 |

| | |
|---|---|
| 1 + 7 = 8 | 7 + 1 = 8 |
| 8 - 7 = 1 | 8 - 1 = 7 |

| | |
|---|---|
| 4 + 3 = 7 | 3 + 4 = 7 |
| 7 - 3 = 4 | 7 - 4 = 3 |

| | |
|---|---|
| 7 + 2 = 9 | 2 + 7 = 9 |
| 9 - 2 = 7 | 9 - 7 = 2 |

| | |
|---|---|
| 5 + 5 = 10 | 5 + 5 = 10 |
| 10 - 5 = 5 | 10 - 5 = 5 |

| | |
|---|---|
| 8 + 0 = 8 | 0 + 8 = 8 |
| 8 - 0 = 8 | 8 - 8 = 0 |

| | |
|---|---|
| 4 + 1 = 5 | 1 + 4 = 5 |
| 5 - 1 = 4 | 5 - 4 = 1 |

- Subtracting and undoing

To illustrate first removing and then joining as another kind of doing-undoing, tell a story like the following, making a record on the chalkboard as before.

> Sue took six cookies to eat while she was reading. As she walked out of the kitchen, she thought six might be more than she would want, so she put two of the cookies on a plate. How many cookies did she have left? (4) Then, as she walked down the hall, she decided that she was pretty hungry after all--so hungry that she really did want six cookies. What could she do?

Use flannel board and individual set materials to show removing and then joining to do and undo. As you record what is done, or give directions for "acting out" the situation, use opportunities to review vocabulary needed.

Give several examples of removing and joining. Use manipulative materials as needed, and record the equations on the chalkboard. Encourage children to generalize about removing and joining, as they did about joining and removing.

Pupil's book, page 19: At least the first few problems should be done as a class activity. Tell children that the second equation in each box must show how to undo the operation shown in the first. They should notice that the first number in the second equation is the difference in the first.

## Doing and Undoing

Fill the blanks.

| | | |
|---|---|---|
| 5 - 1 = 4<br>4 + 1 = 5 | 6 - 0 = 6<br>6 + 0 = 6 | 3 - 2 = 1<br>1 + 2 = 3 |
| 6 - 5 = 1<br>1 + 5 = 6 | 4 - 2 = 2<br>2 + 2 = 4 | 4 - 3 = 1<br>1 + 3 = 4 |
| 5 - 2 = 3<br>3 + 2 = 5 | 6 - 3 = 3<br>3 + 3 = 6 | 5 - 0 = 5<br>5 + 0 = 5 |
| 6 - 5 = 1<br>1 + 5 = 6 | 3 - 1 = 2<br>2 + 1 = 3 | 4 - 4 = 0<br>0 + 4 = 4 |
| 5 - 3 = 2<br>2 + 3 = 5 | 6 - 4 = 2<br>2 + 4 = 6 | 6 - 2 = 4<br>4 + 2 = 6 |

- Again put a chart like this on the chalkboard:

| | |
|---|---|
| | |

Display a set of 9 objects on the flannel board--2 objects of one kind, 7 of another kind. Have each child display a similar set on his desk: 9 objects, 2 of one kind and 7 of another.

Remove the set of 7 objects from the set of 9 objects. Have the children do the same with their sets. Ask for the equation associated with this action and record the equation in the chart on the chalkboard.

| | |
|---|---|
| 9 - 7 = 2 | |

Ask children how they could undo the removing they have done (by joining the set of 7 to the set of 2). Request the equation for this action and record the equation in the chart.

| | |
|---|---|
| 9 - 7 = 2<br>2 + 7 = 9 | |

Now consider the situation in which the set of 2 is removed from the set of 9, and then joined to the remaining set of 7. Perform the manipulation and record the equations in the chart.

| | |
|---|---|
| 9 - 7 = 2<br>2 + 7 = 9 | 9 - 2 = 7<br>2 + 7 = 9 |

Use several other similar examples for illustrative purposes, then proceed with page 20 of the pupil's book.

Pupil's book, page 20:

Here the chart is to be completed with a set of related equations. Some children may have to draw pictures or use manipulative materials.

★ Pupil's book, pages 21 and 22:

(Optional) These pages may be used by more able pupils.

## Subtracting and Undoing

Finish each set of equations.

| | |
|---|---|
| 9 - 5 = 4 | 9 - 4 = 5 |
| 4 + 5 = 9 | 5 + 4 = 9 |

| | |
|---|---|
| 10 - 7 = 3 | 10 - 3 = 7 |
| 3 + 7 = 10 | 7 + 3 = 10 |

| | |
|---|---|
| 7 - 2 = 5 | 7 - 5 = 2 |
| 5 + 2 = 7 | 2 + 5 = 7 |

| | |
|---|---|
| 10 - 4 = 6 | 10 - 6 = 4 |
| 6 + 4 = 10 | 4 + 6 = 10 |

| | |
|---|---|
| 9 - 8 = 1 | 9 - 1 = 8 |
| 1 + 8 = 9 | 8 + 1 = 9 |

| | |
|---|---|
| 8 - 4 = 4 | 8 - 4 = 4 |
| 4 + 4 = 8 | 4 + 4 = 8 |

| | |
|---|---|
| 6 - 0 = 6 | 6 - 6 = 0 |
| 6 + 0 = 6 | 0 + 6 = 6 |

| | |
|---|---|
| 7 - 4 = 3 | 7 - 3 = 4 |
| 3 + 4 = 7 | 4 + 3 = 7 |

★Write 4 equations using 3 numbers.

3 1 4

3 + 1 = 4

1 + 3 = 4

4 - 3 = 1

4 - 1 = 3

5 1 6

5 + 1 = 6

1 + 5 = 6

6 - 1 = 5

6 - 5 = 1

5 2 3

5 - 2 = 3

5 - 3 = 2

2 + 3 = 5

3 + 2 = 5

4 0 4

4 + 0 = 4

0 + 4 = 4

4 - 0 = 4

4 - 4 = 0

★ Write 4 equations using 3 numbers.

| 6 6 0 | 2 3 1 |
|---|---|
| 6-6 =0 | 2+1=3 |
| 6-0 =6 | 1+2 =3 |
| 0+6=6 | 3-1 =2 |
| 6+0=6 | 3-2=1 |

Write as many equations as you can.

| 3 3 6 | 2 4 2 |
|---|---|
| 3+3=6 | 2+2=4 |
| 6-3=3 | 4-2 =2 |
|  |  |
|  |  |

## I-6. Partitioning sets into equivalent subsets

Objectives: To learn how to partition sets into equivalent subsets and how to count by 2's, 5's and 10's.

Vocabulary: Remainder set; (Review) partition, equivalent.

Materials: Twenty counting blocks, materials for flannel board, yarn, 100 small, like objects (e.g., 100 pennies or 100 disks), materials for children, at least 25 objects per child, hundreds-square paper ("hundreds-square" paper is available commercially or it can be made by ruling a duplicator master into a grid of 10 rows and 10 columns), chart, made as shown below, on chalkboard.

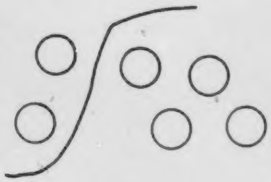
| Number of starting set | Number of each subset | Number of equivalent subsets | Number of remainder set |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

Suggested Procedure:

Review the word partition. You may wish to remind children of the usual use of the word: putting a partition in a room to separate the space into two or more spaces.

We can partition a set into two subsets.

Illustrate by putting a set of objects on the flannel board and using yarn to show a partition:



We also may be able to partition a set into equivalent subsets.

Place 20 counting blocks on a table or desk or objects on flannel boards where all can see. Pull three blocks to one side.

How many members are in this subset? (3.)

Make another subset equivalent to this one, Jim.

Can you make another?

Continue. When the sixth set of 3 has been made, point to the two blocks left over.

Is this set smaller than each of the others? Since it is, we will give it a special name. We call it the remainder set.

Let's make a record of what we are doing.

Use the chart on the chalkboard and have children tell what should be entered in each section.

Using the same 20 blocks, pull a set of 4 blocks to one side, and continue as before.

Do we have any blocks left over this time? (No.) If we want to show that there are no members in the remainder set, what will we write here? (Zero.)

Have the children use their set materials and complete pupil's book, page 23 under teacher direction as long as guidance is needed.

Pupil's book, page 23: Direct children to count out 17 objects and put the rest of their materials out of the way.

What is the number of your starting set? (17.)

How many members are you to have in each subset? (3.)

Count out subsets of 3 members each, and see how many subsets you can form. (5.)

How many are in the remainder set? (2)

- Counting by 2's, 5's and 10's

Tell children to partition their materials into subsets of two members each.

If you have many sets of two members each, can you count by twos to see how many were in the starting set? You start with two. Do you need to say "three", or can you just think three and say "four"?

What will you say next? (Six)

You skip, or think, one counting number and say the next.

What will you say if you have a remainder set of one member? (The next counting number.)

Sometimes, if you know how to count by twos, it's faster to count a lot of things that way than it is to count them by ones.

- Display the set of 100 pennies or disks and show how to pull to the side two at a time. Give practice in counting by twos as needed. Children may line up by twos and one child may count them.

## Partitioning Sets

| Number of starting set | Number of each subset | Number of equivalent subsets | Number of remainder set |
|---|---|---|---|
| 17 | 3 | 5 | 2 |
| 10 | 4 | 2 | 2 |
| 6 | 2 | 3 | 0 |
| 13 | 4 | 3 | 1 |
| 21 | 3 | 7 | 0 |
| 15 | 5 | 3 | 0 |
| 18 | 2 | 9 | 0 |
| 25 | 5 | 5 | 0 |
| 21 | 10 | 2 | 1 |

- Distribute hundreds-square paper. Tell children to make an X in the first box, write 2 in the next, make an X in the next, etc. At another time have them count by twos starting with one: 1, 3, 5, etc.

Pupil's book, page 24: Children are to write numerals in the boxes.

- Display pennies as before. Have a child arrange pennies in sets of five and count them by fives.
  Other forms of material may be used as needed. Have a child make sets of ten by putting two sets of five together and counting by tens.

Remove one set of ten and put three single objects with the others. Have the objects counted by tens and after the last ten is counted, ask children to continue, "Ninety, ninety-one, ninety-two, ninety-three." Show how to start with three and count by tens from three. "Thirteen, twenty-three, thirty-three, etc." Give much practice in counting by tens, starting with numbers other than multiples of ten. Give practice in counting backward, also.

Pupil's book, pages 25 and 26: Children are to fill the boxes, counting by fives both forward and backward on page 25, and by tens, starting with various numbers, on page 26.

## Counting by Twos

| | | | | |
|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 |
| 12 | 14 | 16 | 18 | 20 |
| 22 | 24 | 26 | 28 | 30 |
| 32 | 34 | 36 | 38 | 40 |
| 42 | 44 | 46 | 48 | 50 |
| 52 | 54 | 56 | 58 | 60 |
| 62 | 64 | 66 | 68 | 70 |
| 72 | 74 | 76 | 78 | 80 |
| 82 | 84 | 86 | 88 | 90 |
| 92 | 94 | 96 | 98 | 100 |

Counting by Fives

| | | | | |
|---|---|---|---|---|
| 5 | 10 | 15 | 20 | 25 |
| 30 | 35 | 40 | 45 | 50 |
| 55 | 60 | 65 | 70 | 75 |
| 80 | 85 | 90 | 95 | 100 |

Count Back by Fives

| | | | | |
|---|---|---|---|---|
| 85 | 80 | 75 | 70 | 65 |
| 60 | 55 | 50 | 45 | 40 |
| 35 | 30 | 25 | 20 | 15 |

## Counting by Tens

| | | | | |
|---|---|---|---|---|
| 10 | 20 | 30 | 40 | 50 |
| 60 | 70 | 80 | 90 | 100 |

| | | | | |
|---|---|---|---|---|
| 5 | 15 | 25 | 35 | 45 |
| 55 | 65 | 75 | 85 | 95 |

| | | | | |
|---|---|---|---|---|
| 27 | 37 | 47 | 57 | 67 |

| | | | | |
|---|---|---|---|---|
| 49 | 59 | 69 | 79 | 89 |

## I-7. Place value

**Objective:** To review decimal place value notation.

**Vocabulary:** (No new words.)

**Materials:** Counting blocks (52), counting cards 2" by 4" (35--can be made from oaktag or file cards), abacus for teacher demonstration (optional).

**Suggested Procedure:**

> I would like to have this set of counting blocks partitioned into subsets with 10 members each. John, Mary and Bill may work together to partition this set.

It may be necessary to discuss how three children may work together so as to obtain as many sets of ten as possible.

Draw a chart on the board. Read the headings as you write them.

| Number of sets with ten members | Number of sets with one member | Number of members in starting set | |
|---|---|---|---|
| 5 | 2 | 50 + 2 | 52 |
| | | | |
| | | | |

Ask the children to observe the set of counting blocks. Determine the number of equivalent sets of ten members and write 5 in the appropriate column.

Complete the notation on the chart. Write 2 in the second column to indicate the number of equivalent sets of one member.

How could we find out how many members there are altogether in the subsets which have 10 members? (We could count by tens.)

Let's think before we count them that way. If we know we have five tens, does this tell us anything?

Elicit the response that five tens are fifty.

The chart tells us that we have five tens, which we say are fifty. It also shows we have two ones. One name for 50 and 2 more is 50 + 2. (Record on chart.) What is another name for 50 + 2? (52) (Record on chart.)

- Continue with a similar activity using the 35 counting cards. Arrange the cards in sets of tens and sets of ones. Draw another chart on the board and ask the children how many tens and how many ones are in the set. Record the numerals on the chart. Then, ask them to think of how many in all and what we write to show how many in all. Number names should be written in the two ways illustrated.

| Tens | Ones |
|---|---|
| 3 | 5 |

| Number Names | |
|---|---|
| 30 + 5 | 35 |

Continue to develop this chart.

Write 93 on the chalkboard. Ask the children what number it names. Ask the children to tell how many tens and how many ones. (There are 9 tens and 3 ones.)

What is another way to write the name for 93 to show that we have 9 tens and 3 ones? ( 90 + 3)

Write this in the appropriate part of the chart used earlier.

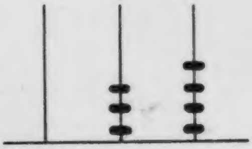
Tell the children that a number can be named in any of these ways. They are names for the same number.

> If I have a set which I tell you has 40 + 9 members, can you tell some other names?
> (4 tens and 9 ones, 49.)

Write these names on the chart.

Continue with this by presenting one of the names for a number which we have discussed here and have children suggest other names. Many of the experiences should be related to counting sets by tens and ones and then recording these three names. Given any one of the three, children should be able to tell the other two names.

- Use the abacus as another way of representing tens and ones. This may be done by using a commercial abacus or by representing one on a chalkboard or flannel board. The position of a bead determines whether it represents a one or a ten. The column on the right represents ones, the column to the left of it represents tens. For instance, this arrangement of beads represents 3 tens and 4 ones, or thirty and four.



If we write a numeral to tell what number is represented, it would be 34. It can also be written 30 + 4. Children should be familiar with all of these ways for writing the number represented on the abacus.

Pupil's book, page 27: For each box, count the number of sets of ten and the number of sets of one. Record the numeral for each in the "tens-ones" chart and then write the expanded and common forms of the numeral to indicate the number of members in the whole set.

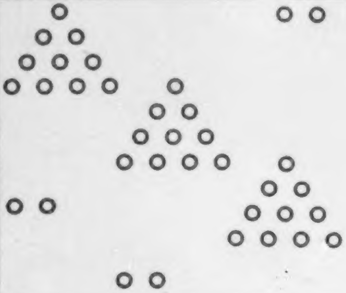
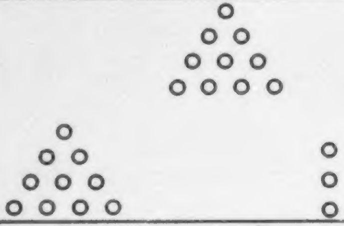
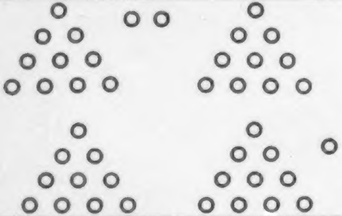
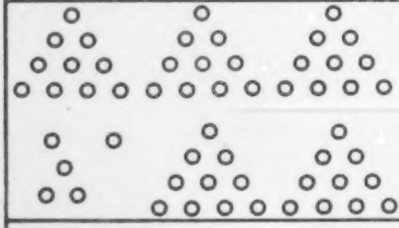
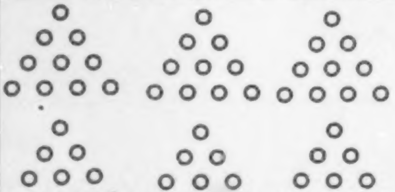
Pupil's book, page 28: Use the first numeral, 36, to illustrate that we may think of this as ____ tens ____ ones or as 30 + 6. Have pupils fill in the appropriate blanks. Then have the children proceed independently with the other examples.

Pupil's book, page 29: Use the first expression, 5 tens 2 ones to illustrate that this may be written as the numeral 50 + 2 or 52. Have the children proceed independently to rewrite each expression as a numeral in expanded and common form.

Pupil's book, page 30: The expanded form is given: 50 + 3. Children are to write the number of tens and ones and the common name for each.

Pupil's book, page 31: For each box, record the number of tens and ones represented on the abacus, then write the common numeral indicated by the number of tens and the number of ones.

**Renaming Numbers**
Fill the blanks.

| | tens | ones | number names |
|---|---|---|---|
|  | 3 | 6 | 30 + 6<br>36 |
|  | 2 | 3 | 20 + 3<br>23 |
|  | 4 | 3 | 40 + 3<br>43 |
|  | 5 | 5 | 50 + 5<br>55 |
|  | 6 | 0 | 60 + 0<br>60 |

**Renaming Numbers**

**Fill the blanks.**

36 30 + 6 3 tens 6 ones

47 40 + 7 4 tens 7 ones

73 70 + 3 7 tens 3 ones

61 60 + 1 6 tens 1 ones

6 0 + 6 0 tens 6 ones

25 20 + 5 2 tens 5 ones

99 90 + 9 9 tens 9 ones

12 10 + 2 1 tens 2 ones

---

84 80 + 4 4 ones 8 tens

22 20 + 2 2 tens 2 ones

30 30 + 0 0 ones 3 tens

75 70 + 5 7 tens 5 ones

## Renaming Numbers

Fill the blanks.

| | | | | | |
|---|---|---|---|---|---|
| 5 | tens | 2 | ones | 50 + 2 | 52 |
| 7 | tens | 0 | ones | 70 + 0 | 70 |
| 4 | tens | 9 | ones | 40 + 9 | 49 |
| 8 | tens | 5 | ones | 80 + 5 | 85 |
| 0 | tens | 4 | ones | 0 + 4 | 4 |
| 3 | tens | 8 | ones | 30 + 8 | 38 |
| 1 | ten | 1 | one | 10 + 1 | 11 |
| 6 | tens | 2 | ones | 60 + 2 | 62 |

| | | | | | |
|---|---|---|---|---|---|
| 2 | tens | 6 | ones | 20 + 6 | 26 |
| 9 | ones | 0 | tens | 0 + 9 | 9 |
| 8 | tens | 6 | ones | 80 + 6 | 86 |
| 4 | tens | 0 | ones | 40 + 0 | 40 |

## Renaming Numbers
Fill in the blanks.

50 + 3 _5_ tens _3_ ones _53_

70 + 1 _7_ tens _1_ ones _71_

30 + 5 _3_ tens _5_ ones _35_

20 + 8 _2_ tens _8_ ones _28_

10 + 7 _1_ tens _7_ ones _17_

60 + 5 _6_ tens _5_ ones _65_

30 + 2 _3_ tens _2_ ones _32_

90 + 6 _9_ tens _6_ ones _96_

---

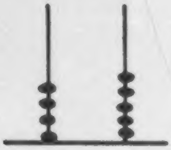
40 + 4 _4_ ones _4_ tens _44_

70 + 8 _7_ tens _8_ ones _78_

10 + 3 _3_ ones _1_ tens _13_

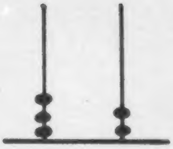
30 + 5 _3_ tens _5_ ones _35_

**Tens and Ones on the Abacus**
**Fill in the blanks.**



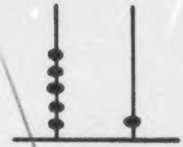
_4_ tens _5_ ones

45



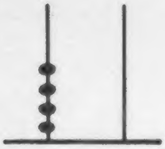
_3_ tens _2_ ones

32



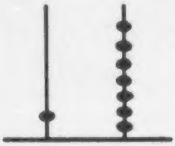
_5_ tens _1_ ones

51



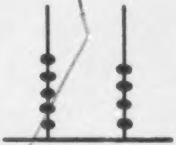
_4_ tens _0_ ones

40



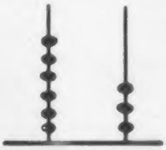
_1_ tens _7_ ones

17



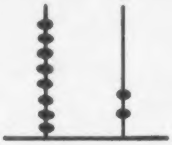
_5_ tens _4_ ones

54



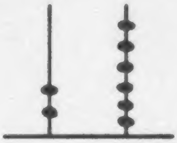
_6_ tens _3_ ones

63



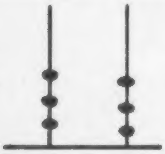
_8_ tens _2_ ones

82



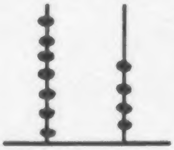
_2_ tens _6_ ones

26



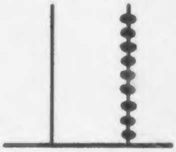
_3_ tens _3_ ones

33



_7_ tens _4_ ones

74



_0_ tens _9_ ones

9

## I-8. The number line

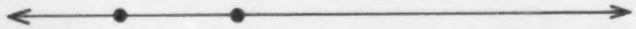
Objective: To review the idea of the number line.

Vocabulary: (Review) number line, point.

Materials: Straightedge.

Suggested Procedure:

Using a straightedge, draw a line on the chalkboard. Mark two points, using an eraser or a strip of paper to space them.



Review the fact that in this drawing of a line the arrows remind us that the line "keeps going" in both directions without end.

> Here we have pictured two points on this line. Does anyone remember one way you have used points on a line? (Children may think of a ruler, but someone will probably mention the number line.) If we say that this (indicate the segment between the marked points) is the distance between points to be marked on the number line, where can I put another point?

Have a child mark other points equally spaced along the line, using the eraser or strip of paper to measure off segments. Be sure a point is marked to the left of the original point.

> These points don't have numbers assigned to them yet. Where shall we start to number the points?

A child will probably indicate the point farthest to the left.

> What number shall we give this point? What shall I write for the next point?

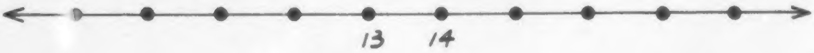
Children will probably assign the number 0 or 1 to the first point shown, but in any event, they are to be brought to the realization that the first marked point can be assigned any number, and that it is only after the second point is marked that choice is no longer possible.

When all points marked have been numbered, ask,

> Could we have more points numbered? (Yes, if we drew a longer picture of the line.)

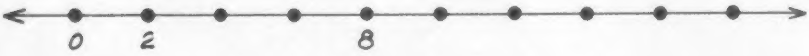
Erase the numerals, make a paper pattern of the unit segment used, fold it in half, and mark points halfway between the points previously marked.

> I'm going to number this point 13 and the next one 14. Who can write numerals for some other points shown?



> Does 1 have a place on the number line? (Yes, but we don't have a picture of it.)
>
> Does 99 have a place on the number line? (Yes,etc.)

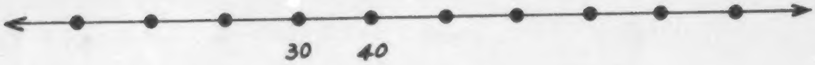
Erase the numerals, and number the points as shown below.

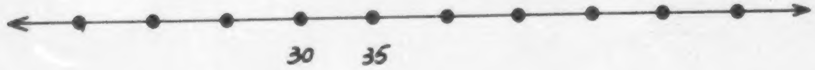


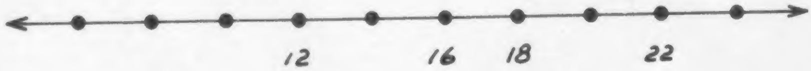
> What number goes with the point that follows the point numbered 2? (4.)

Discuss the fact that the labels now show counting by 2's rather than counting by 1's.

- Erase numerals and have points named using the following as guides.



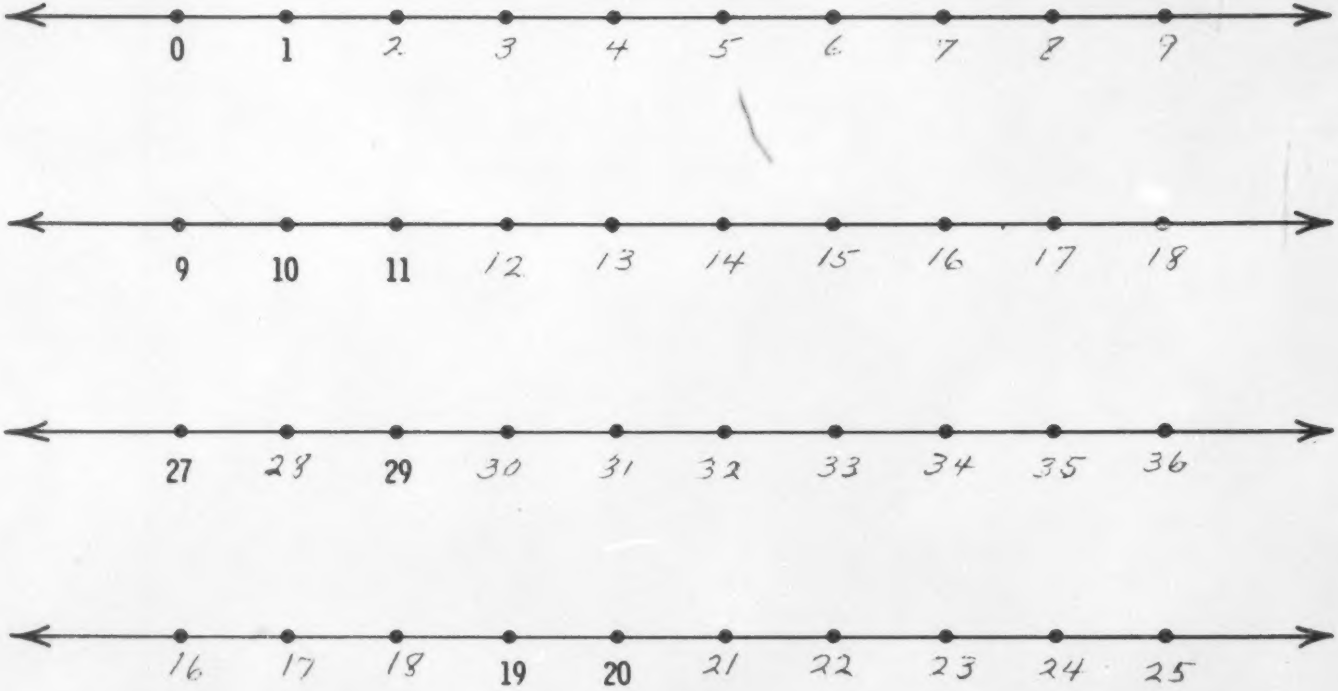




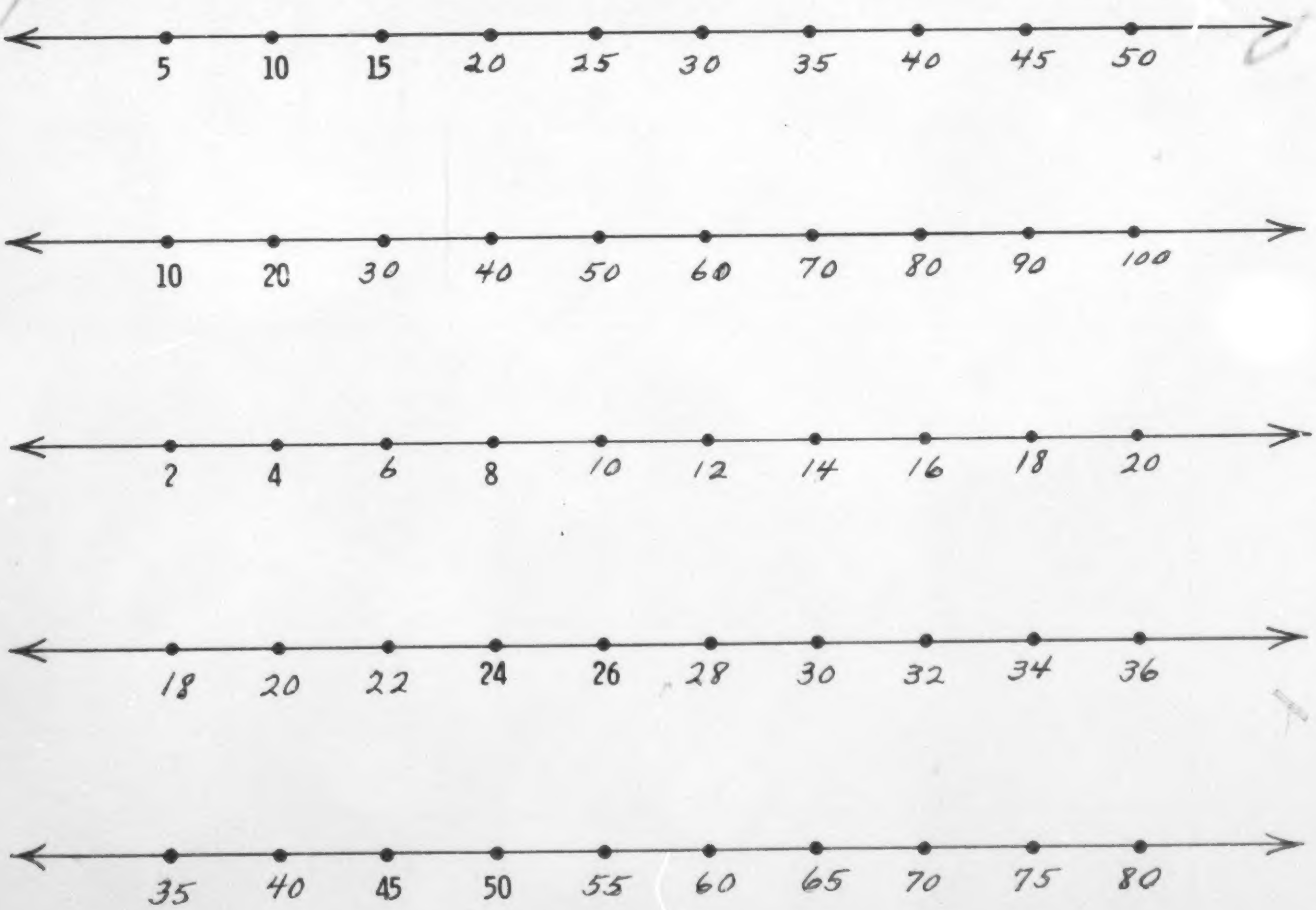
Pupil's book, pages 32 and 33: Children are to write numerals missing on the pictures of the number line. Extra help may be needed on page 33, where intervals other than 1 occur.

★ Pupil's book, pages 34 and 35: (Optional) More able pupils may use the chart on 34 to count by 3's, then fill in the missing numerals on 35.

## The Number Line


0 1 2 3 4 5 6 7 8 9
9 10 11 12 13 14 15 16 17 18
27 28 29 30 31 32 33 34 35 36
16 17 18 19 20 21 22 23 24 25

# The Number Line



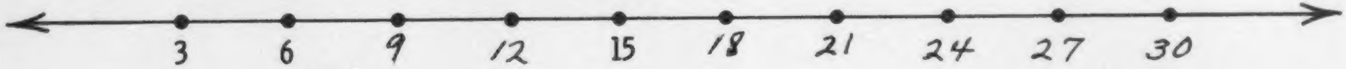
5 10 15 20 25 30 35 40 45 50
10 20 30 40 50 60 70 80 90 100
2 4 6 8 10 12 14 16 18 20
18 20 22 24 26 28 30 32 34 36
35 40 45 50 55 60 65 70 75 80

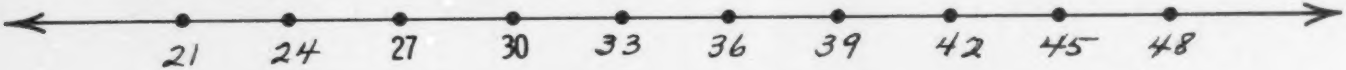
## ★ Counting

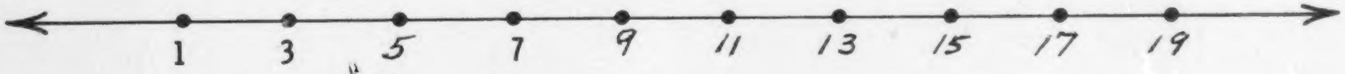
3's

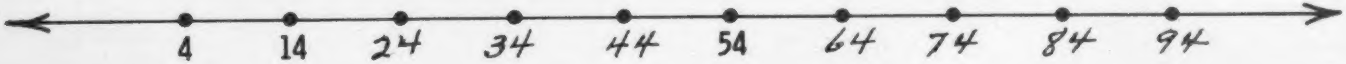
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 3 | | | 6 | | | 9 | |
| | 12 | | | 15 | | | 18 | | |
| 21 | | | 24 | | | 27 | | | 30 |
| | | 33 | | | 36 | | | 39 | |
| | 42 | | | 45 | | | 48 | | |
| 51 | | | 54 | | | 57 | | | 60 |
| | | 63 | | | 66 | | | 69 | |
| | 72 | | | 75 | | | 78 | | |
| 81 | | | 84 | | | 87 | | | 90 |
| | | 93 | | | 96 | | | 99 | |

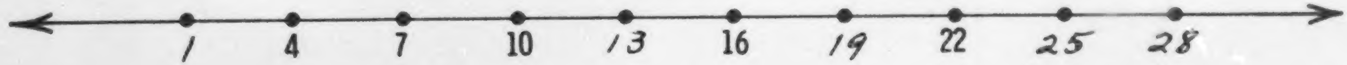
## ★The Number Line


3
6
9
12
15
18
21
24
27
30


21
24
27
30
33
36
39
42
45
48


1
3
5
7
9
11
13
15
17
19


4
14
24
34
44
54
64
74
84
94


1
4
7
10
13
16
19
22
25
28

## I-9. Comparing numbers

Objectives: To visualize the greater than and less than relations by using the number line.

To introduce the symbols $<$ (less than), and $>$ (greater than).

Vocabulary: Mathematical sentence, greater than, less than.

Materials: (None.)

Suggested Procedure:

Draw the number line on the board in preparation for the class.

> Let's look carefully at the number line. What did we mark as the place where the steps begin? (0)
>
> As we move one step from the left to the right on our number line, what do you notice about the numbers? (They become one greater.)
>
> As we move one step from the right to the left on our number line, what do you notice about the numbers? (They become one less.)

Using your hands, indicate the points labeled 6 and 4 on the number line. Say "6 is greater than 4". Ask if anyone can think of another way of comparing 6 and 4. (4 is less than 6.) Repeat with several other pairs of numbers, having the children express both the "greater than" and the "less than" statements.

Tell the children that we can write "4 is greater than 2" this way:

$$4 > 2,$$

writing this on the board. Ask a child to frame with his hands the symbol for "greater than".

Have the children observe that this symbol "points toward" the numeral for the smaller number. Next write

$$7 > 5$$

on the board. Ask a child to read it and frame the symbol for "is greater than". Again point out that the symbol "points toward" the numeral for the number that is less.

Write $>$ on the board.

> When you see this symbol do you say
> "is less than" or "is greater than"?
> (Is greater than.)

Write $<$ on the board.

> When you see this symbol do you say
> "is less than" or "is greater than"?
> (Is less than.)

On the board write several mathematical sentences using both $<$ and $>$. Ask children to read these to the class.

Write numerals for several pairs of numbers with a space between the numerals. Ask children from the class to write the correct symbol between the numerals. Then ask them to read the statement to the class. Have the children notice that the sentence $4 < 6$ is not the same as the sentence $6 > 4$.

Pupil's book, page 36: Draw a ring around each mathematical sentence that is correct. Do the first two boxes across the top of the page with the children so that they will understand that in some boxes both statements are correct while in other boxes only one statement is correct.

*Pupil's book, page, 37*: Children write the mathematical sentences indicated, using numerals and > and < symbols.

*Pupil's book, page 38*: Direct pupils to complete this page by writing > or < to make correct statements.

## Order of Numbers

$3 < 1$
[✓ $3 > 1$]

[✓ $5 > 2$]
[✓ $2 < 5$]

[✓ $4 < 5$]
[✓ $5 > 4$]

[✓ $7 < 9$]
$9 < 7$

$7 < 5$
[✓ $5 < 7$]

[✓ $4 > 2$]
[✓ $2 < 4$]

$0 > 6$
[✓ $6 > 0$]

$2 > 3$
[✓ $3 > 2$]

## Order of Numbers

| | |
|---|---|
| 5 is greater than 1. | $5 > 1$ |
| 1 is less than 5. | $1 < 5$ |
| 4 is less than 6. | $4 < 6$ |
| 1 is greater than 0. | $1 > 0$ |
| 10 is greater than 8. | $10 > 8$ |
| 2 is less than 5. | $2 < 5$ |
| 9 is less than 10. | $9 < 10$ |
| 3 is greater than 1. | $3 > 1$ |
| 6 is greater than 2. | $6 > 2$ |
| 7 is less than 9. | $7 < 9$ |
| 5 is less than 7. | $5 < 7$ |
| 8 is less than 10. | $8 < 10$ |
| 2 is greater than 1. | $2 > 1$ |

## Order of Numbers

Write > or <

| | | |
|---|---|---|
| $3 > 2$ | $70 < 78$ | $34 > 32$ |
| $8 < 9$ | $52 < 57$ | $60 < 80$ |
| $6 > 3$ | $36 > 31$ | $21 < 29$ |
| $1 < 5$ | $62 < 68$ | $12 < 20$ |
| $5 > 2$ | $24 > 20$ | $69 < 71$ |
| $9 > 4$ | $10 < 30$ | $81 < 86$ |
| $3 < 8$ | $47 > 44$ | $58 > 53$ |
| $7 > 1$ | $48 < 53$ | $61 > 39$ |

## I-10. Even and odd numbers

**Objective:** To introduce the idea of odd and even numbers.

**Vocabulary:** Even, odd.

**Materials:** A flannel board, materials for flannel board (apples and pears), small objects for each child, cards with numerals, (1 through the number of children in the class, and one numeral on each card), number line on the chalkboard, large newsprint chart as shown:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 |
| 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 |
| 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 |
| 80 | 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 |
| 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 |

**Suggested Procedure:**

Remind children of times when they may have tried to choose up sides to make two teams so that each team would have the same number of members.

> Sometimes you came out even, and sometimes you had one person left over. Could you tell *before* you chose that you would or wouldn't come out even?

The answers to the last question should reveal the extent to which children already understand the difference between odd and even numbers, and the use of procedures suggested below will depend on the needs of the class.

You may wish to designate eight children to stand at the front of the room and then assign them to two teams, one on the left and one on the right of the room. Children should notice that with 8 you "come out even". Do the same with a group of 7 and observe that there is one left over.

Distribute numeral cards. Direct each child to count out from his set materials the number of objects shown on his card and put the rest away.

> Let's see what happens when we try to separate these objects into two subsets with the same number of objects in each subset.
>
> When will we come out even and when will we have one left over? Let's make a chart to remind us of what we find out.

Make the following tabulation on the chalkboard:

| Even | Not even | |
|---|---|---|
| 8 | 13 | |
| 22 | 11 | etc. |

Record results of pupils' attempts to make two equivalent subsets with their materials.

When all have finished, explain that numbers that come out even are called even numbers, but if we have one left over we sometimes call it the "odd one" so we call all numbers that are not even odd numbers. Erase "Not even" and write "Odd".

- Use the hundreds chart and ring with red the even numbers recorded on the board, checking them off the chalkboard listing as you do so. Have the even numbers read in order from the chart. Point to the last one ringed and ask what the next even number would be.

> Is there any even number before 2 on the chart?

(Children may be hesitant about suggesting 0.)
You can't imagine separating a set with no members into two sets, but at least you know there wouldn't be one left over. If there isn't an odd one, the number must be even.

List the even numbers on the chalkboard in this way:

0 10 20 30
2 12 22 32
4 14 24 34
6 16 26 .
8 18 28 .

If no child notices the pattern, 0, 2, 4, 6, 8 call attention to it. Ring the odd numbers on the hundreds chart with blue. Have them read aloud, in order, and list:

1 11 21 31
3 13 23 33
5 15 25 35
7 17 27 .
9 19 29 .

What would the next odd number be?

- Direct attention to the number line and ask children to imagine a cricket or other small creature who starts on 0 and skips over one point each time he jumps. He would land first on 2, then on 4, etc.

What points can he touch? (The points with even numbers.) Could he land on 14? Why? (Because 14 is an even number.) 52? etc.

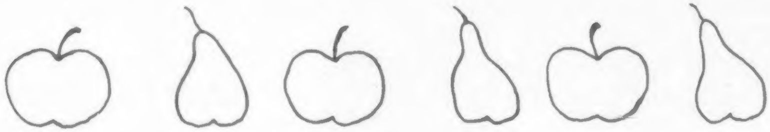
Could he land on 35? 67?
Suppose he started on 25? Would he land on 19? On 48? Why?

Have a child count by twos starting with 2. Stop him at 16 and say:

° What even number comes before 16? What did you say just before you said "sixteen"?

Have a child count by twos starting with 1 and stopping at 15. Ask for the odd number just before 15.

If children have trouble with the idea of the odd number just before or after a given odd number, put such objects as felt apples and pears on the flannel board:



Indicate one of the apples and have a child point to the apple before it (i.e., just to the left of it); indicate a pear and have a child point to the pear that comes after it, etc.

Pupil's book, pages 39 and 40: Children first complete the even number chart and may refer to it when doing the "before and after" exercises with even numbers.

Pupil's book, pages 41 and 42: Children complete the odd number chart and then the "before and after" exercises with odd numbers.

★ Pupil's book, pages 43 and 44: (Optional) More able pupils may use these pages to discover that the sum of two even numbers is an even number; the sum of two odd numbers is an even number; the sum of an even number and an odd number is an odd number; when 2 is added to an even number the sum is the next even number; when 2 is added to an odd number, the sum is the next odd number.

## Even Numbers

Fill in the boxes.

| | | | | |
|---|---|---|---|---|
| 0 | 2 | 4 | 6 | 8 |
| 10 | 12 | 14 | 16 | 18 |
| 20 | 22 | 24 | 26 | 28 |
| 30 | 32 | 34 | 36 | 38 |
| 40 | 42 | 44 | 46 | 48 |
| 50 | 52 | 54 | 56 | 58 |
| 60 | 62 | 64 | 66 | 68 |
| 70 | 72 | 74 | 76 | 78 |
| 80 | 82 | 84 | 86 | 88 |
| 90 | 92 | 94 | 96 | 98 |

## Even Numbers

Which even number comes next?

| | |
|---|---|
| 8 | 10 |
| 32 | 34 |
| 4 | 6 |
| 46 | 48 |

| | |
|---|---|
| 60 | 62 |
| 98 | 100 |
| 22 | 24 |
| 18 | 20 |

What even number comes before?

| | |
|---|---|
| 12 | 14 |
| 4 | 6 |
| 28 | 30 |
| 6 | 8 |

| | |
|---|---|
| 0 | 2 |
| 92 | 94 |
| 12 | 14 |
| 88 | 90 |

Odd Numbers

Fill in the boxes.

| | | | | |
|---|---|---|---|---|
| 1 | 3 | 5 | 7 | 9 |
| 11 | 13 | 15 | 17 | 19 |
| 21 | 23 | 25 | 27 | 29 |
| 31 | 33 | 35 | 37 | 39 |
| 41 | 43 | 45 | 47 | 49 |
| 51 | 53 | 55 | 57 | 59 |
| 61 | 63 | 65 | 67 | 69 |
| 71 | 73 | 75 | 77 | 79 |
| 81 | 83 | 85 | 87 | 89 |
| 91 | 93 | 95 | 97 | 99 |

## Odd Numbers

What odd number comes next?

9 11
13 15
51 53
5 7

39 41
75 77
87 89
91 93

What odd number comes before?

5 7
1 3
27 29
9 11

59 61
43 45
11 12
49 50

## Even and Odd Numbers

★ Fill the blanks.

2 + 2 = 4

0 + 8 = 8

4 + 2 = 6

2 + 4 = 6

6 + 0 = 6

4 + 2 = 6

0 + 4 = 4

2 + 0 = 2

All the addends above are [✓ even] odd.

All the sums are [✓ even] odd.

---

1 + 3 = 4

1 + 1 = 2

5 + 1 = 6

3 + 1 = 4

1 + 5 = 6

3 + 3 = 6

5 + 3 = 8

5 + 5 = 10

All the addends are even [✓ odd].

All the sums are [✓ even] odd.

7

★ Fill the blanks.

2 + 1 = 3

4 + 1 = 5

0 + 3 = 3

2 + 3 = 5

5 + 0 = 5

3 + 2 = 5

3 + 0 = 3

5 + 2 = 7

---

2 + 2 = 4

4 + 2 = 6

6 + 2 = 8

8 + 2 = 10

10 + 2 = 12

1 + 2 = 3

3 + 2 = 5

5 + 2 = 7

7 + 2 = 9

9 + 2 = 11

★ Further Activities:

More able pupils may be asked to list the members of certain sets of numbers, for instance, the set of odd numbers starting with 9 and ending with 43. You may teach the meaning of three dots in 14, 16, 18, ... 52 and make assignments, to be completed on hundreds-square paper, such as:

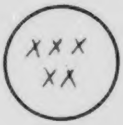
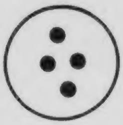
List the members of Set A: 9, 11, 13, ... 43
Set B: 3, 5, 7, ... 21
Set C: 0, 2, 4, ... 24

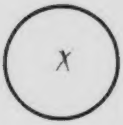
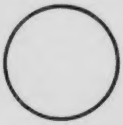
Pupil's book, pages 45 - 49:

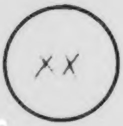
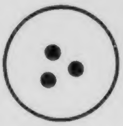
Pages 45 through 49 of the pupil's book may be used for review at this time.

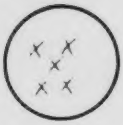
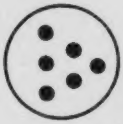
# REVIEW EXERCISES FOR CHAPTER I

Show sets. Use X's.

 has one more member than 

 has one more member than 

 has one fewer member than 

 has one fewer member than 

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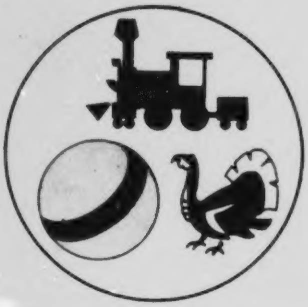
Write > or <.

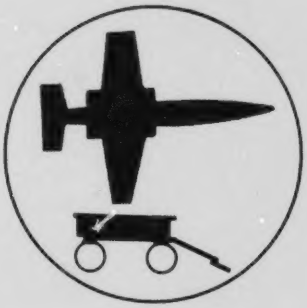
| | |
|---|---|
| 4 < 6 | 39 > 24 |
| 7 > 1 | 48 > 40 |
| 15 < 16 | 90 > 88 |
| 19 < 52 | 12 < 21 |

## Addition and Equations

Join



and

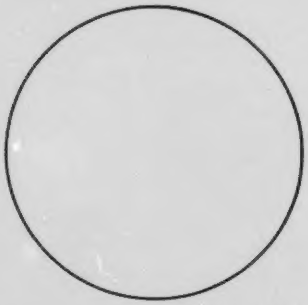


Equations: 3 + 2 = 5

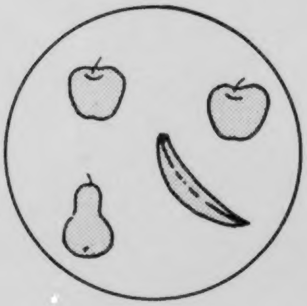
2 + 3 = 5

---

Join



and



Equations: 0 + 4 = 4

4 + 0 = 4

---

Fill the blanks:

4 + 1 = 5

4 + 2 = 6

2 + 4 = 6

6 + 0 = 6

1 + 5 = 6

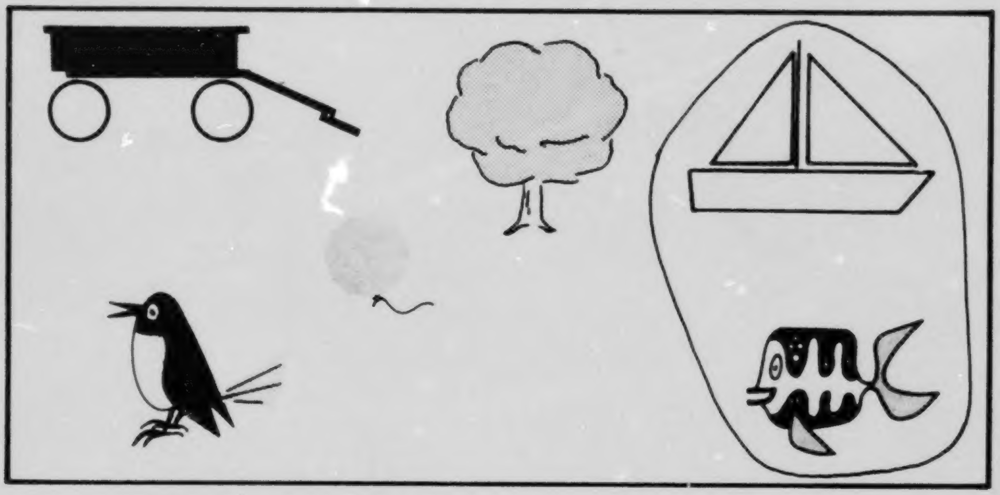
3 + 2 = 5

2 + 3 = 5

3 + 3 = 6

2 + 2 = 4

## Subtraction and Equations



How many are in the starting set? 6

How many are in the set removed? 2

How many are in the set remaining? 4

Equation: 6 - 2 = 4

---

Fill the blanks:

6 - 5 = 1

4 - 2 = 2

5 - 0 = 5

4 - 4 = 0

6 - 2 = 4

3 - 1 = 2

6 - 4 = 2

5 - 3 = 2

5 - 2 = 3

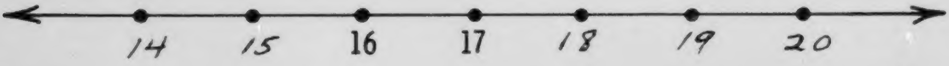
4 - 3 = 1

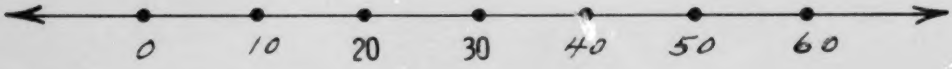
Fill in the blank in the first equation.

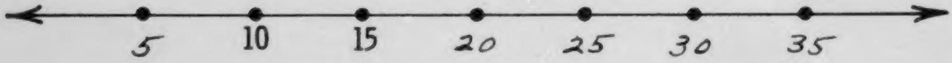
Write an equation to show undoing.

| | | |
|---|---|---|
| 4 + 1 = 5<br>5 - 1 = 4 | 2 + 3 = 5<br>5 - 3 = 2 | 6 - 3 = 3<br>3 + 3 = 6 |
| 3 + 2 = 5<br>5 - 2 = 3 | 5 - 5 = 0<br>0 + 5 = 5 | 3 - 0 = 3<br>3 + 0 = 3 |

Write the numerals.


14
15
16
17
18
19
20


0
10
20
30
40
50
60


5
10
15
20
25
30
35

Fill the blanks.

6 8 10 12 14 16 18

7 9 11 13 15 17 19

10 20 30 40 50 60 70

15 20 25 30 35 40 45

Draw rings to show even numbers.

7 [✓ 36] 13 [✓ 22] 15 [✓ 62]

49 [✓ 28] [✓ 16] [✓ 0] 57 81

What odd number comes before?

13 15 7 9 19 21

89 91 11 13 67 68

What even number comes after?

26 28 38 40 98 100

54 56 0 2 81 82

## Chapter II

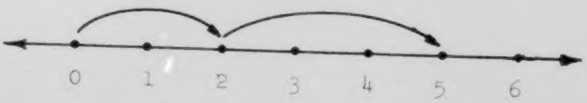
## ADDITION AND SUBTRACTION: REVIEW

### Background

In Section II-1 the device for representing numbers 0, 1, 2, ... by means of equally spaced points on the number line is used to help visualize addition. For instance, the result

$$2 + 3$$

of "taking 2 and adding to it 3" is pictured on the number line as taking 2 steps to the right (beginning at 0) and then taking 3 more steps to the right, thus arriving at 5 as suggested below.



It is to be noted that even an equation like

$$39 + 3 = 42$$

is easily seen on the number line (just take 3 steps to the right, beginning at 39) without any need to appeal to "regrouping" or place-value.

Section II-1 also makes use of partitions to help review the addition facts, particularly for sums of 7, 8, 9 and 10. Corresponding to partitions of sets, we may also speak of partitions of numbers. The partitions of the number 7, for example, are the various pairs of whole numbers whose sums are 7, namely

7, 0
6, 1
5, 2
4, 3
3, 4
2, 5
1, 6
0, 7

Notice that these are ordered pairs. That is, we list separately, for instance, the pair 6, 1 and the pair 1, 6. One reason for this is that later we shall wish to make a point of using (especially in Sections II-5 and II-7) the commutative property of addition, in accordance with which

$$6 + 1 = 1 + 6.$$

Section II-3 introduces an addition table to record systematically the addition facts for sums through 10 (and perhaps a bit beyond, when you feel that your class, or a part of it, is ready for this). Here you should make sure that pupils really do understand how to use the chart: for instance, that the sum which results from taking 4 and adding 3, is found in the table at the intersection of row 4 and column 3, as shown below.

Second Number

First Number

| + | 0 | 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|---|---|
| 0 | | | | | | | |
| 1 | | | | | | | |
| 2 | | | | | | | |
| 3 | | | | | | | |
| 4 | | | | 7 | | | |
| 5 | | | | | | | |
| | | | | | | | |

Specifying a location in a table by giving its row and column actually amounts to using geometric coordinates, an idea which will be carried further in Book III, Chapter III: Describing Points by Numbers. Indeed, it is this idea which lies at the heart of what is called analytic geometry.

The review of subtraction, in Section II-4, uses familiar ideas and teaching devices. For instance, to clarify the subtraction equation

$$7 - 2 = 5$$

a set of 2 objects is removed from a set of 7 objects, the set remaining consisting of 7 - 2 or 5 objects. (We will refer to this set as the remaining set, or the set that remains. However, we will not think of it as the remainder set.) We may also partition a set of 7 into a set of 5 and a set of 2 in order to visualize the above subtraction equation.

Section II-5, involves the inverse ("doing and undoing") relationship between the operations of first adding and then subtracting the same number. For instance, corresponding to the operation of adding 3 ("doing") we have the operation of subtracting 3 ("undoing").

There are two more comments to be made about Section II-5. First, as noted above, the commutative property of addition $(6 + 1 = 1 + 6$,etc.) is used here. Second, pupil page 78. is marked with a star (★) to show that it is intended mainly for the more able pupils.

Section II-7 concerns the relations greater than (>) and less than (<). Since the children tend to confuse these two symbols, a memory prop is offered: in an expression like

$$8 > 6$$

or

$$6 < 8$$

7

the symbol $>$ or $<$ always "points toward" the numeral for the smaller number (here, 6). Again in this section there is a starred pupil page 86 intended primarily for more able pupils.

Section II-8 involves the associative property of addition, illustrated by equations like $3 + (2 + 4) = (3 + 2) + 4$. After children have become somewhat familiar with this property, it is used along with the commutative property, ($2 + 4 = 4 + 2$, etc.) to give a systematic way of seeing why it is that in a column computation like

$$\begin{array}{c} 3 \\ 2 \\ \underline{4} \end{array}$$

we get the same result whether we add "up" or "down". When we add "down", we start with the 3, add the 2, and then add 4 to their sum. Using parentheses we may symbolize this thus:

$$(3 + 2) + 4.$$

When we add "up", we start with the 4, add the 2, and then add the 3 to the sum of 4 and 2. This may be symbolized using parentheses, thus:

$$(4 + 2) + 3.$$

Now, using general principles (rather than just addition facts for the particular numbers in this example) we wish to see why it is that the final result is the same in both cases. More specifically, we wish to use the associative and commutative properties of addition to justify the equation

$$(3 + 2) + 4 = (4 + 2) + 3.$$

We may do this as follows:

$(3 + 2) + 4 = 3 + (2 + 4)$ (By the associative property.)

$= 3 + (4 + 2)$ (Since $2 + 4 = 4 + 2$ by commutative property.)

$= (4 + 2) + 3$ (Commutative property.)

The last step is justified by the commutative property applied to the two numbers $3$ and $4 + 2$. It is hoped that at least the more able pupils will see that the above is a general argument that would work equally well for *any* three numbers, and not just for the particular numbers $3$, $2$, and $4$.

## II-1. Addition using the number line

Objective: To review the addition facts through sums of 10, the commutative property of addition, and the addition property of zero.

Vocabulary: (No new words.)

Materials: Flannel board materials, number line on chalkboard, chalk of 2 different colors; a set of 10 objects (bottle caps, disks, etc.) for each child.

### Suggested Procedure:

Put a set of 4 objects and a set of 2 objects on the flannel board. Have children tell the number of objects in each set. Then join the set of 2 objects to the set of 4 objects. Have children tell the number of members in the union. Ask for the equation that relates to the union of these two sets. ($4 + 2 = 6$.) Write it on chalkboard under the heading "Equations".

Separate objects on flannel board into sets of 2 and 4 members, and this time join the set of 4 objects to the set of 2 objects. Again ask for the equation, $2 + 4 = 6$, and write it beneath the first equation.

Repeat the procedure with materials on flannel board to show: $5 + 4$ and $4 + 5$, $7 + 1$ and $1 + 7$, $8 + 0$ and $0 + 8$, etc. (With many classes it may be desirable to have children use their own sets of objects. For instance, write $8 + 2$ on the chalkboard, have children form sets, join them, and tell you the number of the union.) Point out that when sets are joined the new set is called the union; when numbers are added, the result is called the sum. Both now and later in the lesson it is hoped that pupils will notice the commutative property of addition, that adding 1 to a number gives the next whole number, and that if 0 is one of the addends, the sum is equal to the other addend.

The helpfulness of these concepts may be demonstrated to children by writing:

52 + 36 = 88

36 + 52 =

Children who understand commutativity will be excited to say "88"!

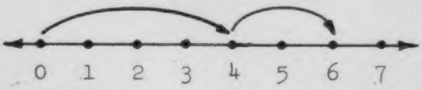
Use several examples of this kind and of the following:

458 + 0 =

62 + 1 =

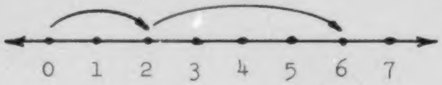
1 + 76 =

- Leave several pairs of equations on the chalkboard, point to the first equation, 4 + 2 = 6, and ask how the number line might be used to illustrate it. We can hope suggestions and demonstrations will be given by children. If necessary, you may need to demonstrate. Note the starting point and also the intervals. The numeral is written at the endpoint of each interval. With children we can refer to these as jumps or steps.



Starting at 0, we first take 4 jumps. Then we take 2 jumps. How many jumps have we taken all together? (6.) How far are we from the starting point? (6 jumps.) What point shows the sum of 4 and 2? (6.)

Use chalk of a different color to illustrate 2 + 4 = 6.



Where do we start? (0.)

How many jumps do we take first? (2.)

Then, how many jumps do we take? (4.)

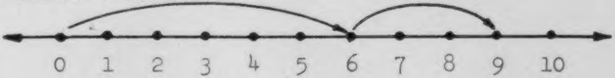
How many jumps are we from the starting point? (6.)

What point shows the sum of 2 and 4? (6.)

Use the same procedure with other pairs of equations on the chalkboard. Have the children show the jumps on the number line, and use different colors for each equation.

When all equations have been illustrated, erase them.

On the number line show:



Ask:

What equation is shown on the number line? (5 + 4 = 9.)

What point corresponds to (shows) the sum? (9.)

When pupils seem to be able to (a) make a number line picture for an equation and (b) give an equation for a number line picture without difficulty, use Pupil's book, pages 50 through 53.

Pupil's book, pages 50 and 51:

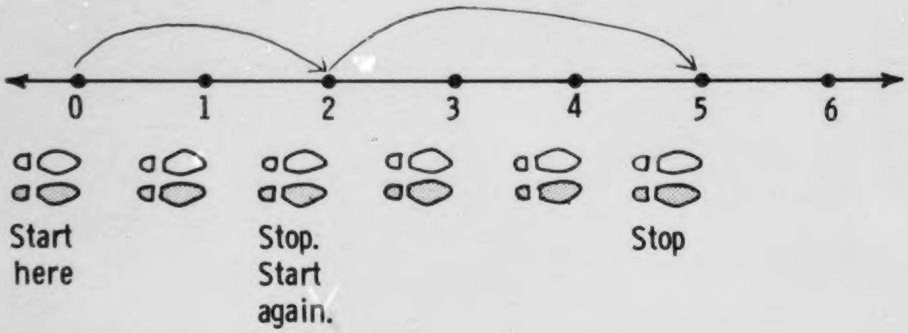
These pages are for use with the group. Some children may be able to use 52 and 53 independently.

Pupil's book, page 54:

Children should review and apply the ideas about 0, 1, and 2 as addends from this and earlier lessons.

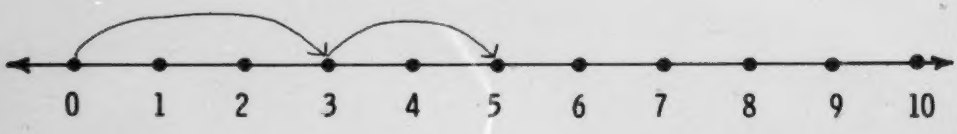
## Using a Number Line When Adding

Show a picture of  2 + 3 = 5.


0
1
2
3
4
5
6
Start here
Stop. Start again.
Stop

What point shows the sum of 2 and 3? 5

Show a picture of 3 + 2 = 5.

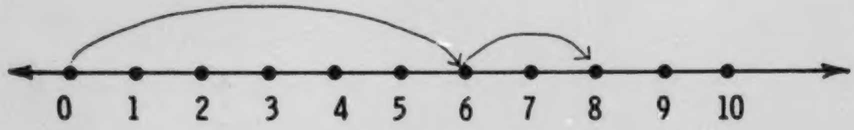

0
1
2
3
4
5
6
7
8
9
10

What is the sum of 3 and 2? 5

Look at both pictures.

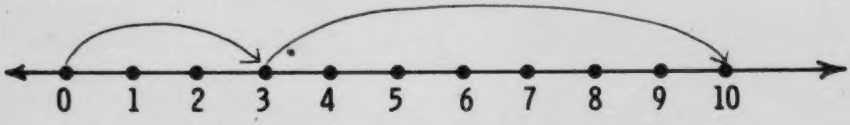
2 + 3 = 3 + 2

Use the number line to show the sum of 6 and 2.



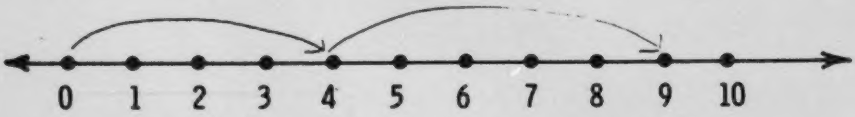
Equation: 6 + 2 = 8.

Use the number line to show the sum of 3 and 7.



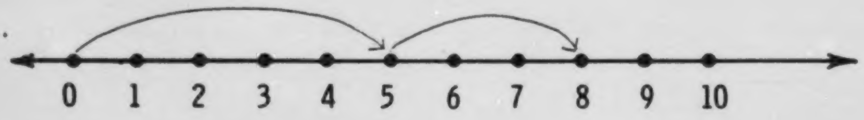
Equation: 3 + 7 = 10

Use the number line to show the sum of 4 and 5.



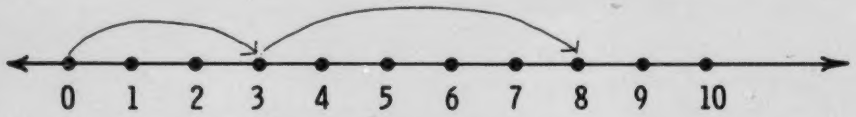
Equation: 4 + 5 = 9

Use the number line to show the sum of 5 and 3.


0
1
2
3
4
5
6
7
8
9
10

Equation: 5 + 3 = 8

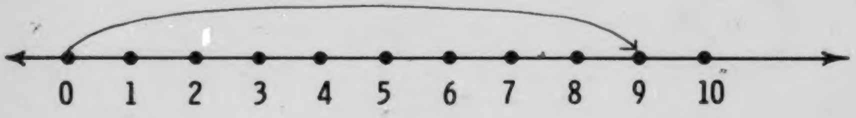
Use the number line to show the sum of 3 and 5.


0
1
2
3
4
5
6
7
8
9
10

Equation: 3 + 5 = 8

5 + 3 = 3 + 5

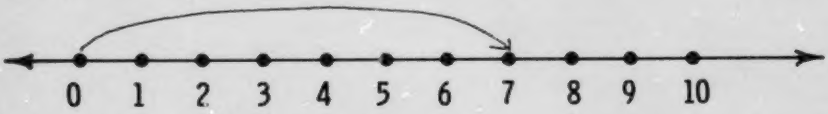
Use the number line to show the sum of 9 and 0.


0
1
2
3
4
5
6
7
8
9
10

Equation: 9 + 0 = 9

What point shows the sum of 9 and 0? 9

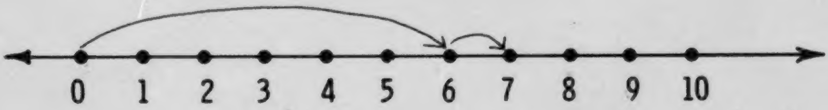
Use the number line to show the sum of 0 and 7.


0
1
2
3
4
5
6
7
8
9
10

Equation: 0 + 7 = 7

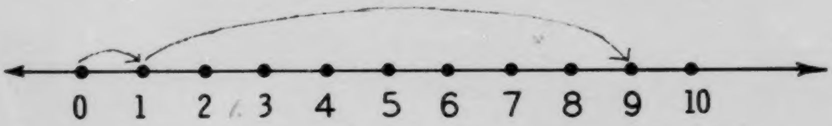
What point shows the sum of 0 and 7? 7

Use the number line to show the sum of 6 and 1.


0
1
2
3
4
5
6
7
8
9
10

Equation: 6 + 1 = 7

Use the number line to show the sum of 1 and 8.


0
1
2
3
4
5
6
7
8
9
10

Equation: 1 + 8 = 9

## Renaming Sums

Fill the blanks.

6 + 0 = 6
6 + 1 = 7
6 + 2 = 8
7 + 0 = 7
7 + 1 = 8
7 + 2 = 9

8 + 0 = 8
8 + 1 = 9
8 + 2 = 10
6 + 0 = 6
6 + 1 = 7
6 + 2 = 8

9 + 0 = 9
9 + 1 = 10
9 + 2 = 11
5 + 0 = 5
5 + 1 = 6
5 + 2 = 7

---

Fill the blanks:

1 + 8 = 9
0 + 9 = 9
1 + 6 = 7
26 + 1 = 27

0 + 7 = 7
2 + 5 = 7
37 + 0 = 37
59 + 1 = 60

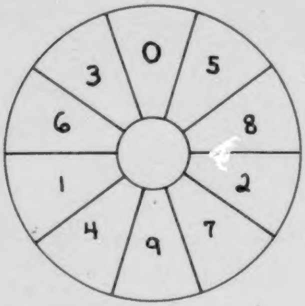
2 + 8 = 10
1 + 9 = 10
68 + 1 = 69
46 + 2 = 48

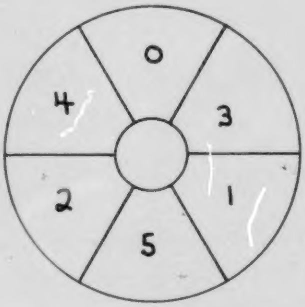
Further Activities:

Use "Drill Doughnuts", of which a detailed description is given below, for added practice at this time and in future lessons.

"Drill Doughnuts" may be used to advantage for additional practice in adding, subtracting, and multiplying both for the basic facts and for encouraging mental computation later on.

For each child in the class, prepare a "Doughnut" cut from cardboard, tagboard, or other heavy stock. Disk is $4\frac{1}{4}$ inches in diameter, center hole is 1 inch in diameter.) Use a red felt pen for writing the numerals on one side and a blue felt pen for the other side.





Give each child a Doughnut and a sheet of newsprint, 9" x 12". Tell children to fold paper in half (to yield two sections on each side of paper, each 9" x 6"). Have Doughnut placed on paper so that zero is at the top and there is room on each section of paper to write numerals around the edge of the Doughnut. Tell children to hold Doughnut still, not to trace around it, but to write on paper through the hole in the middle "5 + ". Beyond the edge of the Doughnut, on the newsprint, they should write the sum of 5 and the number indicated on each section of the Doughnut. Next move Doughnut to another section of the paper. Give the number and operation sign to be written in the center of the Doughnut:

11 -, 5 +, etc. Children again write answers around the Doughnut on the paper. This activity can be repeated using the four sections of the paper.

When all four sections of paper are finished, 40 problems have been done if blue side is used, 24 if red side is used. You can make a key, and since only the center entry and the answers appear for each section, checking papers is easy. Eventually all that is needed in the way of preparation for practice is the following on the chalkboard:

| | |
|---|---|
| 5 X | 7+ |

Red Side

| | |
|---|---|
| 10 - | 12 - |

Blue Side

## II-2. Partitions

Objectives: To partition sets of 7, 8, 9, and 10; and to learn the corresponding addition facts.

Vocabulary: (No new words.)

Materials: Objects for flannel board, yarn, manipulative materials (10 objects and yarn for each child)

Suggested Procedure:

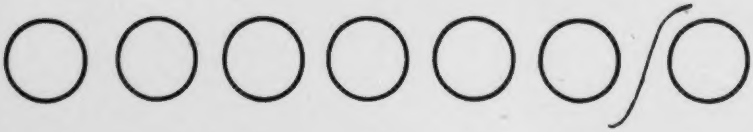
The following procedure is used for sums of seven, eight, nine and ten. If your class is familiar with these sums, you may wish to adapt this material to the extent of review that is necessary.

Partitions of a set of 7

Put a set of 7 objects on the flannel board.

> What is one way we could partition this set of 7 members into just two subsets?

Use yarn to show any partition a child suggests, e.g., 6 and 1.



Explain that the word partition may be used whenever we think of separating a set into subsets, whether they are equivalent subsets or not.

Leave the 7 objects on the flannel board and have children show other ways of partitioning it. Record on chalkboard:

| 7 | |
|---|---|
| 6 | 1 |
| 5 | 2 |
| 4 | 3 |

etc.

Show that when each pair of numbers is added, their sum is 7, illustrating by joining sets. For instance: 7 = 6 + 1. Join a set of 1 object to a set of 6 objects and write 6 + 1 = 7. Children are more familiar with the latter equation and may find 7 = 6 + 1 awkward at first.

Pupil's book, page 55:

Children should show, by a mark on the picture, the partition indicated and complete the equation.

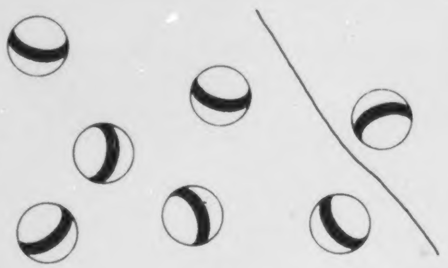
Pupil's book, pages 56 and 57:

Tell children to count the objects in each set, write the number in the first blank, show the partition, and complete the equation.

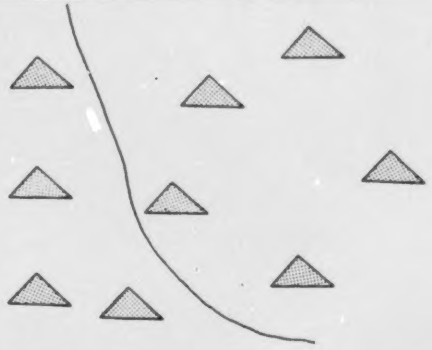
Pupil's book, page 58:

Fill the blanks to show the partitions indicated.

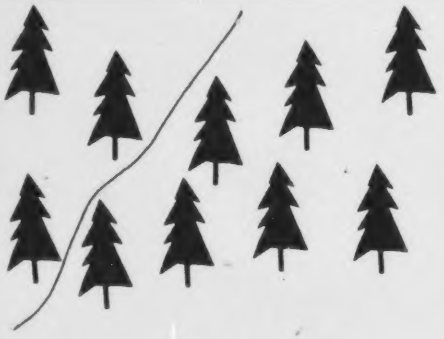
## Partitions of Sets



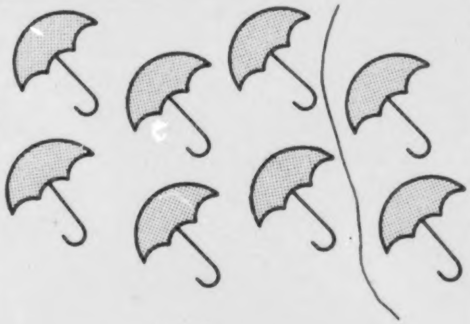
7 = 1 + 6



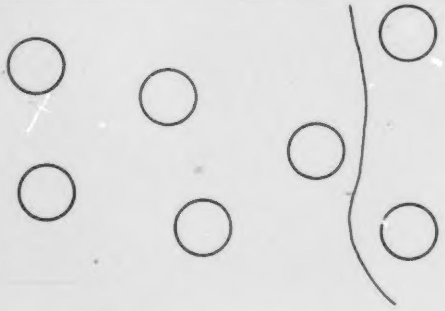
9 = 4 + 5



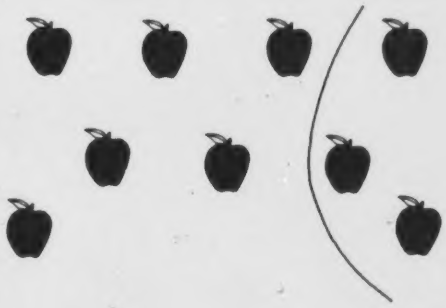
10 = 3 + 7



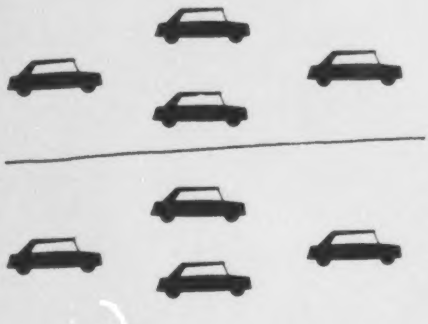
8 = 2 + 6



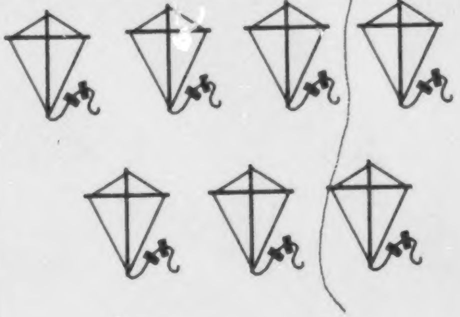
7 = 5 + 2



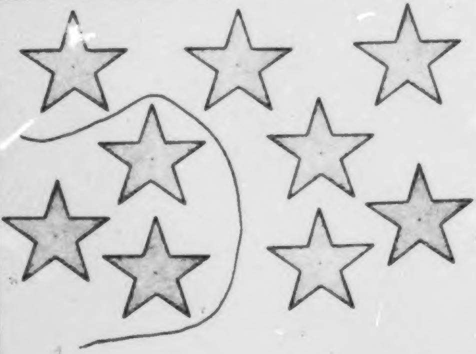
9 = 6 + 3



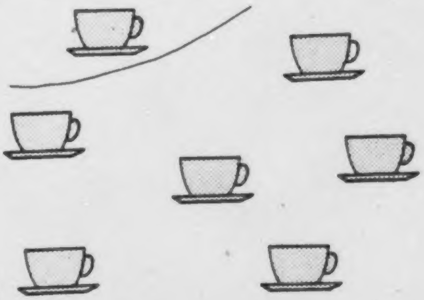
8 = 4 + 4



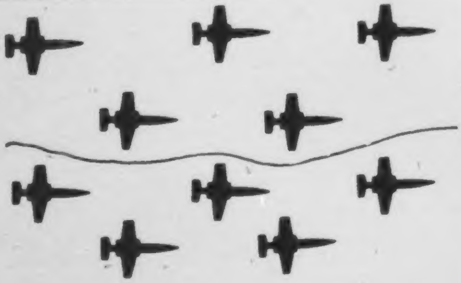
7 = 2 + 5



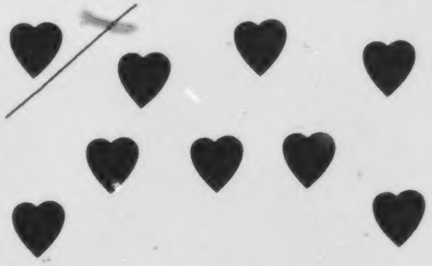
9 = 3 + 6



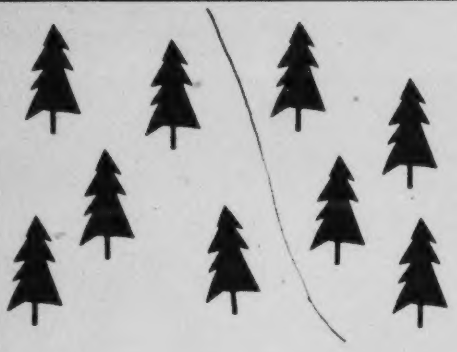
7 = 6 + 1



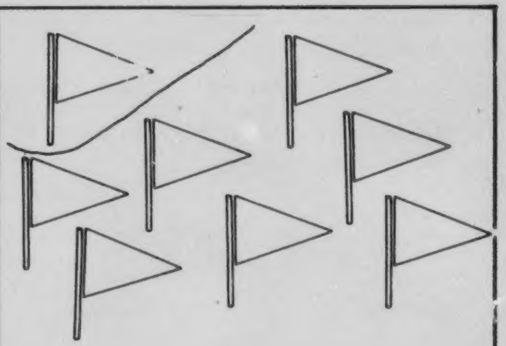
10 = 5 + 5



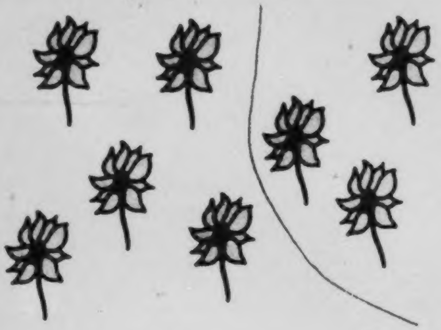
9 = 1 + 8



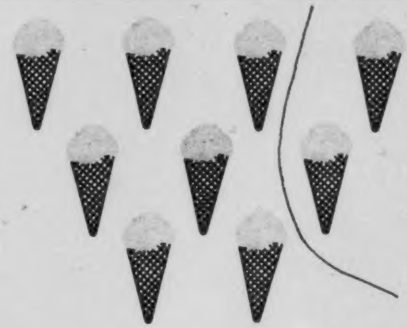
9 = 4 + 5



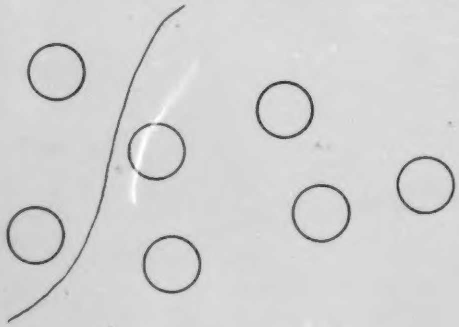
8 = 1 + 7



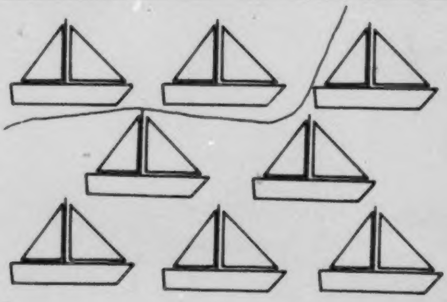
8 = 5 + 3



9 = 7 + 2



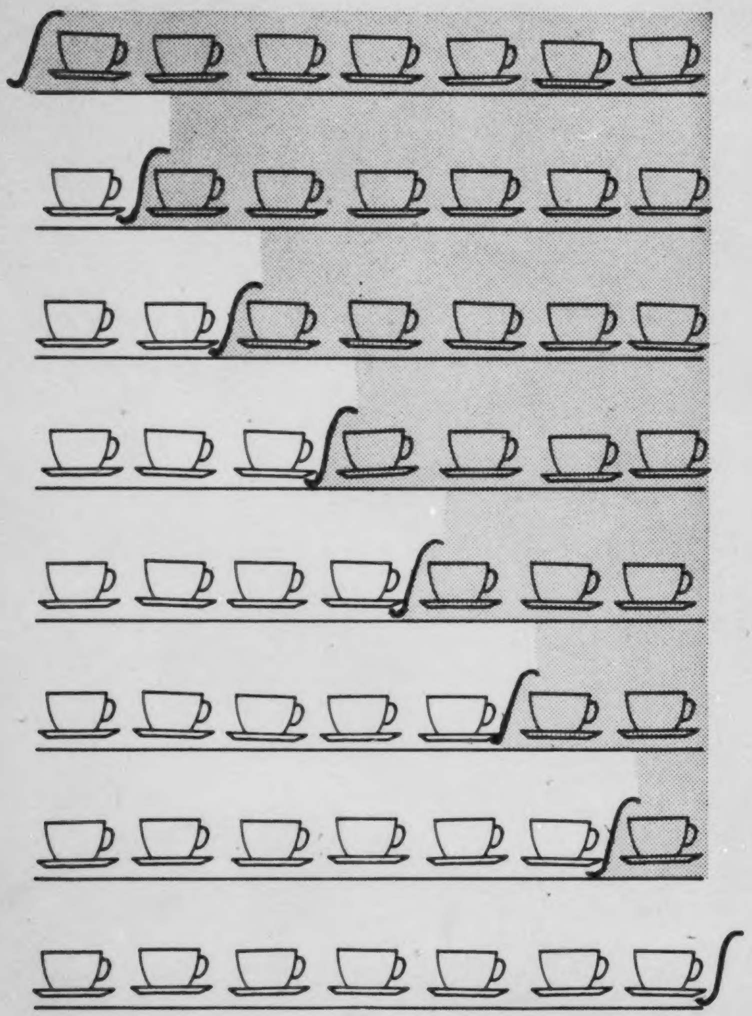
7 = 2 + 5



8 = 6 + 2

## Partitions of Sets of Seven

**Write about the partitions of the sets of seven pictured below.**



7 = 0 + 7

7 = 1 + 6

7 = 2 + 5

7 = 3 + 4

7 = 4 + 3

7 = 5 + 2

7 = 6 + 1

7 = 7 + 0

- Partitions of sets of 8, 9, and 10

  Develop partitions of sets of 8, 9, and 10 similarly. For variety, children may use their own materials and yarn and list partitions on paper, to be recorded later on the chalkboard.

- Generalizations

  Make use of the commutative property, observing that a pair of numbers, such as 5 and 3, have the same sum, regardless of the order in which they are added: (5 + 3 = 3 + 5). Help children discover how many pairs of numbers are possible for a given sum. Let them discover the pattern: for 4 there are 3 pairs (4 and 0; 3 and 1; 2 and 2) and there are 5 possible partitions (4 and 0; 0 and 4; 3 and 1; 1 and 3; 2 and 2). For 5 there are 3 pairs of numbers but 6 possible partitions. Ask how many pairs and how many partitions there are for 6.

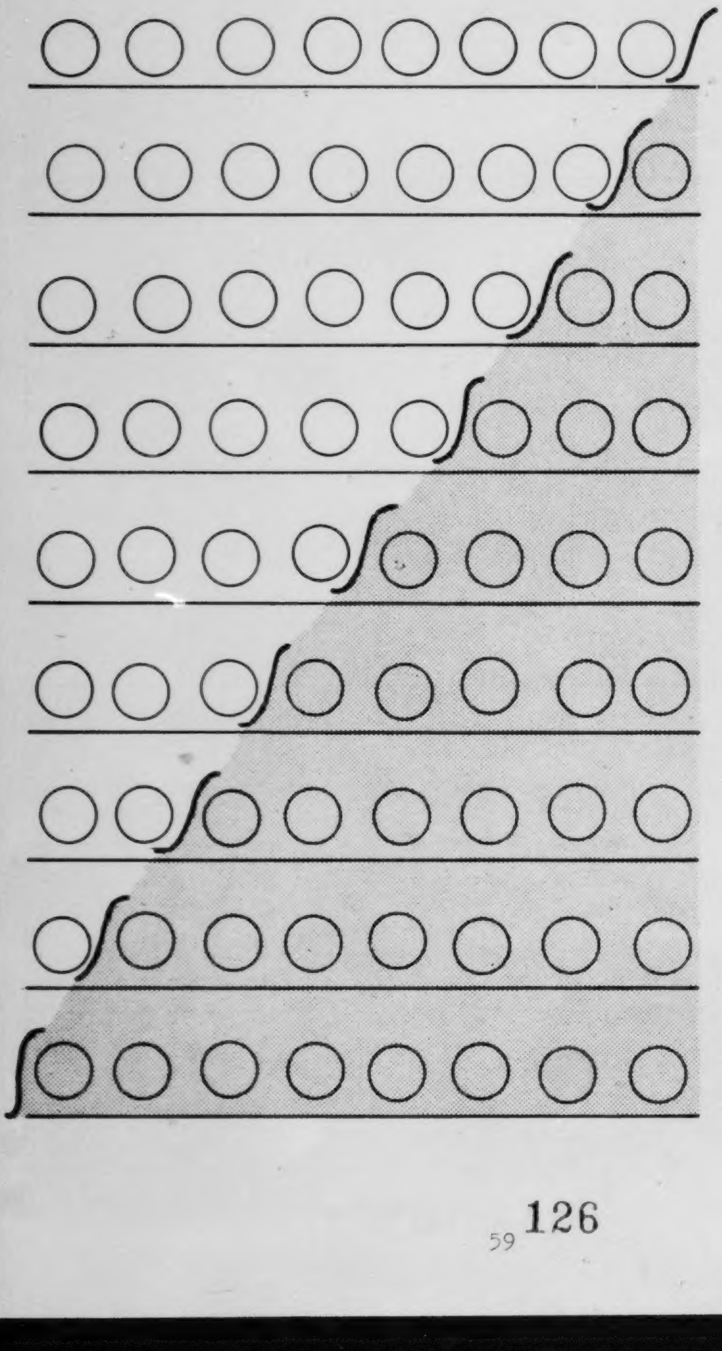
| | |
|---|---|
| Pupil's book, pages 59 - 65: | Partitions of 8, 9, and 10 and addition are developed. |
| Pupil's book, page 66: | Provides practice on facts learned and readiness for the addition table. Children add the number shown at the top of the chart to each number in the left column and enter the sum in the right column. |

You may wish to provide additional practice with charts in the following form:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| + | 7 | 2 | 0 | 8 | 4 | 6 | 5 |
| 3 | | | | | | | |

## Partitions of Sets of Eight

Write about the partitions of the sets of eight pictured below.



8 = 8 + 0

8 = 7 + 1

8 = 6 + 2

8 = 5 + 3

8 = 4 + 4

8 = 3 + 5

8 = 2 + 6

8 = 1 + 7

8 = 0 + 8

## Numbers and Their Sums

Fill in the blanks:

5 + 3 = 8
3 + 5 = 8

2 + 5 = 7
5 + 2 = 7

4 + 3 = 7
3 + 4 = 7

8 + 0 = 8
0 + 8 = 8

6 + 1 = 7
1 + 6 = 7

5 + 1 = 6
1 + 5 = 6

1 + 7 = 8
7+1 = 8

0 + 7 = 7
7+0 = 7

6 + 2 = 8
2+6 = 8

Fill in the blanks:

6 + 1 = 7
1 + 6 = 7

8 = 6 + 2
8 = 2 + 6

7 = 2 + 5
7 = 5 + 2

7 + 0 = 7
0 + 7 = 7

7 = 4 + 3
7 = 3 + 4

6 = 4 + 2
6 = 2 + 4

4 + 3 = 7
3 + 4 = 7

8 = 3 + 5
8 = 5 + 3

7 = 4 + 3
7 = 3 + 4

## Numbers and Their Sums

7
is the sum of

3 and 4
4 and 3
5 and 2
2 and 5
6 and 1
1 and 6
0 and 7
7 and 0

8
is the sum of

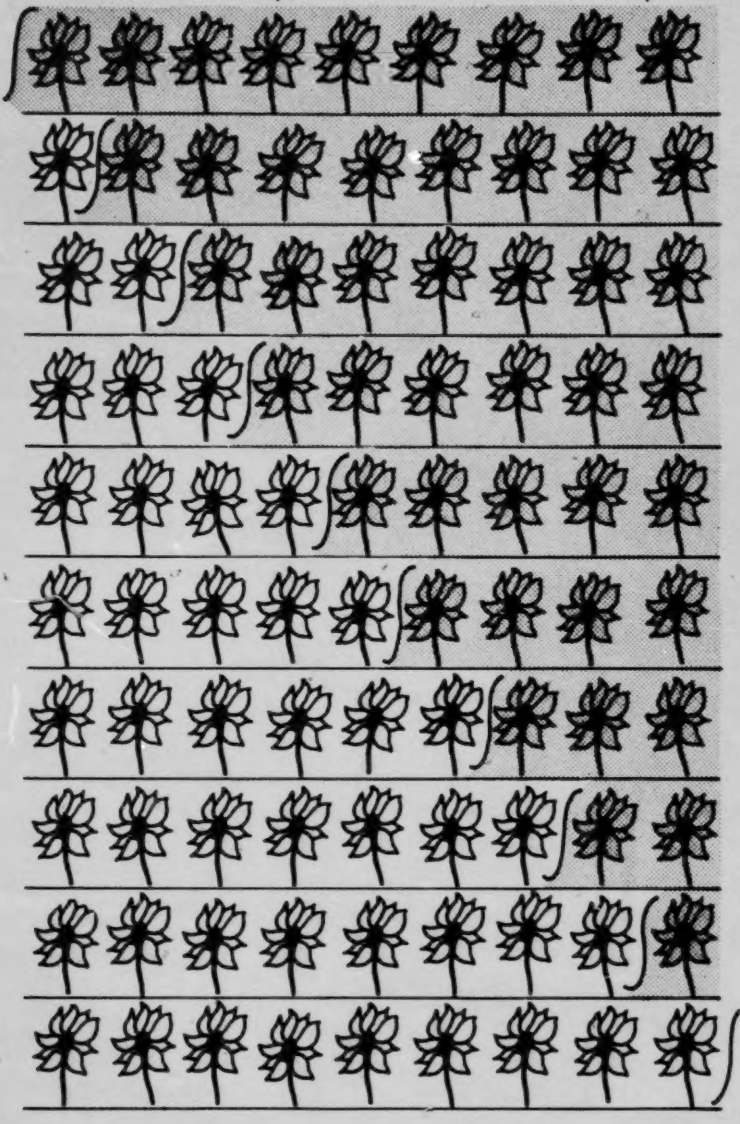
4 and 4
5 and 3
3 and 5
6 and 2
2 and 6
7 and 1
1 and 7
0 and 8
8 and 0

**Fill in the blanks:**

| | | |
|---|---|---|
| 2 + 4 = 6 | 0 + 8 = 8 | 5 + 2 = 7 |
| 1 + 6 = 7 | 5 + 3 = 8 | 1 + 6 = 7 |
| 3 + 4 = 7 | 3 + 3 = 6 | 4 + 4 = 8 |
| 5 + 3 = 8 | 4 + 3 = 7 | 3 + 5 = 8 |

## Partitions of Sets of Nine

Write about the paritions of the sets of nine pictured below.



9 = 0 + 9

9 = 1 + 8

9 = 2 + 7

9 = 3 + 6

9 = 4 + 5

9 = 5 + 4

9 = 6 + 3

9 = 7 + 2

9 = 8 + 1

9 = 9 + 0

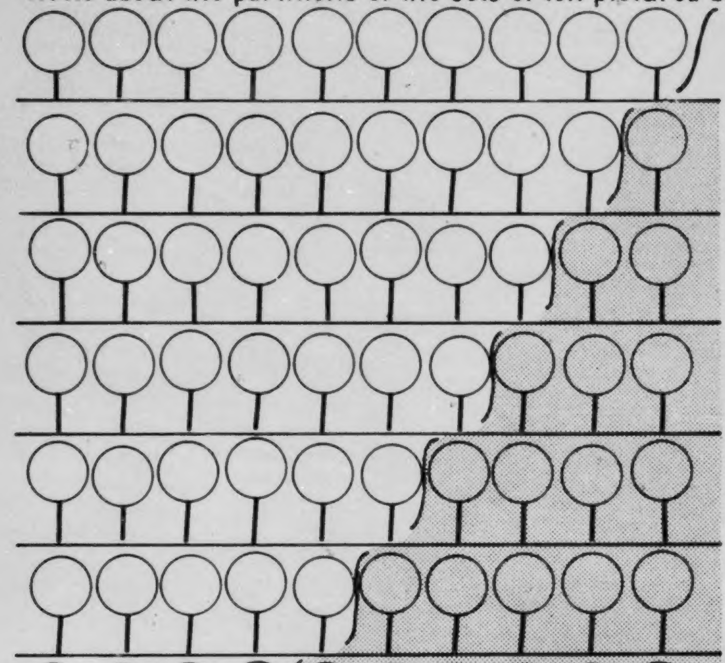
**Numbers and Their Sums**

**Fill in the blanks.**

| | | |
|---|---|---|
| 3 + 4 = 7<br>7 = 3 + 4 | 6 + 2 = 8<br>8 = 6 + 2 | 5 + 4 = 9<br>9 = 5 + 4 |
| 5 + 3 = 8<br>3 + 5 = 8 | 0 + 9 = 9<br>9 + 0 = 9 | 5 + 2 = 7<br>2 + 5 = 7 |
| 4 + 5 = 9<br>9 = 4 + 5 | 6 + 1 = 7<br>7 = 1 + 6 | 6 + 3 = 9<br>9 = 3 + 6 |
| 7 + 0 = 7<br>0 + 7 = 7 | 2 + 6 = 8<br>2 + 6 = 8 | 4 + 4 = 8<br>8 = 4 + 4 |
| 8 + 0 = 8<br>8 = 8 + 0 | 7 + 2 = 9<br>9 = 2 + 7 | 4 + 3 = 7<br>7 = 4 + 3 |

## Partitions of Sets of Ten

Write about the partitions of the sets of ten pictured below.



10 = 10 + 0

10 = 9 + 1

10 = 8 + 2

10 = 7 + 3

10 = 6 + 4

10 = 5 + 5

10 = 4 + 6

10 = 3 + 7

10 = 2 + 8

10 = 1 + 9

10 = 0 + 10

## Numbers and Their Sums

**9**
**is the sum of**

4 and 5
5 and 4
6 and 3
3 and 6
7 and 2
2 and 7
8 and 1
1 and 8
9 and 0
0 and 9

**10**
**is the sum of**

5 and 5
6 and 4
4 and 6
7 and 3
3 and 7
8 and 2
2 and 8
9 and 1
1 and 9
10 and 0
0 and 10

**Fill in the blanks:**

0 + 9 = 9
7 + 2 = 9
6 + 4 = 10
5 + 5 = 10

6 + 2 = 8
1 + 9 = 10
2 + 8 = 10
5 + 4 = 9

7 + 3 = 10
5 + 5 = 10
6 + 3 = 9
4 + 5 = 9

Fill the boxes.

| + | 1 |
|---|---|
| 3 | 4 |
| 9 | 10 |
| 17 | 18 |

| + | 0 |
|---|---|
| 7 | 7 |
| 8 | 8 |
| 92 | 92 |

| + | 2 |
|---|---|
| 8 | 10 |
| 7 | 9 |
| 36 | 38 |

| + | 5 |
|---|---|
| 3 | 8 |
| 5 | 10 |
| 4 | 9 |
| 2 | 7 |

| + | 4 |
|---|---|
| 6 | 10 |
| 4 | 8 |
| 2 | 6 |
| 5 | 9 |

| + | 6 |
|---|---|
| 3 | 9 |
| 2 | 8 |
| 4 | 10 |
| 0 | 6 |

## II-3. The addition table

**Objective:** To develop understanding of how an addition table is constructed and used.

**Vocabulary:** Table, row, column.

**Materials:** An addition chart on chalkboard or tagboard.

**Suggested Procedure:**

Remind children that when equations are used to show sums, several equations may be used to show the same sum. If they wished to list all the equations they have been writing for sums from 1 through 10, it would take much time and space. (You may wish to list the equations for 6: 6 + 0 = 6, 0 + 6 = 6, 5 + 1 = 6, 1 + 5 = 6, 4 + 2 = 6, 2 + 4 = 6, 3 + 3 = 6.) Say that they are going to learn a way of showing the sums through 10 without so much writing.

Draw on the chalkboard:

First Number

| + | | | | | |
|---|---|---|---|---|---|
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |

Explain that the left side of the table will be used to show the first number added. For any equation that starts 0 +, you will write the sum in the row that begins with 0. The top (here write in 0, 1, 2, 3, 4 across the top of the table) will show the second number added. Write "second number" at the top.

Second Number

First Number

| + | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | | | | | |
| 1 | | | | | |
| 2 | | | | | 6 |
| 3 | | | | | |
| 4 | | | | | |

For instance, if you want to show the equation $2 + 4 = 6$, you look for the first number, 2. You then find the second number, 4, at the top and go down that column. (Move your hand down the 4 column.) Ask:

Where do you think you will write the sum?

Move your left hand along row 2 as you move your right hand down column 4. Show that the 6 at the "crossroads" shows the sum of 2 and 4. Have a child show where the sum should be written for $4 + 2 = 6$. Use the chalkboard model for several more pairs of numbers, having a child point out the place for the sum and write it in that place. Then point to one of the entries and have a child give the equation that goes with that location. Repeat or extend as necessary.

Pupil's book, page 67:

Use as a class exercise and check children's understanding. Children write sums only for those equations shown. Then they should complete the table independently.

★Pupil's book, page 70:

Children are to write the equation that is appropriate for each number indicated by a ring.

Pupil's book, page 68:

Pupils are to fill in the unshaded part of the addition table. Some will need supervision.

- Generalizations

When the table has been completed, use your teacher-made table in class discussion that will help children discover that:

(a) The numbers in row 0 are the same as the numbers across the top; and the numbers in column 0 are the same as the numbers down the left side (corresponding to the fact that when 0 is one addend, the sum is the other addend).

(b) As we proceed along a diagonal from lower left to upper right, the sums remain the same (corresponding to a decrease of 1 in the first addend and an increase of 1 in the second).

(c) As we proceed along a diagonal from upper left to lower right, the sums increase by 2 (corresponding to an increase of 1 in each of the two addends).

Show children how to use the addition table to find the sum of two numbers or either addend of an equation if the sum and the other addend are known. If the first addend is missing, they look at the top, find the second addend, and look down that column to the sum, and then go to the left side to find the first addend. If the second addend is missing, they look at the left side for the first addend, go along that row to the sum, and then go up to the top to find the second addend.

The addition table may be used with page 69 in Pupil's book either as a group activity or as additional work for individual pupils.

## An Addition Table

Second Number

First Number

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | ⑤ | 6 |
| 1 | 1 | 2 | 3 | ④ | 5 | 6 | 7 |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 4 | 5 | ⑥ | 7 | 8 | 9 | 10 |
| 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 6 | 7 | 8 | ⑨ | 10 | 11 | 12 |

What equation goes with the 5 in the table? 3 + 2 = 5

Show: 6 + 3 = 9 in the table.

Show: 1 + 3 = 4 in the table.

Show: 4 + 2 = 6 in the table.

Show: 0 + 5 = 5 in the table.

Now finish the table.

## An Addition Table

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | ⑥ | 7 | 8 | 9 | 10 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
| 2 | 2 | 3 | 4 | 5 | ⑥ | ⑦ | ⑧ | 9 | ⑩ | | |
| 3 | 3 | 4 | ⑤ | ⑥ | 7 | ⑧ | 9 | ⑩ | | | |
| 4 | ④ | ⑤ | 6 | 7 | ⑧ | 9 | ⑩ | | | | |
| 5 | 5 | 6 | ⑦ | ⑧ | ⑨ | ⑩ | | | | | |
| 6 | 6 | ⑦ | 8 | 9 | 10 | | | | | | |
| 7 | 7 | 8 | ⑨ | 10 | | | | | | | |
| 8 | 8 | 9 | ⑩ | | | | | | | | |
| 9 | 9 | 10 | | | | | | | | | |
| 10 | 10 | | | | | | | | | | |

Fill the boxes in the table for:

2 + 8
2 + 6
2 + 4
2 + 5

3 + 5
3 + 2
3 + 3
3 + 7

4 + 6
4 + 4
4 + 1
4 + 0

5 + 4
5 + 2
5 + 3
5 + 5

0 + 6
8 + 2
7 + 2
6 + 1

Now finish the table.

## Using the Addition Table

Use the table on page 68 to help you fill in the blanks.

5 + 4 = 9

3 + 5 = 8

2 + 7 = 9

6 + 2 = 8

0 + 9 = 9

2 + 8 = 10

9 + 1 = 10

5 + 2 = 7

4 + 6 = 10

3 + 7 = 10

4 + 5 = 9

7 + 3 = 10

8 + 2 = 10

6 + 3 = 9

5 + 3 = 8

7 + 1 = 8

7 + 2 = 9

0 + 7 = 7

8 + 2 = 10

6 + 0 = 6

3 + 4 = 7

4 + 4 = 8

### ★ An Addition Table

| + | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|---|---|---|---|---|---|---|---|
| 34 | 68 | 69 | 70 | 71 | (72) | 73 | 74 |
| 35 | 69 | 70 | (71) | 72 | 73 | 74 | 75 |
| 36 | 70 | 71 | 72 | 73 | 74 | 75 | 76 |
| 37 | 71 | 72 | 73 | 74 | 75 | 76 | 77 |
| 38 | 72 | (73) | 74 | 75 | (76) | 77 | 78 |
| 39 | 73 | 74 | 75 | 76 | 77 | 78 | (79) |
| 40 | 74 | 75 | 76 | (77) | 78 | 79 | 80 |

What equations go with the numbers shown by the rings?

35 + 36 = 71

38 + 38 = 76

34 + 38 = 72

38 + 35 = 73

40 + 37 = 77

39 + 40 = 79

## II-4. Subtraction

Objective: To review the subtraction facts corresponding to sums through 10, using partitions.

Vocabulary: (Review) subtraction, minus, partition.

Materials: Counters, flannel board and objects, magnetic board and disks, yarn. Set of 10 objects and yarn for each child. Chart, as shown, on chalkboard.

| Number of starting set | Number of set removed | Number of set remaining | Equation |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |

Suggested Procedure:

Put 5 objects on the flannel board. Remove 3 of the objects.

> What did I do? Yes, I removed a set of 3 from a set of 5.
>
> How many members are in the set remaining on the flannel board? We will call this set the set remaining.

Discuss with the children the chart on the chalkboard. Have them decide what to write in each space of the chart and observe that the order of the numbers in the equation column is the same as the order in the first 3 columns. Repeat the procedure with other sets (none greater than 10). Include some instances where the entire set is removed.

Questions should be asked that lead to the generalizations:

(a) When 0 is subtracted from any number, the difference is that number.

(b) When any number is subtracted from itself, the difference is zero.

- Using partitions in subtraction

Place 7 objects on the flannel board. Use yarn to partition the set into a set of 5 and a set of 2. On the chalkboard, write 7 = ___ + ___.

Request a child to say what partition of 7 is shown on the flannel board. (7 = 5 + 2) Remove the set of 2. Discuss the fact that you may write a subtraction equation for this action: 7 - 2 = 5.

Again, put the set of 2 on the flannel board and ask what the remaining set will be if you now remove the set of 5. Remove the set of 5 and discuss the equation: 7 - 5 = 2.

Repeat the above, partitioning the set of 7 in different ways. It may be helpful to have children use manipulative materials and yarn to make partitions of a given set (8, 9, or 10 objects), and record on paper equations similar to those above.

Pupil's book, pages 71 through 74: Point out that at the top of each page the children are(on the left) to show partitions. On the right they are to give subtraction equations which show what would happen if one of the two sets derived from the partition was removed. Instruct children to fill the blank for the equations at the bottom of the page.

## Partitions and Subtraction

7
is the sum of

3 and 4
4 and 3
7 and 0
0 and 7
5 and 2
2 and 5
6 and 1
1 and 6

Subtracting from
7

7 - 3 = 4
7 - 4 = 3
7 - 7 = 0
7 - 0 = 7
7 - 5 = 2
7 - 2 = 5
7 - 6 = 1
7 - 1 = 6

| | | |
|---|---|---|
| 6 - 4 = 2<br>6 - 2 = 4 | 7 - 5 = 2<br>7 - 2 = 5 | 5 - 3 = 2<br>5 - 2 = 3 |
| 7 - 3 = 4<br>7 - 4 = 3 | 5 - 1 = 4<br>5 - 4 = 1 | 7 - 0 = 7<br>7 - 7 = 0 |

## Partitions and Subtraction

8
is the sum of

2 and 6

5 and 3

8 and 0

1 and 7

7 and 1

4 and 4

3 and 5

6 and 2

0 and 8

Subtracting from
8

8 - 2 = 6

8 - 5 = 3

8 - 8 = 0

8 - 1 = 7

8 - 7 = 1

8 - 4 = 4

8 - 3 = 5

8 - 6 = 2

8 - 0 = 8

| | | |
|---|---|---|
| 8 = 3 + 5<br>8 - 3 = 5 | 7 = 6 + 1<br>7 - 6 = 1 | 8 = 6 + 2<br>8 - 6 = 2 |
| 7 = 4 + 3<br>7 - 4 = 3 | 8 = 4 + 4<br>8 - 4 = 4 | 8 = 8 + 0<br>8 - 8 = 0 |

## Partitions and Subtraction

9
is the sum of

1 and 8
3 and 6
0 and 9
7 and 2
4 and 5
6 and 3
9 and 0
8 and 1
5 and 4
2 and 7

Subtracting from
9

9 - 1 = 8
9 - 3 = 6
9 - 0 = 9
9 - 7 = 2
9 - 4 = 5
9 - 6 = 3
9 - 9 = 0
9 - 8 = 1
9 - 5 = 4
9 - 2 = 7

| | | |
|---|---|---|
| 9 = 3 + 6<br>9 - 3 = 6 | 8 = 4 + 4<br>8 - 4 = 4 | 9 = 0 + 9<br>9 - 0 = 9 |
| 9 = 5 + 4<br>9 - 5 = 4 | 9 = 7 + 2<br>9 - 7 = 2 | 8 = 6 + 2<br>8 - 6 = 2 |

## Partitions and Subtraction

10
is the sum of

1 and 9

0 and 10

4 and 6

10 and 0

7 and 3

2 and 8

6 and 4

8 and 2

3 and 7

9 and 1

5 and 5

Subtracting from
10

10 - 1 = 9

10 - 0 = 10

10 - 4 = 6

10 - 10 = 0

10 - 7 = 3

10 - 2 = 8

10 - 6 = 4

10 - 8 = 2

10 - 3 = 7

10 - 9 = 1

10 - 5 = 5

Pupil's book, page 75: This page provides more practice for those who need it.

Pupil's book, page 76: At the top of the page, plus or minus symbols are to be inserted between the first two numerals to make equations. Children should observe that either symbol may be used in problems like 7 0 = 7.

★ At the bottom of the page, pupils should write the related number facts for each set of three numbers.

## More Practice

8 - 1 = 7

4 + 2 = 6

8 - 5 = 3

8 + 0 = 8

7 - 4 = 3

3 + 4 = 7

2 + 7 = 9

10 = 4 + 6

10 = 10 + 0

10 = 7 + 3

9 - 7 = 2

9 - 1 = 8

9 - 4 = 5

7 = 3 + 4

8 = 5 + 3

8 = 2 + 6

4 = 0 + 4

3 = 7 - 4

0 = 5 - 5

5 = 7 - 2

3 + 7 = 10

3 + 6 = 9

10 - 8 = 2

9 = 5 + 4

9 = 2 + 7

9 = 8 + 1

## Writing Equations

Put in + or - to make equations.

7 + 3 = 10
10 − 5 = 5
10 − 8 = 2
6 + 4 = 10
1 + 9 = 10

5 + 2 = 7
7 + 0 = 7
3 + 6 = 9
8 − 5 = 3
2 + 7 = 9

Write 4 equations using the numbers:

| 5, 4, 9 | 7, 3, 10 | 3, 5, 8 |
|---|---|---|
| 5 + 4 = 9 | 7 + 3 = 10 | 3 + 5 = 8 |
| 4 + 5 = 9 | 3 + 7 = 10 | 5 + 3 = 8 |
| 9 - 4 = 5 | 10 − 7 = 3 | 8 − 3 = 5 |
| 9 - 5 = 4 | 10 − 3 = 7 | 8 − 5 = 3 |

## II-5. Missing addends

**Objective:** To learn a way of finding missing addends using the doing-undoing principle.

**Vocabulary:** (Review) addend.

**Materials:** Crayons, paper bag, children's set materials.

**Suggested Procedure:**

To find missing addends is to use the "doing-undoing" principle, and the commutative property.

Put 3 crayons into a paper bag without letting children see how many there are. Have a child put 5 more crayons into the bag. Discuss with the children how they could find out how many crayons were in the bag at first. Empty the bag and have the crayons counted.
Write:

$$\_\_\_ + 5 = 8.$$

Ask what would undo the adding of 5.
(Subtracting 5.)

Write:

$$8 - 5 = \_\_\_ .$$

Show that 3 completes both equations.

Next put 4 crayons into a paper bag. Without letting children count them, put 5 more crayons into the bag. Have someone count the number of members in the union, and write:

$$4 + \_\_\_ = 9.$$

Next, recall the idea that joining is commutative; i.e., if you had started with the second set and then had joined the set of 4 to it, the union would still have had 9 members. Write:

$$\_\_\_ + 4 = 9.$$

Ask what would undo the adding of 4. (Subtracting 4.)
Write:

$$9 - 4 = 5.$$

Show that 5 completes both the equations.

Provide for more practice of this kind for your class--perhaps have children work together using manipulative materials; one child counts out a set, the second joins to it a set the number of which only the second child knows, and the first child must determine the number of the set joined.

After much of this kind of practice, help children to generalize (if they don't make the generalization) that to find a missing addend the other addend is subtracted from the sum. They need not be required to write the series of 3 equations in order to solve the problem; (i.e., 4 + ___ = 9, ___ + 4 = 9, 9 - 4 = ___.)

With most classes, subtraction problems of the type: 7 - ___ = 6 will be approached by thinking of a set of 7 as partitioned into sets of 6 and 1 and realizing that if one of the sets remains it must be the other that was removed. However, more able children may enjoy solving the problem by undoing and commutativity. They may be led to see that if

7 - ___ = 6, then

6 + ___ = 7 (undoing)

___ + 6 = 7 (commutativity)

7 - 6 = 1 (undoing)

- Present orally problems such as the following, and have children give the appropriate equations:

    1. Alice's mother had more pennies in her purse than she wanted to carry. She put 6 of them into a little box. Alice's father saw the box and put all the pennies from his pocket into the box, too. He told Alice she could have all the pennies. When she counted them she found that there were ten. How many pennies had her father put into the box?

       6 + ___ = 10

2. Ray knew that he had 6 marbles. Billy came over to play and some of his marbles got mixed up with Ray's. Billy wanted to get back as many marbles as he had brought. When they counted the marbles, they found 9. How many of Billy's marbles were mixed with Ray's?

$$6 + \_\_\_ = 9$$

3. Sheldon's mother put 10 cookies on a plate. Sheldon ate some of them but he forgot how many. When they counted the cookies that were left, they found 5. How many had been eaten?

$$10 - \_\_\_ = 5$$

Pupil's book, page 77: Discuss the first set of equations with the children, to make sure they see the use of commutativity and undoing. They may see rather quickly that the same answer goes in all three blanks of a set. At the bottom of the page, they may or may not wish to write 2 more equations in order to fill the blanks.

Pupil's book, page 78: ★ May be used with more able pupils. At the top of the page are two equations using the numbers in each problem at the bottom of the page. One of the 2 equations contains the information needed to fill the blanks. The child will have to decide which information is helpful to him in order to fill in the blanks at the bottom.

## Writing Equations

Fill in the blanks.

3 + 5 = 8
5 + 3 = 8
8 - 3 = 5

2 + 7 = 9
7 + 2 = 9
9 - 2 = 7

4 + 6 = 10
6 + 4 = 10
10 - 4 = 6

6 + 3 = 9
3 + 6 = 9
9 - 6 = 3

7 + 3 = 10
3 + 7 = 10
10 - 7 = 3

5 + 2 = 7
2 + 5 = 7
7 - 5 = 2

Write 2 more equations if you need them.

2 + 6 = 8
6 + 2 = 8
8 - 2 = 6

5 + 5 = 10
10 - 5 = 5

1 + 8 = 9
8 - 1 = 9
9 - 1 = 8

4 + 3 = 7
3 + 4 = 7
7 - 4 = 3

3 + 7 = 10
7 + 3 = 10
10 - 3 = 7

2 + 8 = 10
8 + 2 = 10
10 - 2 = 8

★ Fill in the blanks at the bottom of the page.

Some of the equations at the top of the page can be used to help you.

42 - 16 = 26

16 + 42 = 58

100 + 81 = 181

100 - 81 = 19

119 + 375 = 495

119 + 256 = 375

98 + 56 = 154

98 - 56 = 42

139 - 41 = 98

139 + 41 = 180

432 + 750 = 1182

318 + 432 = 750

79 + 94 = 173

15 + 79 = 94

35 - 26 = 9

26 + 35 = 61

---

56 + 42 = 98

41 + 98 = 139

26 + 16 = 42

750 - 318 = 432

61 - 35 = 26

81 + 19 = 100

79 + 15 = 94

375 - 256 = 119

## II-6. Problem solving

**Objective:** To develop skill in using equations for solving story problems.

**Vocabulary:** (No new words.)

**Materials:** Story problems printed on tagboard. (This makes it possible to keep the problems and refer to them later.) Flannel board and sets of symbol cards and counters for children. Charts and numeral cards as specified in the Suggested Procedure.

**Suggested Procedure:**

We suggest that charts such as the ones shown below be used in the development of this section. These charts may be drawn on the chalkboard or made on tagboard. If the latter is prepared, then numeral cards might be used to record answers to questions about problems that are discussed.

A. Joining one set to another set.
Number of one set: _____
Number of other set: _____
Number of union: _____
Equation: ____________________

B. Removing from a set one of its subsets.

Number of starting set: _____
Number of subset removed: _____
Number of subset remaining: _____

Equation: ____________________

C. Partitioning a set into two subsets.

Number of set partitioned: _____

Number of one subset: _____

Number of other subset: _____

Equation: ____________________

Display the following story problem and ask a child to read it.

Nine children were playing.

Four children went home.

How many children were still playing?

Ask the children which one of these three things is illustrated by the problem:

A. Joining one set to another set.
B. Removing from a set one of its subsets.
C. Partitioning a set into two subsets.

Identify chart B and use it for further discussion of the problem. Lead to the observation that when a subset is removed from a set, we are concerned with the starting set, the subset removed, and the subset remaining. We know the number of members for two of these sets and must find the number of members for the third set.

Help children to understand that in this problem we are told the number of the starting set ("Nine children were playing.") and the number of the subset removed ("Four children went home."). We are asked to identify the number of the subset remaining ("How many children were still playing?"). Record 9 and 4 in the chart, and leave the other space blank. Ask for an equation that we may use with this problem: $9 - 4 = \_\_\_\_$, or $9 - 4 = \square$. Ask for the result of subtracting 4 from 9 (i.e., 5); record this in the chart, and interpret it in relation to the problem: Five children were still playing.

The charts you have prepared may be used in a similar way in the discussion of other story problems which follow. However, be flexible in the way in which you use the charts. They are not intended to provide a standard "system" for problem solving work. Rather they represent one strategy that may be helpful on many occasions. Look for, and use to advantage, various approaches that children may take with the same story problem.

- Display the following story and have it read.

Mother needs 8 eggs.

She has only 5 eggs.

How many eggs must she get?

Proceed with a discussion of the problem and its solution, using the charts. Some children may think in terms of chart A; others, in terms of chart C. Good! Use both ways of approaching the problem.

- The following problem, which should be displayed on tagboard, will need to be approached somewhat differently since it serves a somewhat different purpose.

John had _____ marbles.

David gave _____ marbles to John.

How many marbles did John have then?

Read the story, pausing and indicating the blank as it fits into the problem. Mention that later we will write a numeral in each blank, but now we want to think about the problem without any numbers.

Ask children to relate the problem to one of the three ideas they have been working with:

A. Joining one set to another set.

B. Removing from a set one of its subsets.

C. Partitioning a set into two subsets.

Emphasize that we cannot answer the question asked without knowing the number of marbles John had at first and the number of marbles David gave to John.

Substitute this sentence for the last one in the original story problem:

Find how many marbles John then had.

Help children to see that although the form of the sentence is changed, the essence of the problem remains the same. The same "question" is asked, but not explicitly as a question.

Now select two children and designate one to be John and the other to be David. Suggest that they choose as many marbles (use disks or other materials for marbles) as they wish John to have and a set of marbles that David will give to John. Complete the first two sentences in the story by writing the numerals to show how many marbles, e. g., 4, 6.

Have the sets joined as was suggested. Ask the children to show the equation, using their individual cards. ($4 + 6 = \square$ .) Suggest that a child write the equation on the board so each child may check to see if his own equation is right. Also use the appropriate chart (B). Have the correct numeral written in the box ($4 + 6 = \boxed{10}$). Ask one child to read the question asked in the problem and ask another child to give a sentence that answers the question. (How many marbles does John have? John has 10 marbles.)

Suggest to the children that they think of some story problems; choose four to write on the board. It may be necessary to clarify and reword some of the statements. Number the problems 1, 2, 3, and 4.

Pupil's book, page 79:

Direct the children to read problem number 1. Discuss procedures that may be used to solve the problem. Ask the children to write the equation on the dark line toward the top of the box numbered 1, and to write on the dotted lines a sentence for the problem.

## Solving Problems

1. ______________________

2. ______________________

3. ______________________

4. ______________________

- In this lesson we are interested in focusing attention on the problem form by relating each problem to another problem already solved. For this reason the numbers have been deliberately omitted when the new problems are first presented.

Display the following story problems which were printed on tagboard earlier in this section. Review the problems and write the equations used to solve them.

1. John had __4__ marbles.

   David gave __6__ marbles to John.

   How many marbles does John have?

2. Nine children were playing.

   Four children went home.

   How many children were still playing?

3. Mother needs 8 eggs.

   She has only 5 eggs.

   How many eggs must she get?

I am going to read some story problems. When I come to a place where we need to use a number, I am going to show a blank card instead of telling you the number. Think about the story. What are you asked to find? What do you know that will help you solve the problem? When I have finished reading a story, tell which of these problems we have already solved will help you in solving the new problem.

Read the problem, showing the blank cards as you pause whenever a number is needed.

How many beach balls does the family have?

John had ____ old beach balls.

Mother bought ____ new beach balls for the family.

(This is like the first problem. We may join two sets and find how many members all together.)

Jack has ____ cents.

He wants to buy a pencil which costs _____ cents.

How many cents will he need to get from Mother?

(This is like the third problem. We may think of a set to join with the one set in order to have a set with more members.)

There were ____ clams in a pail.

Mother took ____ clams from the pail.

How many clams are in the pail?

(This is like the second problem. We may think of removing a subset from the starting set. We are to find how many remain in the first set.)

Conclude the activity by asking the children to suggest numbers to be used in the problems. For example:

How many old beach balls shall we say John had? (5.)

How many new beach balls did Mother buy? (2.)

Ask a child to show the equation and give the answer to the problem.

(5 + 2 = __7__. The family had seven beach balls.)

- Write the following story on the chalkboard:

  Jim has 8 balloons.

  Ellen has 5 balloons.

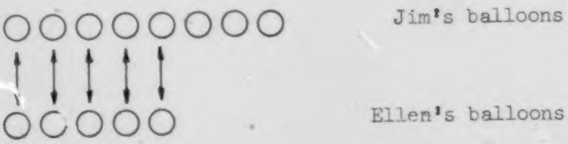
  How many more balloons does Jim have than Ellen has?

The children may say, "Jim has 3 more balloons than Ellen has". Our aim is to help them discover and be able to make generalizations from the set operation that leads to the answer. (When asked how they know that Jim has 3 more than Ellen the child may say, "You take the 5 balloons from the 8 balloons and you have 3 balloons left". It is safe to assume that for many children at this stage it means "taking Ellen's 5 balloons from Jim's 8 balloons".)

Ask a child to use felt disks on a flannel board to show sets equivalent to Jim's set of balloons and Ellen's set of balloons.

How can we compare the two sets?
(We can pair the members of the sets.)

A child should then be asked to pair the members of one set with the members of the other set.



Discuss the action and help the children visualize that if you remove 5 of Jim's balloons to match the set of balloons Ellen has, the the number of balloons that are left in Jim's set is the number of balloons which Jim has more than Ellen has. The equation could be written 8 - 5 = ___. Discussion should bring out that 8 is the number of members in the set of balloons that Jim has, 5 is the number of the subset of Jim's balloons which matches the set of balloons that

Ellen has, and 3 is the number of members in the subset of Jim's balloons which remains after the subset of 5 has been removed.

The equation might be written 5 + 3 = 8.

In this case the discussion should lead to understanding that 5 is the number of the subset of Jim's balloons which matches the set of Ellen's balloons, the number 3 is the number of members in the subset of balloons which is joined with the subset of 5 balloons to make a set of 8 balloons, and 8 is the number of balloons in the set of balloons which Jim had to start with.

- Leave the first story problem and the equations on the board. Write the following story problem on the chalkboard:

Bob has 7 darts.

Sam has 5 darts.

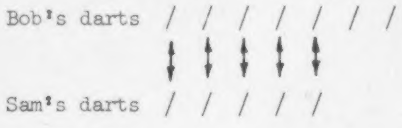
How many fewer darts does Sam have?

Discuss the question, the information given, and how the problem might be solved.

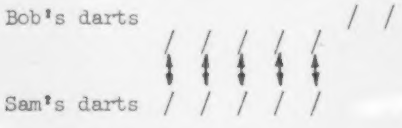
Use objects to show sets which are equivalent to Bob's set of darts and Sam's set of darts.

Compare the sets and write the equations.

For example, 7 = 5 + ___. You can pair Sam's darts with Bob's darts for all but 2 of Bob's darts.



7 - 5 = ___. You can pair Bob's darts with Sam's darts for all but 2 of Bob's darts.



The children might say: Sam has 2 fewer darts because he lacks 2 of having enough to be able to pair his darts with Bob's darts. Sam has 2 fewer because he would have to get 2 more to be able to pair his darts with Bob's darts.

Erase the words "fewer" and "Sam" in the last sentence of the problem and complete the question to read, "How many more darts does Bob have?" Discuss the question, bringing out the ideas that if Sam has 2 fewer, then Bob has 2 more. If Bob has 2 more, then Sam has 2 fewer.

Go back to the story about the balloons.

> If Jim has 3 more balloons than Ellen has, how many fewer balloons does Ellen have than Jim has?

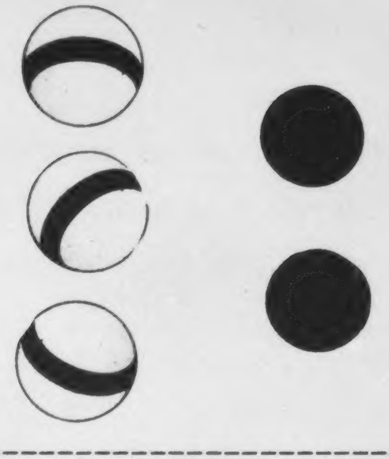
Pupil's book, pages 80, 81, 82, and 83:

Read the problems for the class before expecting the children to solve the problems independently.

Further Activities:

1. The children should be encouraged to write original story problems and to draw pictures and write equations for them.
2. Children might be given an equation and asked to think about a story to go with the equation. Some children will enjoy writing the story and sharing it with the class.
3. During short "break" periods tell short, story problems to the children. Let one child write the equation, another child complete it, a third child repeat the question posed by the problem, and a fourth child give the answer.

## Using Equations



Tom had 3 red balls.

He bought 2 blue balls.

Then how many balls did he have?

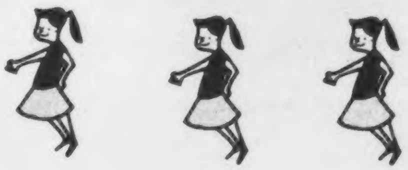
3 + 2 = 5

Tom had 5 balls.

Susan was playing house.

Betty and Linda came to play with her.

How many girls were playing house?



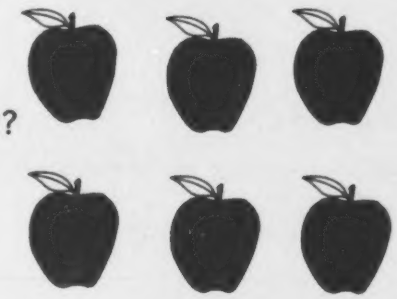
1 + 2 = 3

Three girls were playing house.

Six ripe apples were on a tree.

he birds ate two of them.

How many of the apples are still on the tree?

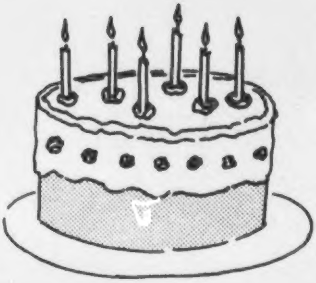


6 − 2 = 4

4 apples are still on the tree.

## Using Equations

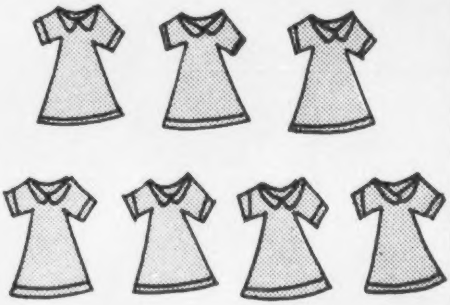
Bill's birthday cake had 6 lighted candles.
Bill blew out 4 of them.
How many candles were burning then?



6 - 4 = 2

2 candles were burning.

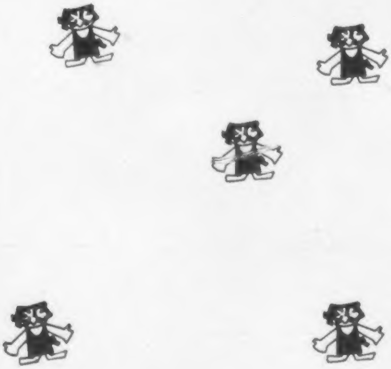
Judy has 7 dresses.
Only one dress is blue.
How many dresses are not blue?



7 - 1 = 6

6 dresses are not blue.

Beth has 5 dolls.
Jean has 8 dolls.
How many dolls must Beth get to
have as many as Jean?



5 + 3 = 8

Beth must get 3 dolls.

## Solving Problems

Peggy has 3 cookies.
Sarah has 6 cookies.
How many more cookies
does Sarah have than Peggy?

Draw Peggy's cookies.

X X X

Draw Sarah's cookies.

X X X
X X X

3 + 3 = 6

Sarah has 3 more cookies.

---

Sue has 9 cents.
John has 5 cents.
How many fewer cents does
John have than Sue?

Draw Sue's cents.

X X X
X X X
X X X

Draw John's cents.

X X X
X X

9 − 5 = 4

John has 4 fewer cents.

## Solving Problems

Ellen has 8 jacks.

Beth has 4 jacks.

How many jacks must Beth get

to have as many as Ellen?

Draw Ellen's jacks.

X X X X

X X X X

Draw Beth's jacks.

X X X X

4 + 4 = 8

Beth must get 4 jacks.

---

Bill has 9 balls.

John has 3 balls.

How many more balls

does Bill have than John?

Draw Bill's balls.

X X X

X X X

X X X

Draw John's balls.

X X X

9 - 3 = 6

Bill has 6 more balls.

## II-7. Comparing numbers: Review and extension

Objective: To review the relations greater than and and less than and the symbols for them, and to compare numbers expressed as sums or differences

Vocabulary: (Review) greater than (>), less than (<)

Materials: Flannel board materials, children's manipulative materials.

Suggested Procedure:

Put 9 objects on the left side of the flannel board. On the right side put 8 objects. Ask which set has more members. (You may wish to pair members of the 2 sets to show clearly that the set on the left has 1 more member than that on the right.) On the chalkboard, write

$$9 \quad 8$$

> What can we say about these numbers? Are they equal? (No.) Is one greater than the other? (Yes.) Tell in a sentence, which number is greater than the other. (Nine is greater than eight.). Do you remember a symbol we can use to say "is greater than"? (>)

If children have difficulty in remembering which symbol to write, use the > symbol and continue.

> Is there another sentence we can use so that we can name the number 8 first? (Eight is less than nine.) There is a different symbol that means "is less than". (<)

On the chalkboard, write:

$$8 < 9$$

> The symbol looks a little like the head of an arrow. Which numeral does it point to, in both sentences? (That for the smaller number, 8.) Whether we start with the smaller number or the greater, the pointed end of the arrow is always toward the numeral for the smaller number.

On the chalkboard, write several more pairs: e.g., 7 2; 6 3; 7 7; 4 10. For each pair of numbers have a child put the symbol which shows which of these numbers is the greater. They should notice that for the pair 7 7 the equals sign is needed.

- Separate the 9 objects on the flannel board into two sets one of 4 members and one of 5. Write:

$$4 \quad 5$$

Ask what sign would be used between these numerals when you join the set of 4 and the set of 5. Insert the plus sign. Discuss the fact that the symbols $4 + 5$ and 9 name the same number.

If we know that 9 is greater than 8, we can say that $4 + 5$ is greater than 8 because 9 and $4 + 5$ are names for the same number.

Write:

$$(4 + 5) > 8$$

Have children think of other names for the number 9 and write statements such as:

$$(6 + 3) > 8$$
$$8 < (7 + 2)$$
$$(1 + 8) > 8$$
$$8 < (10 - 1)$$

Use many more examples of this sort, including some involving a subtraction expression. If necessary, use flannel board or manipulative materials to illustrate.

Pupil's book, page 84:

Children may need to write $3 + 2$ as $5$ and $5 > 4$ before they can write $(3 + 2) > 4$. If this is necessary, suggest that they do so.

After checking to make sure that the use of inequality symbols is understood, show that on both sides of the symbols other names for numbers may be used; e.g., $(2 + 5) > (10 - 4)$.

Pupil's book, page 85:

May be used as was page 84.

★ Pupil's book, page 86:

(Optional) Children have to decide whether to add or subtract in order to make a true statement. In some instances, symbols will have to be inserted at two places.

## Comparing Numbers

$6 > 4$ Six is greater than four.

$4 < 6$ Four is less than six.

Add or subtract. Put in >, < or =.

| | | |
|---|---|---|
| $3 + 2 > 4$ | $6 = 3 + 3$ | $10 - 8 < 4$ |
| $6 - 4 > 1$ | $6 > 3 + 1$ | $6 + 3 < 10$ |
| $3 + 3 < 7$ | $5 < 4 + 2$ | $5 > 7 - 4$ |
| $2 + 1 > 0$ | $8 < 5 + 4$ | $4 = 9 - 5$ |
| $4 + 2 > 5$ | $10 > 6 + 3$ | $8 = 5 + 3$ |
| $2 + 2 < 6$ | $4 < 3 + 4$ | $7 + 3 > 9$ |

## Comparing Numbers

7 > 3     5 < 9     3 + 2 = 4 + 1

---

Put in <, >, or = .

| | | |
|---|---|---|
| 6 + 1 = 1 + 6 | 6 + 0 > 0 + 0 | 5 + 3 = 1 + 7 |
| 3 + 4 > 2 + 4 | 7 - 5 = 1 + 1 | 4 + 6 > 7 + 2 |
| 5 + 2 = 7 + 0 | 3 + 4 = 2 + 5 | 8 - 7 > 9 - 9 |
| 2 + 2 < 3 + 3 | 7 - 0 > 6 - 4 | 5 + 0 > 6 - 3 |
| 1 + 5 < 6 + 1 | 6 + 3 > 6 - 3 | 7 + 2 > 5 + 3 |
| 2 + 3 = 5 + 0 | 4 + 5 = 7 + 2 | 2 + 6 < 3 + 7 |
| 4 + 3 > 1 + 3 | 10 - 8 < 6 + 2 | 4 + 4 = 3 + 5 |

## Comparing Numbers

Put in + or -.

$10 - 5 > 6 - 4$

$3 + 7 = 8 + 2$

$5 + 4 > 10 - 2$

$8 - 4 < 10 - 4$

$5 + 3 = 10 - 2$

$7 + 3 > 5 + 4$

$6 - 2 > 10 - 7$

$5 - 5 = 6 - 6$

$9 + 6 > 5 + 4$

$3 + 6 < 9 + 1$

$10 - 3 < 4 + 4$

$9 - 2 < 6 + 4$

$7 + 2 > 3 + 5$

$8 - 1 < 2 + 6$

$6 + 1 > 1 + 5$

$4 + 3 = 5 + 2$

$1 + 9 > 7 - 3$

$2 + 8 < 9 + 2$

## II-8. Using the associative and commutative properties

**Objective:** To understand and use the associative property of addition, and then use it together with the commutative property.

**Vocabulary:** Associative, parentheses, column.

**Materials:** Materials for flannel board.

**Suggested Procedure:**

### The associative property of addition

Let us think about the sum of three numbers

$$5 + 2 + 3$$

that I have written on the chalkboard. How can we compute the sum of these numbers? (Suppose they suggest adding 2 to 5.) What would that give? (7.) And then what would we do? (Add 3 to 7.) What would we have then? (10.) We can write

$$\begin{aligned} 5 + 2 + 3 &= 7 + 3 \\ &= 10 \end{aligned}$$

Is there another way of adding these three numbers? Suppose we first add the 2 and the 3. What will this give? (5.) Now we add this 5 to the 5 we already had. What do we get then? (10.) So we can also write

$$\begin{aligned} 5 + 2 + 3 &= 5 + 5 \\ &= 10 \end{aligned}$$

can't we?

In the first way of adding, we found that

$$5 + 2 = 7$$

and then we found that

$$7 + 3 = 10.$$

In the second way of adding, we found that

$$2 + 3 = 5$$

and then we found that

$$5 + 5 = 10.$$

Which way do you think was easier? (The first, the second, neither, etc. It might be brought out that in this particular example the addition facts used in the second way are perhaps simpler and more familiar.)

In the sum

$$5 + 2 + 3$$

when we want to add the 2 and the 3 first, we have a special way of showing this. Do you know what it is? We put parentheses around the 2 and the 3, like this:

$$5 + (2 + 3)$$

When we want to add the 5 and the 2 first, how can we show this? Like this:

$$(5 + 2) + 3.$$

When we finish adding all three of these numbers, do we get the same sum no matter which numbers---the 2 and the 3, or the 5 and the 2--we add first? (Yes.) What sum do we get in both cases? (10.)

When we put parentheses around the 2 and the 3 like this

$$5 + (2 + 3)$$

we say that the 2 and the 3 are associated or grouped together. "Associate" is a big word. Can you think of any other examples of associations, perhaps where people, instead of numbers, are associated or grouped together? (Parent-teacher association, building and loan association, etc.) Review again why is it that we associate the 2 and the 3 in

$$5 + (2 + 3).$$

(To show that the 2 and the 3 are to be added first.)

- Let's practice with another example:

$$6 + 2 + 1.$$

If we put parentheses in like this

$$6 + (2 + 1),$$

which two numbers are associated? (The 2 and the 1.) Which two numbers do we add first? (The 2 and the 1.) So we can write

$$\begin{aligned} 6 + (2 + 1) &= 6 + 3 \\ &= 9. \end{aligned}$$

After adding all three numbers this way, what sum do we get? (9.)

If instead we now put the parentheses in like this

$$(6 + 2) + 1,$$

which two numbers are associated? (The 6 and the 2.) Which two numbers do we add first? (The 6 and the 2.) So this time we write

$$\begin{aligned} (6 + 2) + 1 &= 8 + 1 \\ &= 9. \end{aligned}$$

After adding all three numbers this way, what sum do we get? (9.) Is this the same sum we got before? (Yes.)

Do you think that in adding any three numbers, it makes any difference whether we begin by adding the first two or begin by adding the last two? (No. It doesn't matter which we add first.) This is called the associative property of addition, because in adding three numbers it doesn't matter whether we associate the first two or the last two of these numbers. That is, the associative property of addition says that in adding three numbers we get the same sum no matter whether we begin by adding the first two or begin by adding the last two numbers.

In writing a sum of three numbers we usually leave parentheses off altogether, like this

$$3 + 4 + 1.$$

We can do this, because the associative property of addition tells us that the sum will be the same no matter whether we put parentheses like this

$$3 + (4 + 1)$$

or like this

$$(3 + 4) + 1.$$

Pupil's book, page 87: You will undoubtedly find it helpful to work with the class for the first few examples.

Pupil's book, page 88: Pupils are to insert parentheses and fill in the blanks. They must first decide, from the position of the blanks, which two numbers are to be added first.

## Adding Three Numbers

Fill in the blanks.

| | |
|---|---|
| (2 + 4) + 1 = 6 + 1<br>= 7 | (4 + 5) + 1 = 9 + 1<br>= 10 |
| (3 + 1) + 2 = 4 + 2<br>= 6 | 2 + (4 + 1) = 2 + 5<br>= 7 |
| 4 + (5 + 1) = 4 + 6<br>= 10 | 5 + (3 + 2) = 5 + 5<br>= 10 |
| 2 + (4 + 4) = 2 + 8<br>= 10 | 3 + (4 + 2) = 3 + 6<br>= 9 |
| (5 + 3) + 2 = 8 + 2<br>= 10 | (1 + 7) + 2 = 8 + 2<br>= 10 |

## Adding Three Numbers

Use ( ). Fill in the blanks.

$(3 + 4) + 2 = \underline{\ 7\ } + 2$

$= \underline{\ 9\ }$

$2 + (5 + 3) = 2 + \underline{\ 8\ }$

$= \underline{\ 10\ }$

$(2 + 1) + 6 = \underline{\ 3\ } + 6$

$= \underline{\ 9\ }$

$(5 + 1) + 3 = \underline{\ 6\ } + 3$

$= \underline{\ 9\ }$

$5 + (1 + 3) = 5 + \underline{\ 4\ }$

$= \underline{\ 9\ }$

$(1 + 7) + 2 = \underline{\ 8\ } + 2$

$= \underline{\ 10\ }$

$(2 + 5) + 3 = \underline{\ 7\ } + 3$

$= \underline{\ 10\ }$

$3 + (4 + 2) = 3 + \underline{\ 6\ }$

$= \underline{\ 9\ }$

$1 + (7 + 2) = 1 + \underline{\ 9\ }$

$= \underline{\ 10\ }$

$2 + (1 + 6) = 2 + \underline{\ 7\ }$

$= \underline{\ 9\ }$

- **Using the commutative and associative properties of addition**

After the children understand the associative property, proceed to some problems involving both the associative and the commutative properties. On the chalkboard write:

$$2 + 3 + 2 =$$

> As you know, we can add the first two numbers and then add the last to their sum.

Write:

$$\begin{aligned}(2 + 3) + 2 &= 5 + 2\\ &= 7\end{aligned}$$

> We can also add the sum of the last two numbers to the first.

Under the above equation, write:

$$\begin{aligned}2 + (3 + 2) &= 2 + 5\\ &= 7\end{aligned}$$

> Would it be possible to add these three numbers in any other order?

A child will probably suggest adding 2 and 2 first. Leave space below the second equation and write:

$$(2 + 2) + 3 =$$

> How do we know we can do this?

If there is no response, write, between the second and third equations:

$$2 + (2 + 3)$$

Point out that $2 + 3 = 3 + 2$, using the expression "commutative property" if you have introduced it, and then that the associative property lets you add 2 and 2 first.

- Column form

Explain that we often write problems in an "up and down", or "column" form, rather than across. When we do this we do not use plus signs between numerals. Write:

$$\begin{array}{r} 3 \\ 1 \\ \underline{5} \end{array}$$

Let's add 3 and 1 and 5 by starting at the top of the column. $3 + 1 = 4$. Next we add 5 to the 4. What is the sum? Write it here.

Write:

$$\begin{array}{r} 3 \\ 1 \\ \underline{5} \\ 9 \end{array}$$

Now let's start at the bottom of the column. $5 + 1 = 6$. Add 3 to 6. What is the sum? The sum is the same no matter in which order we add.

Let us look at this more closely. When we start at the top of the column, what do we do first? We take 3 and add 1, getting

$$3 + 1 = 4.$$

Next, what do we do? We add 5, getting

$$\begin{aligned}(3 + 1) + 5 &= 4 + 5 \\ &= 9\end{aligned}$$

When we start at the bottom of the column, what do we do first? We take 5 and add 1, getting

$$\begin{aligned}(5 + 1) + 3 &= 6 + 3 \\ &= 9.\end{aligned}$$

So when we start at the top of the column, we get 9 as the sum

$$(3 + 1) + 5,$$

and when we start at the bottom, we get 9 as the sum

$$(5 + 1) + 3.$$

★ (Optional) Could we have used the associative and commutative properties to show that

$$(3 + 1) + 5 = (5 + 1) + 3?$$

Let's see. (You will have to take the lead here):

$$\begin{aligned}(3 + 1) + 5 &= 3 + (1 + 5) && \text{(associative property)}\\ &= 3 + (5 + 1) && \text{(commutative property)}\\ &= (5 + 1) + 3 && \text{(commutative property)}\end{aligned}$$

Would this have worked for any other numbers just as well as for 3, 1, and 5? (Yes)

- Give children practice at the chalkboard in writing problems in column form and in finding the sums. It is important that they form the habit of keeping the column straight. Have them add, starting at the top and writing the sum. Then have them add starting at the bottom.

  If they do not find that the sum is the same, they should be aware of the fact that they must have made a mistake. (Many children are eager to correct their own mistakes.)

Pupil's book, page 89: Tell children to start at the top of each column. When they have written all the answers, they should go back to the first problem, start at the bottom of each column, and see whether they find the sum to be the same.

## Adding Three Numbers

Find the sums. Start at the top. Write your answer.

| | | | | | |
|---|---|---|---|---|---|
| 3 | 6 | 4 | 7 | 3 | 3 |
| 5 | 1 | 1 | 1 | 7 | 5 |
| 1 | 2 | 5 | 2 | 0 | 2 |
| 9 | 9 | 10 | 10 | 10 | 10 |

---

| | | | | | |
|---|---|---|---|---|---|
| 2 | 1 | 3 | 2 | 8 | 4 |
| 4 | 7 | 2 | 4 | 1 | 5 |
| 3 | 2 | 4 | 4 | 1 | 1 |
| 9 | 10 | 9 | 10 | 10 | 10 |

---

| | | | | | |
|---|---|---|---|---|---|
| 3 | 2 | 2 | 1 | 2 | 1 |
| 3 | 7 | 3 | 6 | 2 | 4 |
| 3 | 1 | 5 | 3 | 3 | 3 |
| 9 | 10 | 10 | 10 | 7 | 8 |

---

Think about the sums again.

Start at the bottom this time.

## II-9. Developing problem solving skills

**Objective:** To develop skill in solving problems in physical situations.

**Vocabulary:** (No new words.)

**Materials:** Story problems printed on tagboard (so that they may be saved and referred to later.) Individual sets of counters, paper and pencils and blank cards.

**Suggested Procedure:**

Display the following story printed on tagboard:

> There were 8 cookies on a plate.
>
> Jack ate 5 of the cookies.
>
> Then how many of these cookies did he not eat?

Ask the children what they think about first after they read a story problem. Answers they give should give guidance to the questions you ask in further discussion of the problem. This discussion might proceed as follows:

> What does this story ask us to find?
> (How many cookies were on the plate after Jack ate some.)
>
> What does the story tell us?
>
> (It tells us how many cookies were on the plate at first and how many Jack ate.)

Direct the children to use materials on their desks to make sets equivalent to those described in the story and to find the number that will be used in answering the question that is asked. Have the equation written on the board. 8 - 5 = __ and then write 3 in the blank.

> Is this equation the answer to our problem?

Review the idea that this equation helps us answer the question asked in the problem but the number 8 in the equation does not mean 8 cookies, neither does the number 3. We have to state the answer to the problem in the terms of the question asked. (There were 3 cookies left on the plate.)

- Write the following story on the chalkboard:

> There were 9 trees in Jim's yard.
> Four of the trees died.
> How many of these trees are still alive?

Discuss the question asked and the facts known.

> Is this problem like the last one in any way? (Yes. We removed a subset the same as we did in the last problem.)
>
> Does knowing this make it easier to write an equation that will help us solve the problem? (Yes, because it will be the same kind of equation. It will have different numbers, though.)

Ask one child to write the equation, another to read the question and a third child to give the answer. (There are 5 trees still alive.)

- Display the following story written on tagboard:

> Mother had 5 purses.
> She bought another purse.
> How many purses does she now have?

Discuss the problem, asking about information given and information asked to be given. Indicate the first story printed on tagboard and ask if that problem, and the way they solved it, helps answer the question asked in the new story. (Since the new story is about two sets of purses, and since the sets must be joined to find out how many there are altogether, you cannot use the same method of removing a subset from a set.)

Direct the children to show with materials on their desks a set which is equivalent to the set of 5 purses and a set equivalent to the set of 1 purse. Ask them how the sets should be joined (1 purse is joined to the 5 purses). Then they write on their paper an equation which shows the action. The equation should also be written on the board so the children may check their work (5 + 1 = _6_). The question should be read by a child and the answer given. (Now Mother has 6 purses.)

- Read the following story problems to the children. For each problem have the set operations (joining or removing) and the question identified by a child. Compare the problem with the two examples on the tagboard and determine which equation should be used to find the answer. The answer sentence should then be given.

1. Beth and Mark went fishing.

   They put all the fish they caught in the same basket.
   Beth caught 3 fish and Mark caught 4. fish.
   How many fish did they put into the basket?
   (They put 7 fish in the basket.)

2. The next day Mark caught 5 fish but 1 fish got away.
   How many fish did Mark have then?
   (Mark had 4 fish.)

3. Mark had the 4 fish in the basket when he met Mrs. Brown.
He gave 2 of these fish to Mrs. Brown.
How many fish did Mark have then?
(Mark had 2 fish left.)

4. Mark gave the 2 fish that he still had to his mother.
Father gave 5 fish to Mother.
How many fish did Mother get from Mark and Father?
(Mother got 7 fish from Mark and Father.)

5. Mother needs 8 fish to cook for supper.
She got 7 fish from Mark and Father.
Beth caught 1 fish and gave to Mother.
Does Mother have enough fish to cook for supper?
(Yes, because 7 fish and 1 more fish are 8 fish.)

- Write the following story on the chalkboard:

  Jill had 3 dolls.
  She got some dolls for her birthday.
  Then, how many dolls did she have?

  Discuss the solution to the problem. Observe that it isn't possible to answer the question without knowing how many dolls Jill got for her birthday. The story doesn't tell the fact. They do know, however, that Jill would have more than 3 dolls unless she gave some away.

- Too much information

  Write this story on the chalkboard:

  John had 3 red boats.
  He had 4 red tops.
  Mother gave John another red boat.
  Father gave John a red top.
  How many red boats did John have then?

  Have the entire story problem read. Discuss the question asked in the problem. Encourage the children to discover that not all the facts given are related to the solution of the problem.

  Reread each sentence and help the children determine whether or not that sentence contains information they will need to solve the problem. The sentences which are related to the solution of the problem should be underlined. Conclude by answering the question, "How many red boats did John have then?" (John had 4 red boats.)

  Read the following problems and have the children determine whether there is more than enough information, too little information, or exactly enough information to solve the problem.

  1. Sue read 2 pages this morning.
     This afternoon she read some more pages.
     How many pages did she read?

(You can't answer the question asked in this problem but you do know that the answer is more than 2 pages.)

2. Each of 3 boys had an apple.
   Two boys did not have apples.
   How many boys had apples?

(You do not need the information about the 2 boys who did not have apples. The answer is that three boys had apples.)

3. Bill spent 3¢ for candy and 5¢ for gum.
   How much did Bill spend for candy and gum.

(Bill spent 8¢ for candy and gum. There is exactly enough information.)

4. David had 24 toys.
   Only 7 of the toys were boats.
   Pat had not boats so David gave Pat 2 boats.
   How many boats did Pat have then?

(Pat had 2 boats. You do not need to remember how many toys David had how how many of his toys were boats. You only have to remember that Pat had no boats before David gave 2 boats to him.)

Further Activities

1. Continue to use oral problems during short "break" periods. Omit needed data in some problems. Provided unneeded data in some problems. Encourage the children to write story problems during free time activities. Children enjoy illustrating the stories they write.
2. Suggest that each child write a story problem. Let the children exchange papers and solve the problem on some other child's paper. They should be encouraged to use both the equation and the sentence which answers the question.

Pupil's book, pages 90, 91, 92, 93, and 94:
May be used now or over a period of time.

## Solving Problems

Write the equation on the dark line.

Write the answer on the dotted lines.

1. Mary saw 4 cookies on a plate.
   Her mother said that there should be 9 cookies on the plate.
   How many more cookies must Mary get?

   9 – 4 = 5

   Mary must get 5 more cookies.

2. Linda and Betty have 8 dolls.
   Linda has 5 dolls.
   How many dolls does Betty have?

   8 – 5 = 3

   Betty has 3 dolls.

3. There are 6 pencils on the desk.
   Ann's teacher needs 10 pencils.
   How many more should Ann get for her?

   10 – 6 = 4

   Ann should get 4 more for her.

## Solving Problems

4. Joe saw a kite in a toy store.

   The kite cost 10 cents. 10 - 3 = 7

   Joe has only 3 cents.

   How much more money does he need to buy the kite?

He needs 7 cents.

5. There are 7 reading books and 2 science books on the table. 7 + 2 = 9

   How many reading and science books are on the table?

There are 9 reading and science books on the table.

6. Pat saw 10 airplanes on the ground at the airport. 10 - 8 = 2

   Eight of these airplanes took off.

   How many of these airplanes were still on the ground?

2 airplanes were still on the ground.

**Problem Solving**

7. Tom had 7 lighted candles on his birthday cake. He blew out 7 of them. How many of the candles on his birthday cake were still lighted?

7-7=0

No candles are still lighted.

8. There were 8 children playing tag. Four of them were girls. How many were boys?

8 - 4 = 4

4 children were boys.

9. Sue picked 8 flowers from the garden. Mother wants 10 flowers. How many more flowers must Sue pick for Mother?

8+2 = 10

Sue must pick 2 more flowers.

Problem Solving

Complete the sentence.

1. Dick had 7 balls.

   He gave some of his balls to Sam.

   How many balls did Dick have then?

I could tell if I knew how many balls he gave to Sam.

2. Mother gave 3 cookies to Bob.

   She gave some cookies to Sue.

   How many cookies did Bob and Sue have?

I could tell if I knew how many cookies she gave to Sue.

3. Sally has 2 doll dresses.

   Peggy has 4 doll dresses.

   How many doll dresses do Sally and Beth have all together

I could tell if I knew how many doll dresses Beth has

4. Joe saw some ducks on a pond.

   Three of these ducks flew away.

   Then how many of these ducks were on the pond?

I could tell if I knew how many ducks Joe saw on the pond

## Solving Problems

Draw lines under the facts you use to solve the problem.

Father has 4 hats.
He has 3 coats.
Father has 1 brown hat.
His other hats are black.
How many black hats does Father have?

4 - 1 = 3

Father has
3 black hats.

---

Jimmy has some new books.
Father gave 3 of these books to Jimmy.
Mother gave 4 cars to Jimmy.
Grandmother gave 4 of these books to Jimmy.
How many books does Jimmy have?

3 + 4 = 7

Jimmy has
7 new books.

---

Jane has 6 candy canes.
She has 4 candy suckers.
She gives 3 of the canes to David.
She gives 2 of the suckers to David.
How many canes does Jane have now?

6 - 3

Jane has
3 candy canes.

---

Four little boys went swimming.
Two mothers went swimming.
Three fathers went swimming.
Five little girls went swimming.
How many children went swimming?

4 + 5 = 9

9 children
are swimming.

# Chapter III

# SETS OF POINTS

## Background

### Introduction

This chapter is devoted to geometry. We shall study what may be called physical geometry--that is, the geometry of the world around us. The study involves a certain amount of abstraction, for the fundamental objects we shall deal with are not things we can pick up or feel or see. We shall think of a point, for example, as an exact location in space. A point, then, has no size or shape or color; it has no physical attributes at all except its location. We indicate a point by making a pencil dot or a chalk dot; but every child will agree that such a dot does not mark an exact location, and he will enjoy imagining the unseeable points.

We may remark that the geometry studied in college courses is of a higher degree of abstraction still. There the fundamental geometric objects like point and line are not defined at all, and the study proceeds deductively from certain formally stated assumptions about them (called axioms).

Our purpose here is to help the pupil observe and describe fundamental geometric relationships. The discussion is intuitive. In the primary grades we are not particularly concerned with formal deductions.

### Point

By a point we mean an exact location--for example, the exact spot at the corner of a room where two walls and the ceiling meet. We indicate points by drawing dots; but we realize that a pencil dot, no matter how small, gives only an approximate location, not an exact one. (In fact, it is clear that a pencil dot on a sheet of paper covers infinitely many points--that is, more than can be counted.) Nevertheless, in order to keep the language simple, we refer to the dots themselves as the actual points.

It is customary to denote points by capital letters.

A point is a fixed location: points do not move. The point at the corner of the ceiling remains even if the whole building falls down. Nevertheless, it must be remembered that fixing a location is a meaningful notion only with respect to some particular frame of reference. Frames of reference in common use are: the sun, the earth, a car, a person, a ruler. A point that is fixed with respect to one frame of reference need not be fixed with respect to a different one. For example, when a ruler is carried across the room, a point on the ruler remains fixed with respect to the ruler but does not remain fixed with respect to the earth.

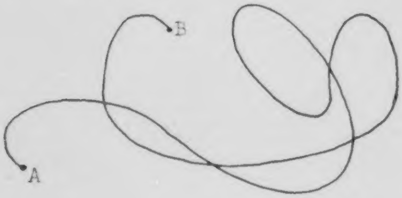
A geometric figure is any set of points.

## Congruence

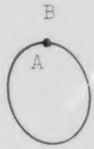
The idea of congruence in geometry is basic. Two geometric figures are said to be congruent provided that they have the same size and shape. A test is whether one will fit exactly on the other. In practice, the objects may not be conveniently movable; then one tests for congruence by making a movable copy of one and checking it against the other. Of course, all such tests, since they involve actual physical objects, often including the human eye, are only approximate. Nevertheless, in order to keep the language simple, we shall say, "The segments $\overline{AB}$ and $\overline{CD}$ are congruent" (rather than seem to be)--just as people say, "Johnny and Jimmy are exactly as tall as each other" (rather than seem to be).

Curve

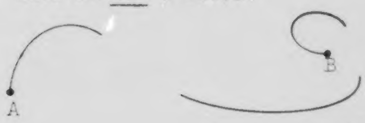
By a curve we mean any set of points followed in passing from a given point A to a given point B. Inherent in this definition is the intuitive notion of continuity; this is a curve:



and so is this:

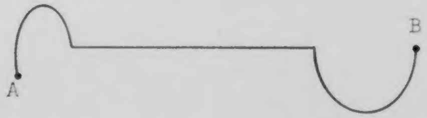


while this is not a curve:

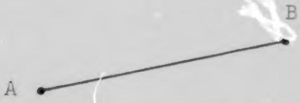


(However, it is a union of three curves.) We agree that a single point is not a curve.

It is also noteworthy that, according to the definition, a curve can be straight (in contrast with everyday usage). This is a curve.



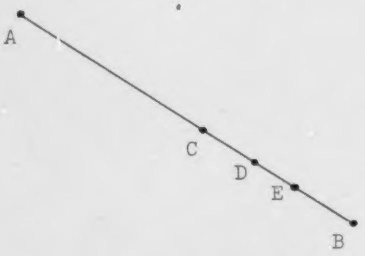
and so is this:



## Line Segment

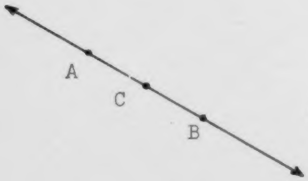
The last picture is an example of a line segment, that is, a straight curve. The endpoints are marked A and B; the line segment is denoted, accordingly, by either $\overline{AB}$ or $\overline{BA}$. Again, we agree that a single point is not a line segment.

Observe that a line segment can always be expressed in many different ways as a union of other line segments. For example, the line segment $\overline{AB}$ shown here is the union of the line segments $\overline{AC}$ and $\overline{CB}$, the union of the line segments $\overline{AD}$, $\overline{AE}$, and $\overline{CB}$, etc.



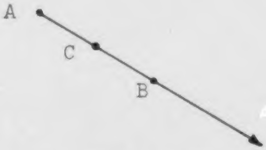
## Line

When a line segment is extended infinitely far in both directions, we get a line. Such extensions are only conceptual, of course, not practical. A line has no endpoints. No matter how far out we go in either direction along a line, still more of the line will lie ahead. The infinite extent is indicated by arrows. The line containing points A and B is denoted by $\overleftrightarrow{AB}$. The line shown contains points A, B, and C; some names for this line are, therefore, $\overleftrightarrow{AB}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BC}$, etc.

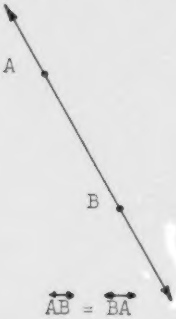


Note that, although $\overline{AB}$ and $\overline{AC}$ are different line segments, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ are the same line.

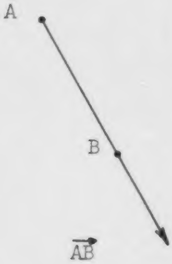
Just as a line is the infinite extension of a line segment in both directions, a ray is the infinite extension of a line segment in one direction. A ray therefore has a single endpoint. The infinite extent of a ray is indicated by an arrow. The ray with endpoint A and containing another point B is denoted by $\overrightarrow{AB}$. The ray shown has endpoint A and contains points B and C; some names for this ray are, therefore, $\overrightarrow{AB}$ and $\overrightarrow{AC}$.



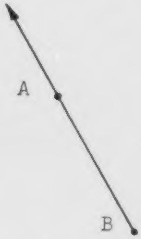
Note that, although $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ are the same line, $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays:



$\overleftrightarrow{AB} = \overleftrightarrow{BA}$



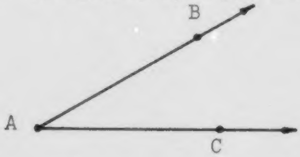
$\overrightarrow{AB}$



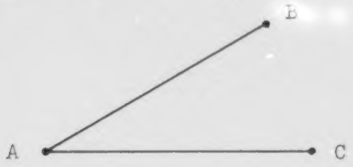
$\overrightarrow{BA}$

## Angle

By an angle we mean the union of two rays having the same endpoint. (We exclude the case in which the two rays are part of the same line.) The common endpoint is called the vertex of the angle. The plural of "vertex" is "vertices". The angle formed by rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is denoted by $\angle BAC$ or $\angle CAB$.

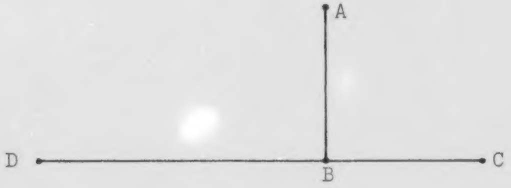


Two segments with a common endpoint determine an angle: segments $\overline{AB}$ and $\overline{AC}$ with common endpoint A determine the angle $\angle BAC$ with vertex A:

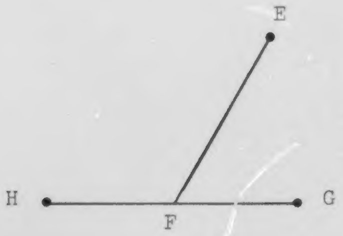


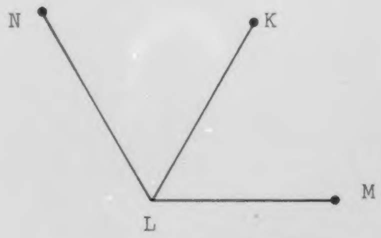
## Right Angle

An angle is called a right angle if "two of them can fit together to form a line". In the diagram, $\angle ABC$ is congruent with $\angle ABD$, and the three points C, B, and D lie on a line; therefore, $\angle ABC$ and $\angle ABD$ are right angles.



Note that there are two parts to the definition: the part concerning congruence, and the part concerning the line. In the next diagram, $\angle EFG$ and $\angle EFH$ form a line but are not congruent, while $\angle KLM$ and $\angle KLN$ are congruent but do not form a line.



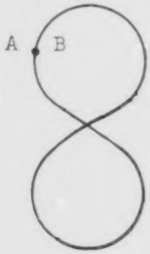


## Plane

When a flat surface such as a table top, wall, or sheet of glass, or even this sheet of paper, is extended infinitely in all directions, we get a plane. Notice that if two points of a line lie in a given plane then the entire line is contained in the plane. Two intersecting lines determine a plane. In the teaching material, the infinite extent of the plane is not stressed.

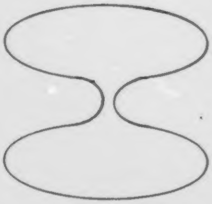
## Closed Curve, Simple Closed Curve

We have called a curve any set of points followed in passing from a given point A to a given point B. When the points A and B coincide, the curve is said to be closed.



A closed curve

A closed curve that lies in a plane and does not cross itself is simple.

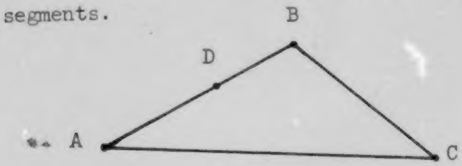


A simple closed curve

A simple closed curve has the interesting property of separating the rest of the plane into two subsets, an inside or interior (the subset of the plane enclosed by the curve) and the outside or exterior. Any curve connecting a point of the interior with a point of the exterior necessarily intersects the simple closed curve. (It may be of interest that this seemingly obvious fact is actually quite hard to prove.)

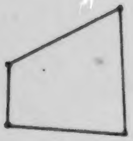
Polygon

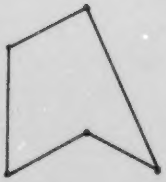
An important class of simple closed curves is the class of polygons. A polygon is a simple closed curve that is a union of line segments. Recall that a line segment can always be expressed in many different ways as a union of line segments. Hence a polygon, too, can be expressed in different ways as a union of line segments.

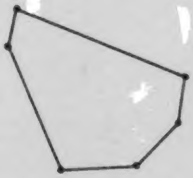


The union of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ =
the union of $\overline{AD}$, $\overline{DB}$, $\overline{BC}$, and $\overline{CA}$.

If we look at the various line segments in a polygon, we notice that they are of two kinds: those that are contained in other line segments, and those that are not contained in other line segments. For example, in the picture above, $\overline{AD}$ is of the first kind, since it is contained in the line segment $\overline{AB}$. On the other hand, $\overline{AB}$ is of the second kind, since it is not contained in any line segment except itself. Line segments of this second kind are called sides: a line segment in a polygon is called a side if it is not contained in any other line segment in the polygon. The polygon shown has three sides: $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$. A polygon of three sides is called a triangle. A polygon of four sides is a quadrilateral; of five sides, a pentagon; of six, a hexagon. (The last two names are not used in the teaching material.)


Quadrilateral


Pentagon

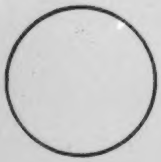

Hexagon

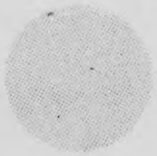
It may be observed that two consecutive sides of a polygon--that is, two sides with an endpoint in common--never lie on the same line. The endpoints of the sides are vertices (singular: vertex) of the polygon. The vertices of the triangle shown on page 198 are A, B, and C.

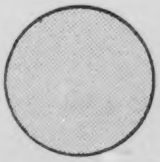
Rectangles are special kinds of quadrilaterals. Squares are special kinds of rectangles.

## Region

The union of a simple closed curve and its interior is called a region. We refer to a triangular region, rectangular region, or circular region, etc., indicating that the simple closed curve is a triangle, rectangle, or circle, etc. For example, an ordinary sheet of paper is a rectangular region; the edges of the paper form a rectangle.


Circle


Interior


Circular region

## III-1. Points

Objective: To develop the idea of a point.

Vocabulary: (No new words.)

Materials: Pencil, needle.

Suggested Procedure:

Use the playground as well as the classroom. You may wish to ask the class, before a play period, to look for examples of points on the playground; you may wish to take the class outside for at least part of the mathematics class; or you may ask questions after the children have come in from a play period regarding what they might have seen.

To emphasize the concept of a point as an exact location, you may mention the points at various corners of the building, the point in the air a child can reach when he jumps, a spot on a ball, the point marked by a speck of dust. Indicate a point in the air with the tip of your pencil. Indicate a point with the tip of the needle. Mark a point with chalk on the board.

We can think of a point as something that does not move or as something that does move. The way we think about it depends on what is important to us at the time. Have a child jump in the air with outstretched hand. Indicate with the tip of the pencil the point in the air that the child reached. "This is the point Peter jumped to." Clearly, we think of this point as staying there in the air, not as being carried by Peter's finger tip back to his desk. Ask the children to look at that place in the air, now marked by the pencil tip, and to keep looking after you take the pencil away.

> Can you still look at the place and think about the point where my pencil tip was?

> Yes, the point I was showing you is still right there. (But no one can see it.)
>
> The point Peter jumped to is an exact location in the room.

Now indicate a point on Peter's finger. "Here is the point on Peter that reached higher than any other part of him when he jumped." Clearly, we are now thinking of the point as staying with Peter. It is an exact location on Peter's finger.

Discuss the point marked by a spot on a ball. Sometimes we think of the point as remaining where it was even after the ball is kicked away, but at other times we like to think of the point as moving with the ball. The way we think about it depends on what we are interested in at the time: the location on the ground or the location on the ball.

Discuss the point marked by a speck of dust. What happens to the point when the speck of dust is blown away? (It still remains.)

Mark a point on the chalkboard. Explain to the children that the chalk mark is not really an exact location but covers many different points. (In fact, it covers more points than anyone can count.) Label the point A. Explain that we use capital letters to name points. Mark another point and label it M. Explain to the children that different points are given different names.

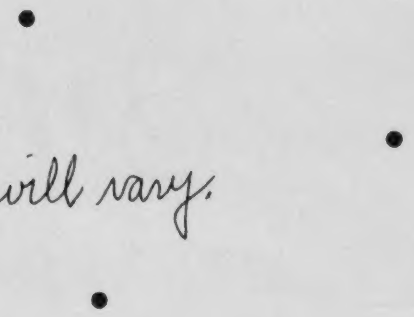
Pupil's book, page 95: Points

Ideas

> A mark is used to show a point.
>
> Points are named by capital letters.
>
> In any one example, different points are named by different letters.

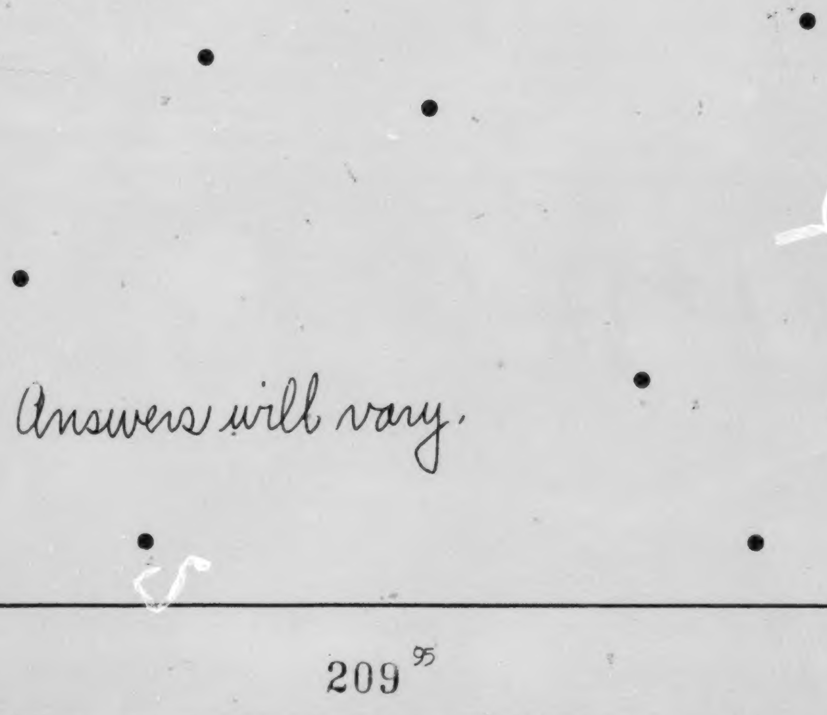
## Points

1. Name the points A, B, and C.



Answers will vary.

2. Name the points.



Answers will vary.

## III-2. Curves, line segments

**Objectives:** To develop the idea of a curve as a set of points.

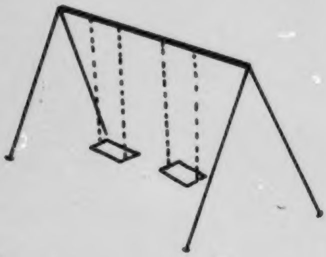
To develop the idea of a line segment as a special kind of curve.

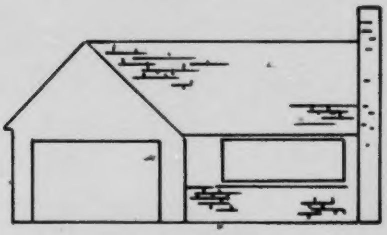
**Vocabulary:** Curve, line segment.

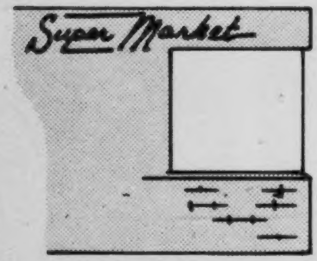
**Materials:** (None.)

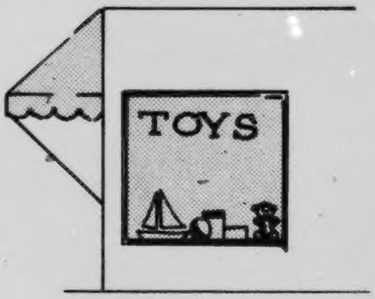
**Suggested Procedure:**

Sketch on the board a "map" similar to the following illustration:









Discuss the sketch as a "map" of the place where Sally lives.

> Sally often goes from her home to the grocery store. On the way she stops at the playground to swing. When she comes home from the grocery store she goes past the toy store and looks in the window.

As you mention each of the places where Sally stops, mark a point with a dot on the sketch.

Ask a child to draw a path that Sally could have taken. (The path should include the points which have already been indicated by dots.) We call it a curve. A curve is a set of points. A curve is any set of points followed in passing from one given point to another given point.

> When Sally was on her way to the store, she saw a dog.

Mark a point on the curve to show where Sally might have been when she saw the dog.

> On the way home Sally found a nickel.

Mark a point on the curve to show where Sally might have been when she found the nickel. Ask several children to suggest other experiences Sally might have had and mark the points where Sally might have been when they occurred.

> All of these are points of the curve Sally followed.
>
> Can we mark other points of this curve?
>
> One day Sally's mother was in a hurry for some eggs so she asked Sally to go directly to the store.

Use a straightedge to draw this path.

This is a special kind of curve. It is called a line segment. A line segment is a straight curve between two points. This one goes from Sally's house straight to the store.

When Sally took this path to the store she saw a beetle. Suppose this is the point where Sally was when she saw the beetle. (Mark the point.)

Indicate several points on the line segment.

There are points from one end of this line segment to the other end. A line segment is a set of points.

Mark two points on the chalkboard and label them A and B. (The points should be at least 24" apart.) Direct a child to draw a curve from A to B. Then use a straightedge and draw a line segment from A to B. (It is assumed that the child's curve wanders enough so that the difference between the first curve and the line segment is observable.)

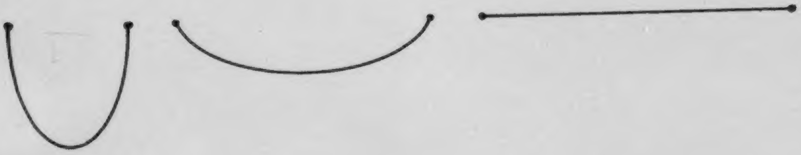
Indicate the path drawn by the child and ask if it is a curve. (Yes.)

Indicate the line segment and ask if it is a curve. (Yes.)

What other name can we give this curve? (Line segment.)

Discuss with the children that the line segment is a special kind of curve. Every line segment is a curve, but not every curve is a line segment.

Hold up a piece of string, one end in each hand. Let the string hang. This string shows a curve. Pull slowly until it is taut. Each position along the way shows a curve. The last position shows a special kind of curve: a line segment.



A line segment is named by its endpoints. Draw a line segment and label its endpoints D and Q. This line segment has two names: $\overline{DQ}$ and $\overline{QD}$. Give other examples; make some horizontal, some vertical, some slanting.

Pupil's book, page 96: Curves

Ideas

A curve is a set of points followed in passing continuously from one given point to another given point.

Pupil's book, page 97: Curves and line segments

Ideas

A line segment is a curve. It is a special kind of curve.

Pupil's book, pages 98 - 100: Line segments

Ideas--page 98.

A line segment is named by its endpoints. Each line segment has two names.

Ideas--page 99.

Many line segments can have a single endpoint in common.

Ideas--page 100.

There is only one line segment with two given endpoints.

Each line segment contains other line segments as subsets. (This idea need not be emphasized.)

## Curves

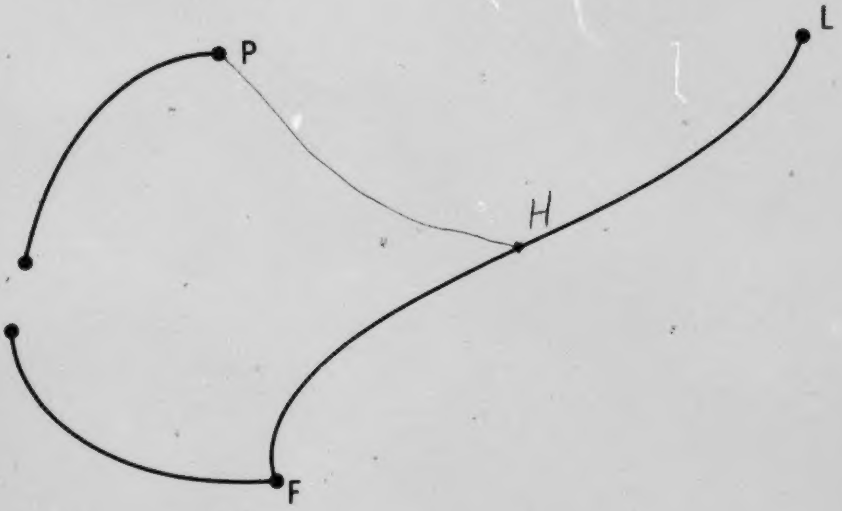
Put a ring around the answer:

Is there a curve shown going from F to L? [✓ Yes] No

Is there a curve shown going from F to P? Yes [✓ No]

Mark another point on the curve from F to L. Name it H.

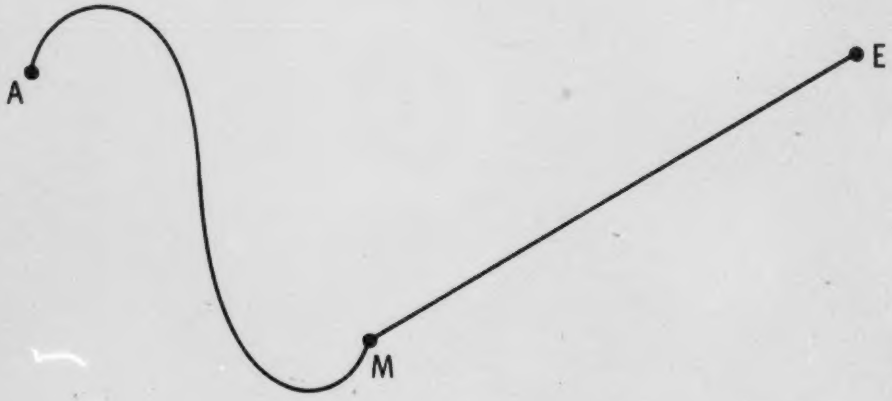
Draw a curve from P to H.



## Curves and Line Segments

Is there a curve shown going from A to M? [✓ Yes] No

Is there a curve shown going from M to E? [✓ Yes] No



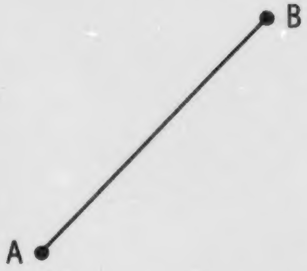
Is there a line segment shown going from M to E? [✓ Yes] No

Name its endpoints. E M

Is there a line segment shown going from A to M? Yes [✓ No]

## Line Segments

1. Here is a picture of a line segment.



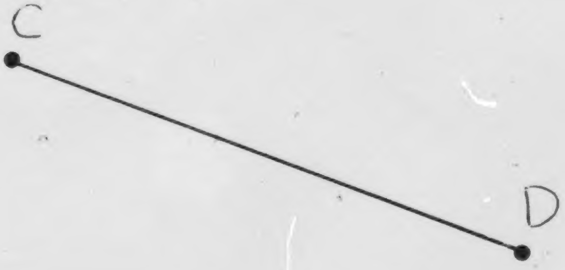
One name for this line segment is $\overline{AB}$.

Another name for the line segment is $\overline{BA}$.

2. Here is another line segment.

Name one endpoint C.

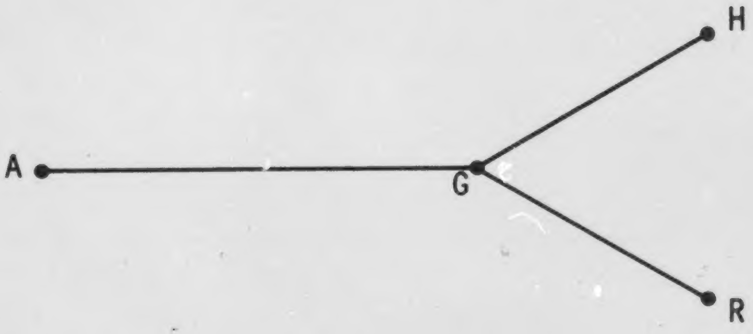
Name the other endpoint D.



Write two names for this line segment. $\overline{CD}$ $\overline{DC}$

## Line Segments

3. Here are some line segments that have G as an endpoint.



Name three different line segments shown.

$\overline{AG}$ $\overline{GR}$ $\overline{GH}$

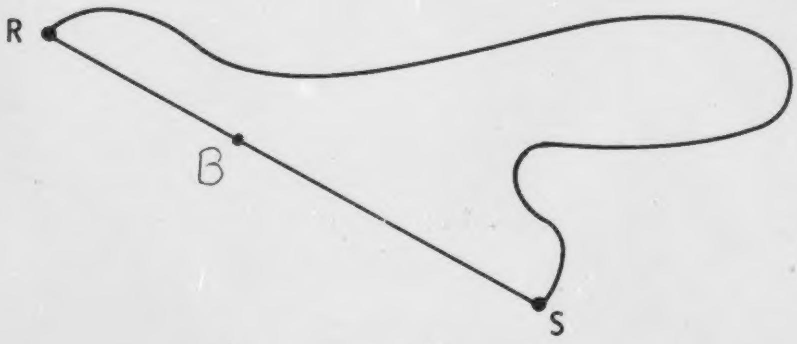
## Line Segments

Is there a line segment shown with endpoints R and S? [✓ Yes] No

Write two names for this line segment. $\overline{RS}$ $\overline{SR}$

Can there be another line segment with endpoints R and S?

Yes [✓ No]



Mark another point on $\overline{RS}$. Name it B.

Is the line segment $\overline{RB}$ shown? [✓ Yes] No

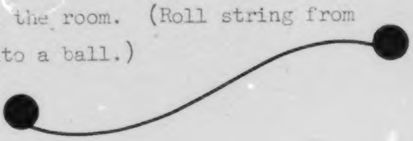
Is the line segment $\overline{SB}$ shown? [✓ Yes] No

## III-3. Lines

Objective: To develop the idea of a line.

Vocabulary: (No new words.)

Materials: Unmarked oaktag straightedge (one for each child), masking tape, a piece of string longer than the room. (Roll string from each end into a ball.)



Suggested Procedure:

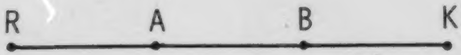
Ask two children to come to the front of the room. Give one ball of string to each child. Use two pieces of masking tape to mark points on the string and direct the children to pull the string tightly to show a line segment between the two points.

Ask the children to move away from each other a little, unrolling the string as they go. Mark two more points on the string and discuss the fact that you have extended the original line segment to a longer one.

Repeat the procedure, having the children move out to show a still longer line segment (where the string is again pulled taut).

Have the children imagine repeating the process again and again without end (with the help of an inexhaustible supply of string). The result would be a line. Observe that a line has no endpoints.

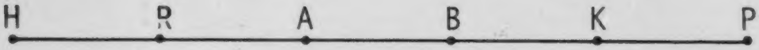
Draw a line segment on the board; label it $\overline{AB}$. Ask the children to imagine a longer line segment that contains $\overline{AB}$ as a part of it. Lead them to suggest extending $\overline{AB}$ in both directions. Draw the extensions, creating a new line segment; label it $\overline{RK}$.



Emphasize that A and B, the endpoints of the original line segment, are not endpoints of the new one (although they are points on it).

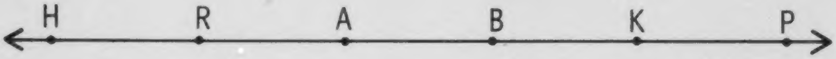
Observe that there are now several line segments whose endpoints have been named, and discuss the relations among them: $\overline{RA}$ is part of $\overline{RK}$, $\overline{RA}$ is not part of $\overline{BK}$, and so on.

Extend $\overline{RK}$ the same way: extend it in both directions to form a line segment $\overline{HP}$.



Emphasize that R and K, the endpoints of the line segment we were just working with, are not endpoints of the new line segment $\overline{HP}$.

Get the children to agree that the process of extending line segments in both directions can, in our imagination, be repeated again and again without end. The result will be a line. Emphasize that a line has no endpoints. (The endpoints of each line segment get "swallowed up" when the line segment is extended.) Explain that we show a line with the help of arrows:



Ask how many lines there can be through the two points A and B that we started with. When the children agree that there can be only one (the one shown), discuss the fact that it is also the only line through A and K, or through R and K, and so on.

Through any two points there is exactly one line. Therefore, any two points on a line may be used to name the line. Introduce the symbol $\overleftrightarrow{AB}$, and agree that other equally good names for the line shown on the board are $\overleftrightarrow{AK}$, $\overleftrightarrow{BA}$, $\overleftrightarrow{PR}$, and so on.

### Pupil's book, pages 101-103: Lines

Ideas

There are many lines through one point.
There is only one line through two points.

Any two points on a line may be used to name the line.

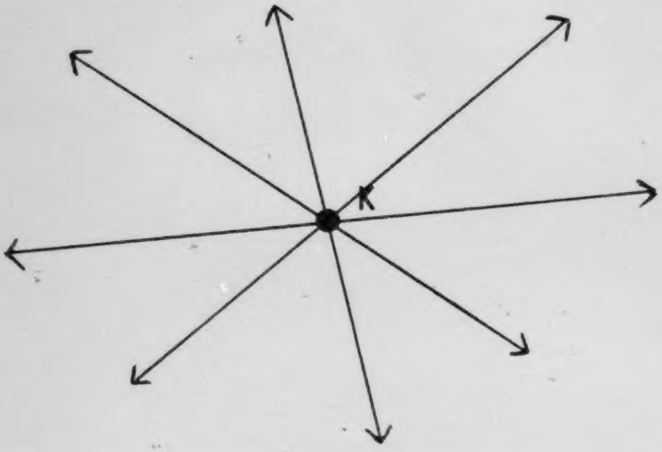
Example 4

A line goes on "forever". The two lines cross, although the parts that can be drawn on the page do not cross.

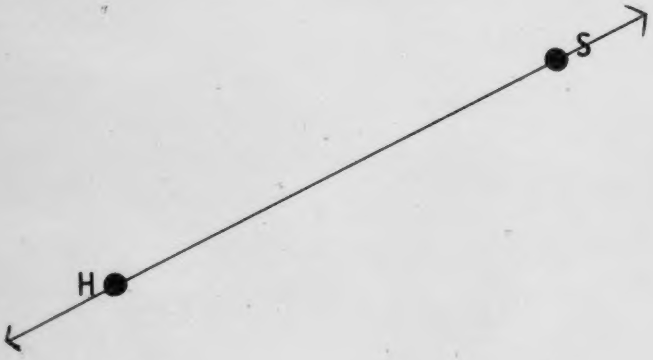
5

## Lines

1. Draw four different lines through the point K.



2. Draw a line through the points H and S.



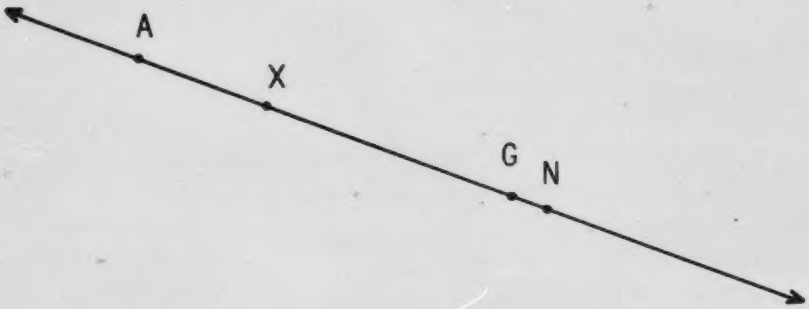
Name this line $\overleftrightarrow{HS}$

Could you draw a different line through H and S? Yes [✓ No]

## Lines

3. Line $\overleftrightarrow{AG}$ is shown below.

Write four more names for this line. $\overleftrightarrow{AX}$ $\overleftrightarrow{AN}$ $\overleftrightarrow{XN}$ $\overleftrightarrow{GN}$



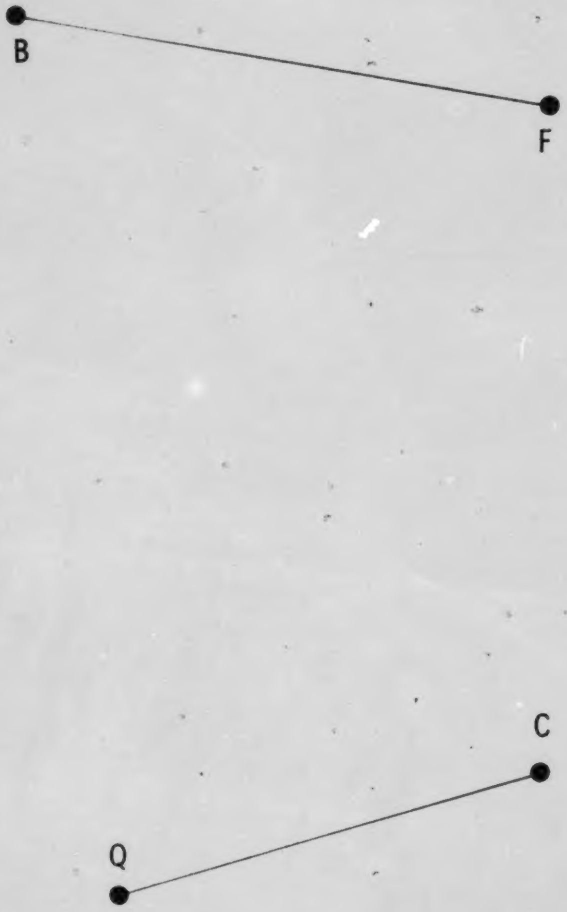
Is line segment $\overline{AG}$ part of line $\overleftrightarrow{AG}$? [✓ Yes] No

Is $\overline{AG}$ part of $\overline{XN}$? Yes [✓ No]

Is $\overline{AG}$ part of $\overleftrightarrow{XN}$? [✓ Yes] No

4. Draw line segments $\overline{BF}$ and $\overline{CQ}$.

Do they cross each other? Yes [✓ No]



Do the lines $\overleftrightarrow{BF}$ and $\overleftrightarrow{CQ}$ cross each other? [✓ Yes] No

### III-4. Closed curves

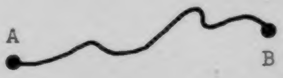
Objective: To introduce the idea of a (plane) simple closed curve.

Vocabulary: Closed curve, simple closed curve, polygon.

Materials: Chalk of several different colors; piece of string or yarn for each child.

Suggested Procedure:

Mark two points on the chalkboard and label them A and B. Tell the children to imagine that they are taking a trip from point A to point B. Draw a curve connecting the two points (but do not draw a line segment) and ask what this path from one point to another is called. (A curve.)



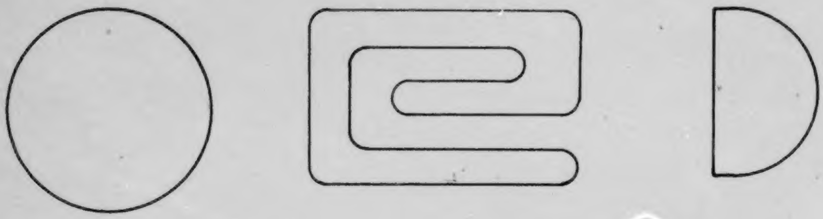
> We have a curve from point A to point B. Now suppose we want to go from point B back to point A by a different path. Who can show how we might do it?

Give a child some chalk of the same color as that previously used.

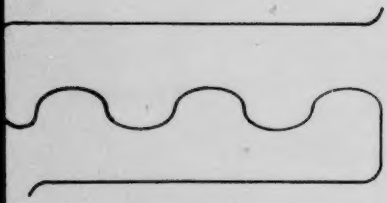
> Where did the trip end? (Just where it started.)

Have other children draw curves showing a round trip between A and B, with each child using a different color of chalk. Tell the class that a curve that begins and ends at the same point is called a closed curve. With your finger or a pointer, trace the various closed curves drawn by the children to show that each curve begins and ends at the same point. If any child has drawn a closed curve that crosses itself, call attention to that one, showing that it does, indeed, cross itself at at least one point. (If no curve that crosses itself has been drawn, use another color to illustrate some that do.) Call attention to the closed curves in which there are *no* crossings. Refer to these as *simple closed curves*. Then have children note the closed curves that are *not simple* closed curves (because of crossings).

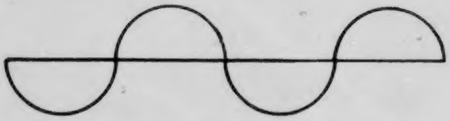
Draw several curves on the chalkboard, some closed and some not, and have the children state which are closed curves. Continue, making some simple closed curves and some closed curves that are not simple. Have the children mark in some way the simple closed curves. Have them label crossing points on closed curves that are not simple.



Three simple closed curves

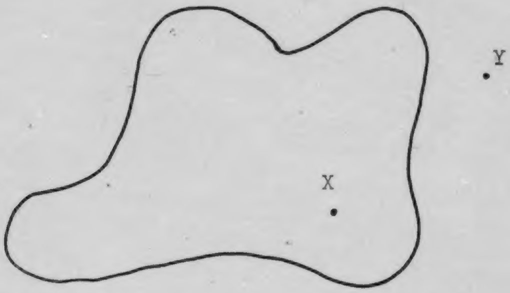


Not closed

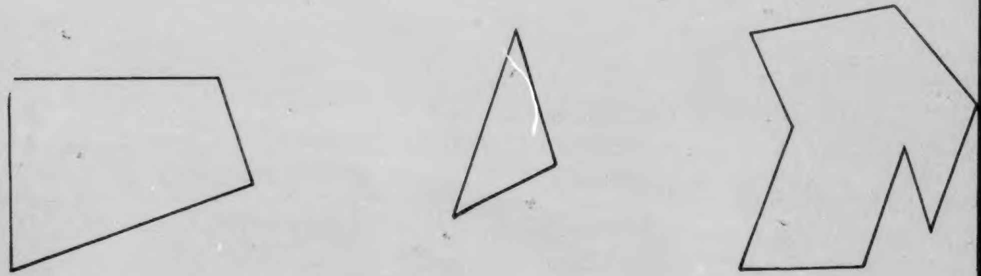


Closed, but not simple

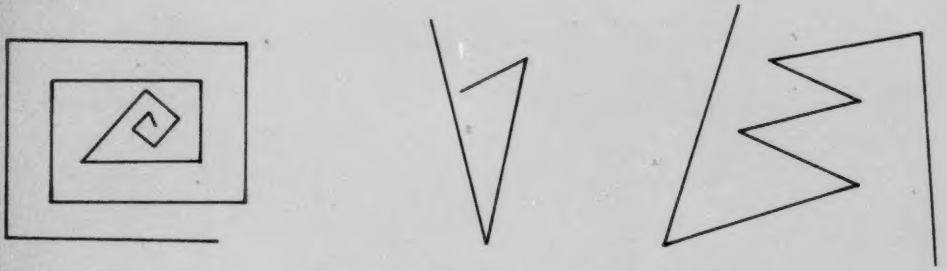
Now develop the idea of inside and outside with the children. Mark two points relating to a simple closed curve--one inside the curve, the other outside the curve. It is impossible (within the plane) to draw a curve between these points that does not cross the simple closed curve at least once.



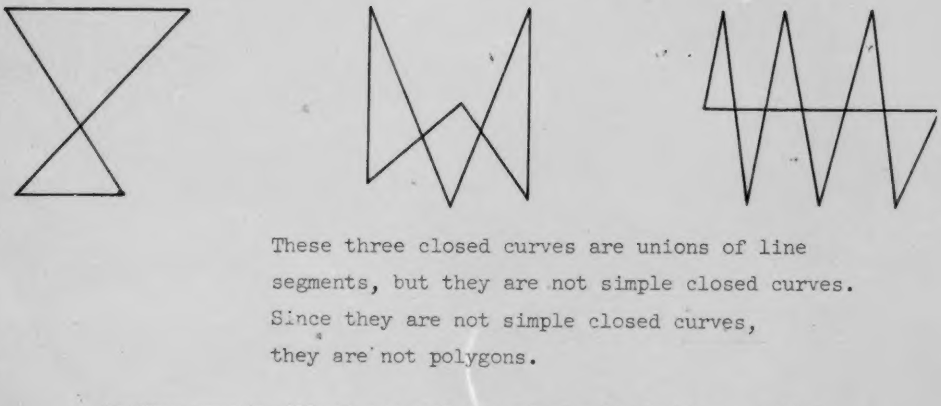
Explain that a simple closed curve that is a union of line segments is called a polygon. Draw polygons of various kinds, and some figures that are the unions of line segments but not simple closed curves. Have children state which are simple closed curves.



These three simple closed curves are unions of line segments. Therefore, they are polygons.

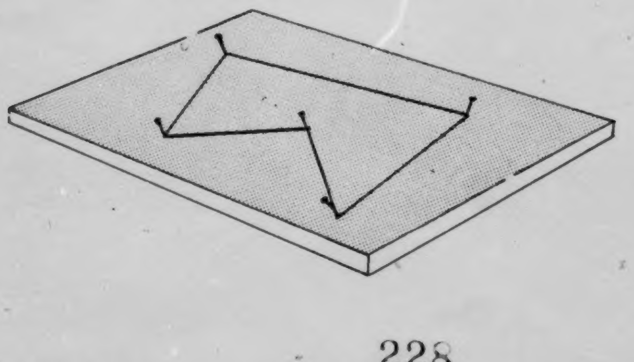


These three curves are unions of line segments, but are not closed curves. Since they are not closed curves, they are not polygons.



These three closed curves are unions of line segments, but they are not simple closed curves. Since they are not simple closed curves, they are not polygons.

Another way to illustrate polygons is by stretching a rubber band around nails in a piece of ceiling tile.



## Pupil's book, pages 104-106: Closed curves

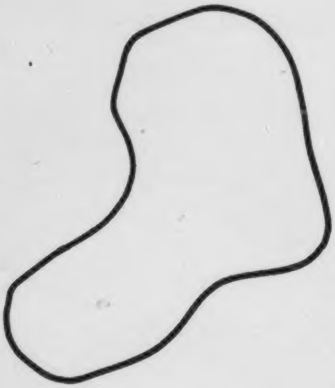
Ideas

A closed curve starts and ends at the same point.

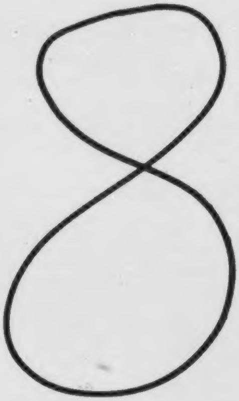
A simple closed curve does not cross itself.

A polygon is a simple closed curve made up entirely of line segments. (Page 106, Example 2 is not a polygon.)

## Closed Curves

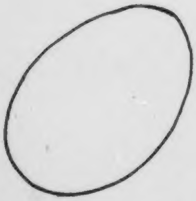


A simple closed curve

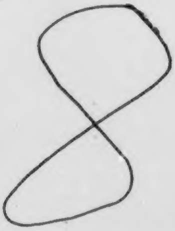


A closed curve but not a simple closed curve

1. Draw a simple closed curve.



2. Draw a closed curve that is not a simple closed curve.



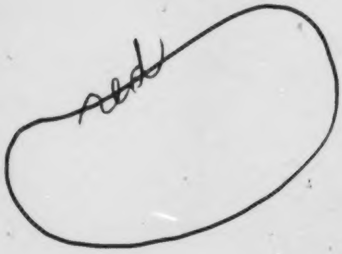
## Closed Curves

With a red crayon, trace each simple closed curve.

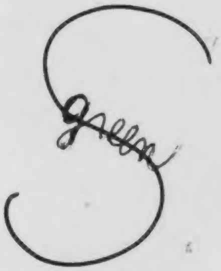
With a blue crayon, trace each closed curve that is not a simple closed curve.

With a green crayon, trace each curve that is not closed.

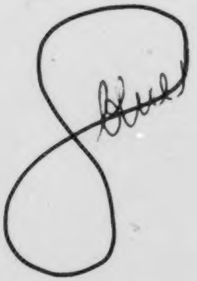
1.



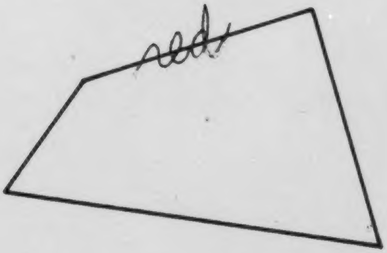
2.



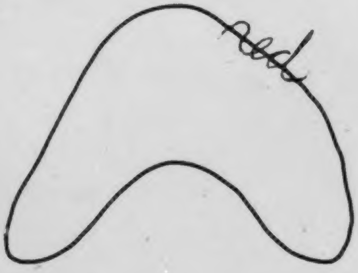
3.



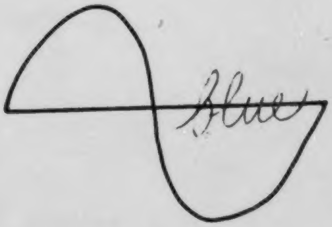
4.



5.



6.

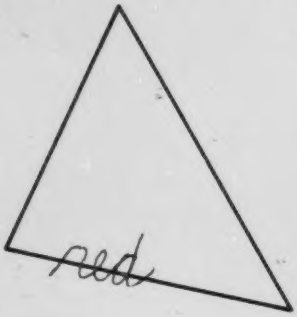


## Closed Curves

With a red crayon, trace each polygon.

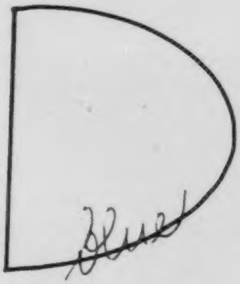
With a blue crayon, trace each simple closed curve that is not a polygon.

1.



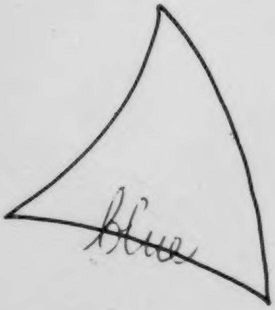
red

2.



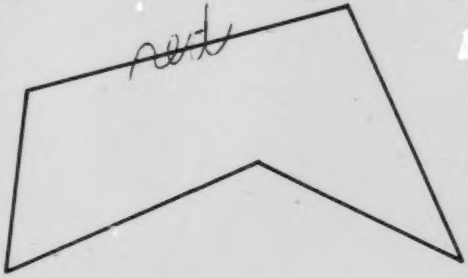
blue

3.



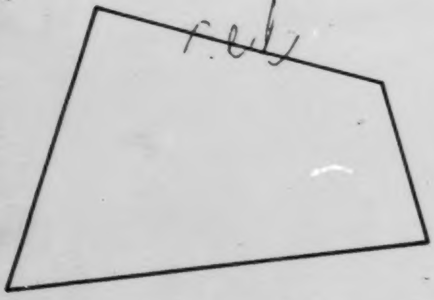
blue

4.



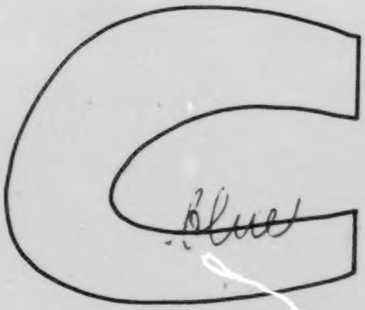
red

5.



red

6.



blue

## III-5. Triangles

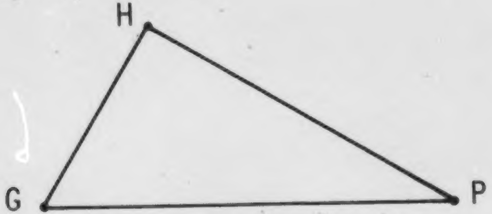
Objective: To develop the understanding that a triangle is a polygon with 3 sides and 3 vertices.

Vocabulary: Side, vertex, vertices; (Review) triangle.

Materials: Straightedge for each child.

Suggested Procedure:

Mark three points G, H, and P on the chalkboard and draw $\overline{GH}$, $\overline{HP}$, and $\overline{GP}$.

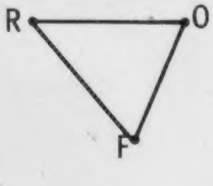


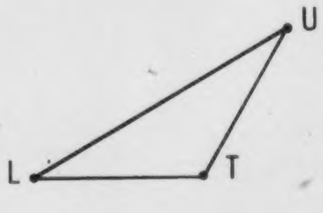
Is this figure a polygon? (Yes.) It is a special kind of polygon called a triangle.

This polygon is a union of three line segments: $\overline{GH}$, $\overline{HP}$, and $\overline{GP}$. A polygon that is a union of three line segments is called a triangle. The line segments are called the sides of the triangle. The sides of this triangle are $\overline{GH}$, $\overline{HP}$, and $\overline{GP}$.

Each endpoint of a side of a triangle is called a vertex of the triangle. The plural of "vertex" is "vertices". The vertices of this triangle are G, H, and P.

Make additional illustrations such as these and label the vertices.



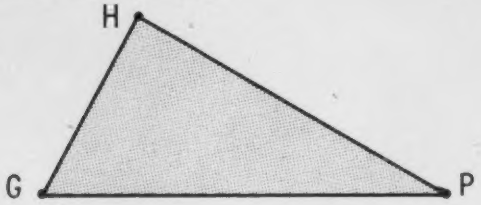


Ask the children what kind of polygons you have drawn. (Triangles.) Have them name the various vertices and sides as you point to them.

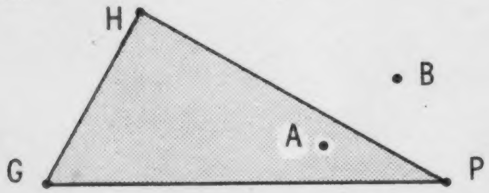
> How many vertices does a triangle have? (Three.)
>
> The vertices of a triangle are used to name the triangle.

Introduce the notation $\triangle$ GHP. Other names for this triangle are $\triangle$ GPH, $\triangle$ HPG, etc. Write the names for the other triangles shown on the board. ($\triangle$ RFO, $\triangle$ UTL, etc.)

Distinguish for the children between the triangle itself (a union of line segments) and the inside of the triangle (the points enclosed by the triangle). Illustrate by shading.



Now mark a point A inside the triangle and a point B outside the triangle, in the approximate positions shown.



Ask the children whether they think a curve could be drawn from A to B without crossing the triangle. (No.) Of course, the line segment from A to B will cross somewhere on $\overline{HP}$; but perhaps some other curve from A can get through? Attempt some, and show the children how you get stopped every time, like a puppy in a fenced-in yard.

Pupil's book, pages 107-108: Triangles

Ideas

A triangle is named by its vertices.

Pupil's book, pages 109-110: Triangles

Ideas

The same line segment may be a side of more than one specified triangle.

Pupil's book, pages 111-112: Triangles

Ideas

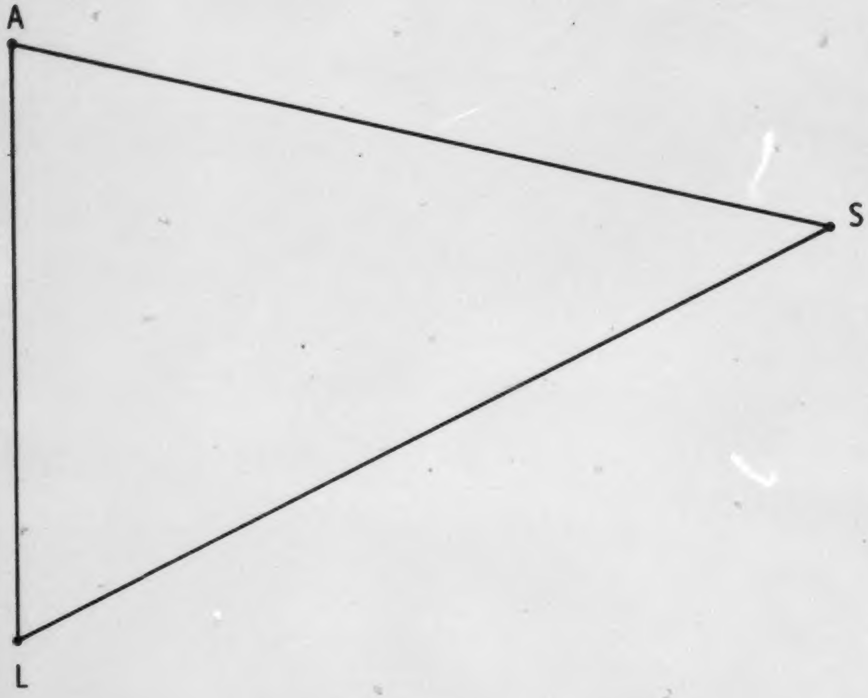
A triangle separates the rest of the plane into two clearly differentiated parts: the inside of the triangle, and the outside.

## Triangles

1. Here is a triangle, $\triangle ASL$.

One vertex is A.

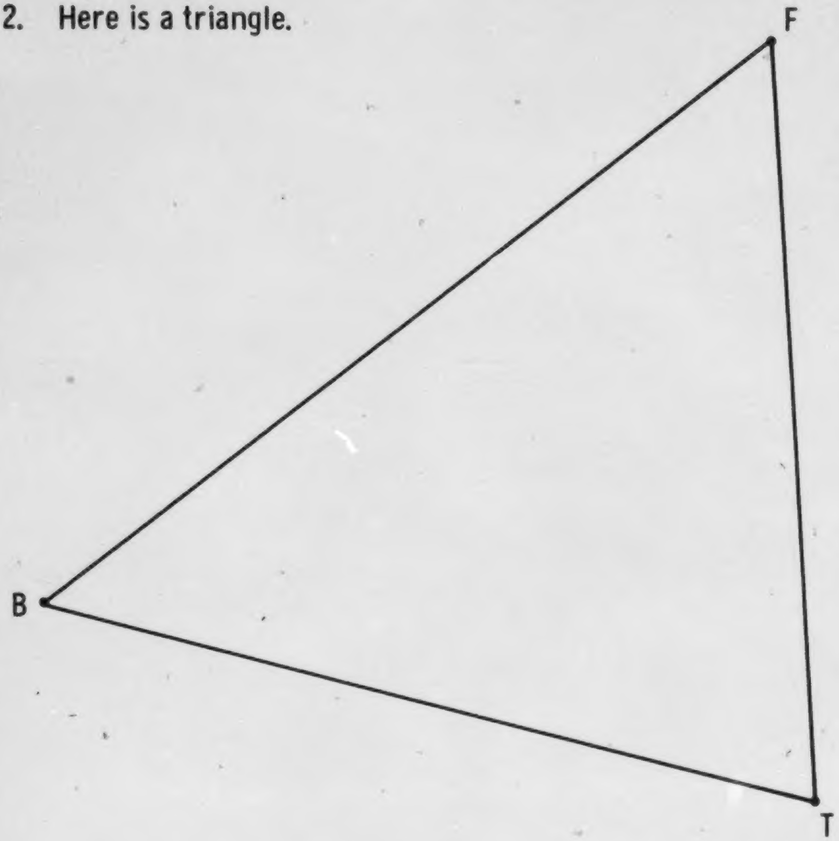
One side is $\overline{AS}$.



The vertices are A, L, S

The sides are $\overline{AS}$, $\overline{AL}$, $\overline{LS}$

2. Here is a triangle.



Name the vertices. B F T

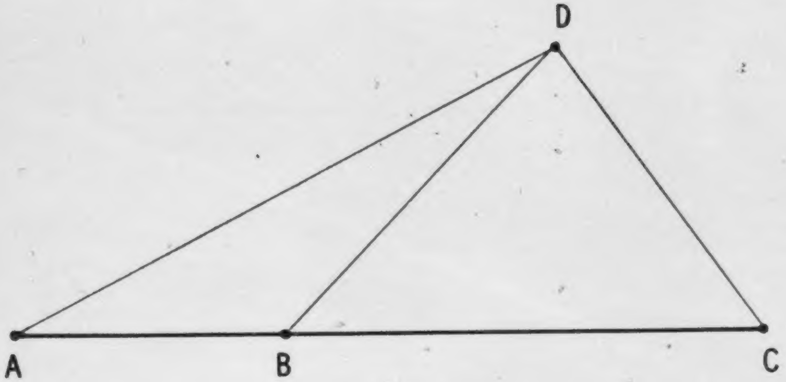
Name the sides. $\overline{BF}$ $\overline{FT}$ $\overline{TB}$

Name the triangle. △BFT

## Triangles

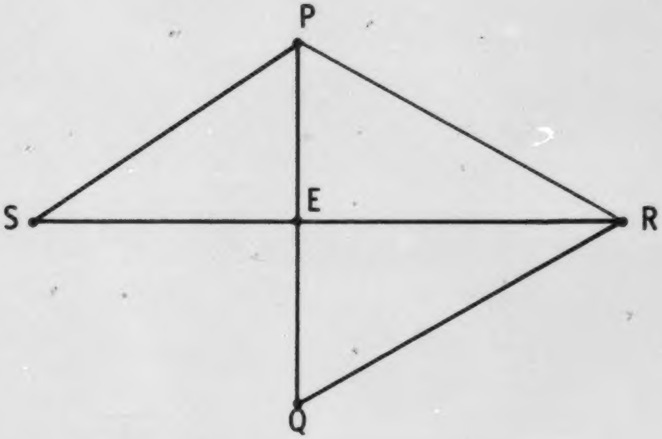
3. The points A, B, and C lie on a line.

Draw $\overline{DA}$, $\overline{DB}$, and $\overline{DC}$.



Name all the triangles drawn. $\triangle ABD$ $\triangle BDC$ $\triangle ADC$

4. $\overline{PQ}$ and $\overline{RS}$ are line segments meeting at the point E.



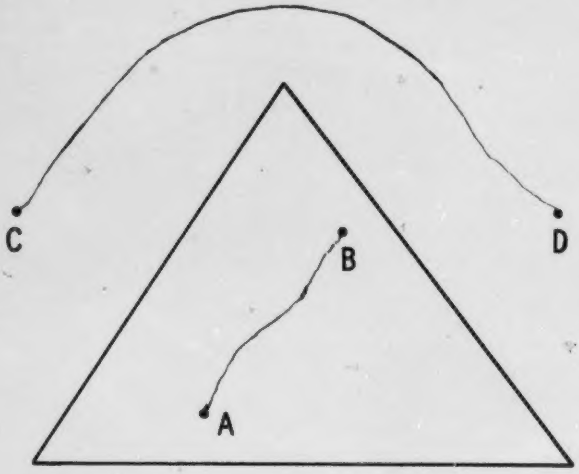
Name the triangles drawn. $\triangle$SPE $\triangle$EQR

Draw the line segment $\overline{PR}$.

Name the new triangles drawn. $\triangle$PER $\triangle$PQR $\triangle$SPR

## Triangles

5. Draw a curve from A to B that does not cross the triangle.
Draw a curve from C to D that does not cross the triangle.



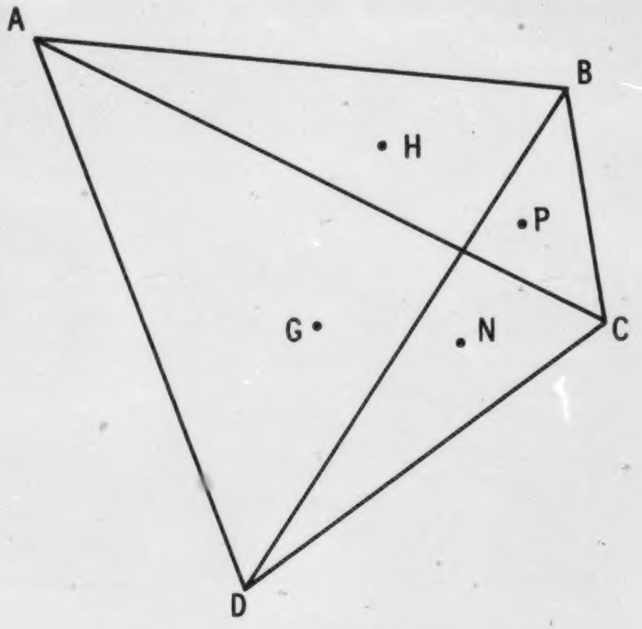
Points A and B are [✓ inside] / outside the triangle.

Points C and D are [✓ inside] / outside the triangle.

Can you draw a curve from A to C that does not cross the triangle? Yes [✓ No]

## Triangles

6.



Name the marked point inside △ABC and inside △DBC. P

Name the marked point outside △ABC and outside △DBC. G

Name the marked point inside △ABC but outside △DBC. H

Name the marked point outside △ABC but inside △DBC. N

## Enrichment Activities

Provide a supply of sticks of various lengths for the children to form into triangles. Include some combinations of three sticks from which no triangle can be constructed; the condition for this to happen is that one of the sticks be as long as or longer than the other two put together. Lead the children to discover this condition for themselves and have them formulate it in words.

## Chapter IV

## ADDITION AND SUBTRACTION: FURTHER FACTS AND TECHNIQUES

Background:

In Section IV-1, the idea of "tens and ones"--that is, the decimal place value system as applied to 2-digit numerals--is reviewed. Pupils learn to use three successive forms of expression, as in the example

$$43, \ 4 \text{ tens and } 3 \text{ ones}, \ 40 + 3.$$

The point of view is that "43" and "40 + 3" are two names for the same number, so that we can write

$$43 = 40 + 3.$$

This renaming is then used, along with the associative property of addition, in computing sums and differences which are "simple" in that no renaming of ones as tens ("carrying") or renaming of tens as ones ("borrowing") is involved.

(Note: Because they do not really suggest the processes involved, the terms "carrying" and "borrowing" are not used.)

Example:

$$\begin{aligned} 43 + 5 &= (40 + 3) + 5 && \text{(Since } 43 = 40 + 3.) \\ &= 40 + (3 + 5) && \text{(By associativity.)} \\ &= 40 + 8 && \text{(Since } 3 + 5 = 8.) \\ &= 48 && \text{(Since } 40 + 8 = 48.) \end{aligned}$$

The sums considered in this section involve only one 2-digit numeral.

Section IV-2 concerns computation of sums and differences where two 2-digit numerals are involved. The text and exercises in this section still do not involve renaming ones as tens ("carrying") nor renaming tens as ones ("borrowing"). The "expanded" form is used:

$$\begin{array}{r} 40 + 3 \\ \underline{-(30 + 5)} \end{array}$$

Parentheses can be used to show that not just the 30 but also the 5 is subtracted. This expanded form will be useful in Chapter VI and later, where more complicated computations, involving renaming of ones as tens and tens as ones, etc., are studied.

Section IV-3 uses partitions of sets of 11 to 14 objects to teach the basic addition and subtraction facts corresponding to sums of 11 through 14. Here the main technique is to have children work with sets of objects, or pictures of these, in order to help them complete equations like

$$8 + 5 = 10 + \_\_\_\_$$

and hence to conclude that

$$8 + 5 = 13.$$

Manipulative objects, or pictures of these, are also used to help children learn subtraction facts like

$$14 - 6 = 8.$$

Here the child may simply think of removing a set of 6 from a set of 14 to get a set of 8. He might also think of a set of 14 as a set of 10 and a set of 4. Then he might think of removing a set of 6 from the set of 10, leaving a set of 4 to be combined with the initial set of 4 to get 8. Alternatively, he might think of removing the initial set of 4 together with 2 from the set of 10, leaving a set of 8.

Section IV-4 extends the study of addition and subtraction facts to those corresponding to sums through 18, and an addition chart through 9 + 9 is systematically constructed.

The "two-step" problems considered in Section IV-5, on problem solving, are clearly presented in the Suggested Procedure for this section. The role of the associative property of addition, and the corresponding use of parentheses, should be noted and clearly understood.

## IV-1. Simple addition and subtraction: one 2-digit numeral

**Objectives:** To review place value for tens and ones, and to compute simple sums and differences involving one 2-digit numeral.

**Vocabulary:** Basic facts.

**Materials:** Sticks (bundles of 10 each, and single sticks.)

**Suggested Procedure:**

Review place value and develop the idea that a number greater than 10 can be renamed as the sum of a number of tens and a number of ones:

What does the number 15 mean? (One ten and five ones.)

How do we write one ten? (10.)

Can we write 15 as 10 + 5? (Yes.)

Is 10 + 5 another name for 15? (Yes.)

How can we rename 23 in this way? (20 + 3.)

Give many opportunities for renaming such numbers by having pupils show on the chalkboard or on their papers: 37 = 30 + 7, 69 = 60 + 9, and including some examples such as 50 + 4 = 54 and 40 + 8 = 48.

**Pupil's book, pages** 113,114:

These pages may be used before the next part of this lesson or just before pupil's book, page 115.

**Other Names for Numbers**

*Fill the blanks.*

32 3 tens and 2 ones 30 + 2

14 1 tens and 4 ones 10 + 4

27 2 tens and 7 ones 20 + 7

43 4 tens and 3 ones 40 + 3

55 5 tens and 5 ones 50 + 5

47 4 tens and 7 ones 40 + 7

24 2 tens and 4 ones 20 + 4

38 3 tens and 8 ones 30 + 8

90 9 tens and 0 ones 90 + 0

56 5 tens and 6 ones 50 + 6

72 7 tens and 2 ones 70 + 2

## Other Names for Numbers

Fill the blanks.

46 = 40 + 6

52 = 50 + 2

27 = 20 + 7

89 = 80 + 9

73 = 70 + 3

61 = 60 + 1

30 = 30 + 0

95 = 90 + 5

14 = 10 + 4

28 = 20 + 8

20 + 3 = 23

90 + 6 = 96

40 + 5 = 45

70 + 2 = 72

30 + 8 = 38

50 + 7 = 57

10 + 8 = 18

60 + 9 = 69

80 + 4 = 84

70 + 0 = 70

- Using the associative property

Show a set consisting of 2 bundles of 10 sticks each and 5 single sticks. Have the number of the set identified (25) and ask a child to join to the set a set of 3 sticks and tell the number of sticks in the union. (28.)

> We have used the associative property of addition when we wanted to add 3 numbers less than 10. Let's see how it will help us in adding a number to another number that is greater than 10.

Write: $25 + 3 = \_\_\_\_$

> Tell me how we can rename 25 as the sum of tens and ones. (20 + 5.)

On the chalkboard show:

$25 + 3 = \_\_\_\_$

$20 + 5 + 3 = \_\_\_\_$

> We know that we can add the last two numbers, and then add their sum to the first.

On the chalkboard, put parentheses around 5 + 3.

> What is the sum of 5 and 3? (8.)

Chalkboard should show:

$25 + 3 = \_\_\_\_$

$20 + (5 + 3) = \_\_\_\_$

$20 + 8 = \_\_\_\_$

> 20 + 8 is another name for what number? (28.)

Write 28 as the sum in each blank above to complete the equations.

Repeat this procedure with several other such sums, having children use sticks, if necessary. When children understand how the associative property is being used, point to one of the expressions in parentheses and say:

> Look at 5 + 3. You have known for a long time that 5 + 3 = 8. Such very simple facts, in which both the numbers to be added are less than 10, are called basic facts.

Discuss the meaning of the words "basic" and "fact". Have children give examples of other basic addition facts they have learned.

Pupil's book, page 115:

> Children are asked to rename the number greater than 10, use the associative property, and write the sum.

## Renaming and Using Basic Facts

Fill the blanks.

35 + 2 = 30 + 5 + 2
35 + 2 = 30 + 7
35 + 2 = 37

41 + 8 = 40 + 1 + 8
41 + 8 = 40 + 9
41 + 8 = 49

93 + 6 = 90 + 3 + 6
93 + 6 = 90 + 9
93 + 6 = 99

85 + 4 = 80 + 5 + 4
85 + 4 = 80 + 9
85 + 4 = 89

52 + 7 = 50 + 2 + 7
52 + 7 = 50 + 9
52 + 7 = 59

26 + 2 = 20 + 6 + 2
26 + 2 = 20 + 8
26 + 2 = 28

16 + 3 = 10 + 6 + 3
16 + 3 = 10 + 9
16 + 3 = 19

77 + 2 = 70 + 7 + 2
77 + 2 = 70 + 9
77 + 2 = 79

32 + 4 = 30 + 2 + 4
32 + 4 = 30 + 6
32 + 4 = 36

53 + 5 = 50 + 3 + 5
53 + 5 = 50 + 8
53 + 5 = 58

- Using the expanded form

Write: $31 + 7 = \_\_\_\_$

Have a child show how to solve the problem, using the associative property.

> We are sometimes going to write problems in a different form. Look at this one:
>
> $$46 + 3 = \_\_\_\_$$
>
> Let's rename the 46 and write 40 + 6. Just below the numeral for the number of ones in 46, let's show how many ones are to be added to 46.
>
> $$\begin{array}{r} 40 + 6 \\ \underline{\qquad 3} \end{array}$$
>
> Now we add the ones: 6 + 3 = 9, and write 9 underneath. We must add 9 to the 40, like this:
>
> $$\begin{array}{r} 40 + 6 \\ \underline{\qquad 3} \\ 40 + 9 \end{array}$$
>
> We know that 40 + 9 = 49, so we can write 49 in the blank in our original problem to complete the equation.

Have children solve several problems using this form.

Compare pairs of examples, such as:

$46 + 3 = \_\_\_\_$ $35 + 2 = \_\_\_\_$ $64 + 5 = \_\_\_\_$

$26 + 3 = \_\_\_\_$ $75 + 2 = \_\_\_\_$ $44 + 5 = \_\_\_\_$

Ask what basic fact is used in each.

Pupil's book, pages 116,117:

> Use the first few problems for class discussion and then allow children to complete the pages independently.

## Addition

32 + 7 = 39

Rename 32: 30 + 2

Add 7: 7

30 + 9 Write 39 to complete the equation.

Fill the blanks.

| | | |
|---|---|---|
| 25 + 4 = 29<br>20 + 5<br>4<br>20+9 | 76 + 3 = 79<br>70 + 6<br>3<br>70 + 9 | 81 + 6 = 87<br>80 + 1<br>6<br>80 + 7 |
| 55 + 2 = 57<br>50 + 5<br>2<br>50 + 7 | 13 + 5 = 18<br>10 + 3<br>5<br>10 + 8 | 98 + 1 = 99<br>90 + 8<br>1<br>90 + 9 |
| 62 + 4 = 66<br>60 + 2<br>4<br>60 + 6 | 74 + 3 = 77<br>70 + 4<br>3<br>70 + 7 | 23 + 2 = 25<br>20 + 3<br>2<br>20 + 5 |

## Addition

Rename, add, and fill the blank.

45 + 1 = 46

40 + 5

1

40+6

14 + 2 = 16

91 + 3 = 94

52 + 5 = 57

31 + 8 = 39

87 + 2 = 89

21 + 5 = 26

73 + 4 = 77

54 + 5 = 59

42 + 6 = 48

84 + 4 = 88

32 + 3 = 35

- The expanded form in subtraction .

Give a child 27 sticks (2 bundles of 10 and 7 single sticks, and ask him to remove 4 sticks. Write:

27 - 4 = _____

> Jimmy had 2 tens and 7 ones in his starting set. He removed 4 ones from the 7 ones. How many does he have in the set remaining? (2 tens and 3 ones.) We can show it in this way:
>
> $$20 + 7 - 4 = 20 + 3 = 23$$
>
> Or we can use this form:
>
> $$\begin{array}{r} 20 + 7 \\ -\ 4 \\ \hline 20 + 3 \end{array}$$

Present several examples of this sort. Have children write and solve problems.

Give several pairs of examples, such as:

36 - 5 = _____
76 - 5 = _____

54 - 1 = _____
84 - 1 = _____

79 - 6 = _____
39 - 6 = _____

Ask what basic fact is used in each.

Pupil's book, pages 118,119:

> Give help as needed on the first few examples. Children should then complete the pages independently.

## Subtraction

| | | |
|---|---|---|
| 19 - 4 = 15<br>10 + 9<br>- 4<br>10 + 5 | 82 - 2 = 80<br>80 + 2<br>- 2<br>80 + 0 | 36 - 3 = 33<br>30 + 6<br>- 3<br>30 + 3 |
| 44 - 1 = 43<br>40 + 4<br>- 1<br>40 + 3 | 68 - 5 = 63<br>60 + 8<br>- 5<br>60 + 3 | 97 - 6 = 91<br>90 + 7<br>- 6<br>90 + 1 |
| 23 - 2 = 21<br>20 + 3<br>- 2<br>20 + 1 | 79 - 3 = 76<br>70 + 9<br>- 3<br>70 + 6 | 38 - 6 = 32<br>30 + 8<br>- 6<br>30 + 2 |
| 75 - 3 = 72<br>70 + 5<br>- 3<br>70 + 2 | 65 - 4 = 61<br>60 + 5<br>- 4<br>60 + 1 | 43 - 3 = 40<br>40 + 3<br>- 3<br>40 + 0 |

## Subtraction

Rename, subtract, and fill the blank.

| | | |
|---|---|---|
| 57 - 5 = 52<br>50 + 7<br>- 5<br>50+2 | 98 - 3 = 95 | 89 - 7 = 82 |
| 24 - 4 = 20 | 18 - 2 = 16 | 79 - 5 = 74 |
| 56 - 2 = 54 | 84 - 3 = 81 | 75 - 2 = 73 |
| 46 - 6 = 40 | 37 - 4 = 33 | 38 - 5 = 33 |

## IV-2. Simple addition and subtraction

**Objective:** To compute simple sums and differences involving two 2-digit numerals.

**Vocabulary:** (No new words.)

**Materials:** Sticks in bundles of ten and single sticks, or theater tickets in strips of ten and single tickets.

**Suggested Procedure:**

Display bundles of sticks, or strips of tickets, but no single sticks or tickets. Discuss the fact that you have a set of 3 tens and a set of 4 tens.

> What will we have if we join the 4 tens to the 3 tens? (7 tens.)
>
> Can we say that 3 tens plus 4 tens is equal to 7 tens? (Yes.)
>
> How can we write this? (30 + 40 = 70.)

Repeat with other sets of tens. Ask what basic fact was used. (3 + 4 = 7.) Use sticks or tickets to show subtraction in the same way: Start with a set of 8 tens, have 5 tens removed and write:

80 - 50 = 30.

In each case, ask what basic fact was used. (Above, 8 - 5 = 3.)

**Pupil's book, page 120:**

> Help children to see the relationship between the pairs of sentences on the left. They should complete the right side of the page independently.

**Pupil's book, page 121:**

> Work with the class using the problems at the top of the page. They should complete the bottom part of the page independently.

## Adding Tens

Fill the blanks.

4 tens and 5 tens are 9 tens.

40 + 50 = 90

3 tens and 2 tens are 5 tens.

30 + 20 = 50

7 tens and 1 ten are 8 tens.

70 + 10 = 80

6 tens and 3 tens are 9 tens.

60 + 30 = 90

2 tens and 5 tens are 7 tens.

20 + 50 = 70

5 tens and 3 tens are 8 tens.

50 + 30 = 80

1 ten and 6 tens are 7 tens.

10 + 60 = 70

20 + 20 = 40
60 + 10 = 70
50 + 40 = 90
40 + 30 = 70
70 + 20 = 90
30 + 60 = 90
20 + 40 = 60
50 + 10 = 60
40 + 40 = 80
20 + 70 = 90
10 + 50 = 60
60 + 20 = 80
40 + 50 = 90
20 + 60 = 80

## Removing and Subtracting Tens

Fill the blanks.

Start with 9 tens. Remove 4 tens. There are 5 tens in the set remaining.

90 - 40 = 50

Start with 7 tens. Remove 2 tens. There are 5 tens in the set remaining.

70 - 20 = 50

Start with 8 tens. Remove 6 tens. There are 2 tens in the set remaining.

80 - 60 = 20

60 - 30 = 30
80 - 20 = 60
90 - 50 = 40
60 - 50 = 10
90 - 70 = 20
80 - 40 = 40

60 - 10 = 50
60 - 40 = 20
90 - 30 = 60
80 - 10 = 70
70 - 40 = 30
90 - 20 = 70

80 - 30 = 50
50 - 20 = 30
20 - 10 = 10
70 - 60 = 10
40 - 20 = 20
50 - 40 = 10

- Display sticks in two sets: a set consisting of 2 tens and 3 ones, and a set of 4 tens. Have a child show that the number of the union of these two sets is 6 tens and 5 ones. Have children give the equation:

$$25 + 40 = 65.$$

Suggest that the problem be written using the expanded form used in Section IV-1. When 25 has been renamed as 20 + 5, ask how many ones are to be added to the 5 ones. (None. No single sticks were joined to the 5 single sticks.)

Show:

$$\begin{array}{l} 20 + 5 \\ \underline{40\phantom{ + 5}} \\ 60 + 5 \end{array}$$

Give several problems of this sort. Show also that 20 + 35 = _____ may be solved by writing:

$$\begin{array}{l} 20 \\ \underline{30 + 5} \\ 50 + 5 \end{array}$$

Emphasize the fact that only the number of tens changes, because no ones are added to ones.

When children understand how to add a multiple of ten to a number of tens and ones, use sticks and show removing 30 sticks from 47 sticks. Give as many examples as necessary, using the form:

$$\begin{array}{rl} & 40 + 7 \\ - & \underline{30\phantom{ + 7}} \\ & 10 + 7 \end{array}$$

Pupil's book, pages 122,123

These pages provide practice. Children are to write problems in expanded form and fill the blanks to complete equations.

## Sums of Numbers

| | | |
|---|---|---|
| 63 + 20 = 83<br>60 + 3<br>20<br>80 + 3 | 45 + 30 = 75<br>40 + 5<br>30<br>70 + 5 | 70 + 18 = 88<br>70<br>10 + 8<br>80 + 8 |
| 39 + 60 = 99 | 14 + 40 = 54 | 56 + 30 = 86 |
| 82 + 10 = 92 | 60 + 15 = 75 | 50 + 48 = 98 |
| 10 + 56 = 66 | 40 + 53 = 93 | 64 + 30 = 94 |

## Subtraction

76 - 50 = 26

70 + 6
-50
20 + 6

43 - 10 = 33

40 + 3
- 10
30 + 3

59 - 40 = 19

50 + 9
- 40
10 + 9

31 - 30 = 1

98 - 60 = 38

24 - 10 = 14

65 - 50 = 15

58 - 30 = 28

64 - 40 = 24

16 - 10 = 6

87 - 50 = 37

97 - 70 = 27

- Have a child use sticks or tickets, arranged as tens and ones to show:

$$36 + 21 = 57$$

Discuss the fact that when he joined 2 tens and 1 one to 3 tens and 6 ones, the number of the union was 5 tens and 7 ones.

Have 36 and 21 renamed, and write:

$$\begin{array}{r} 30 + 6 \\ \underline{20 + 1} \end{array}$$

We add the ones: $6 + 1 = 7$.
We add the tens: $30 + 20 = 50$.
We get $50 + 7$, which is 57.

Present other problems, such as $72 + 24 =$ _____, and have children rewrite the problem, renaming both numbers, and compute the sum. If necessary, let them use sticks or tickets to illustrate.

Pupil's book, page 124:

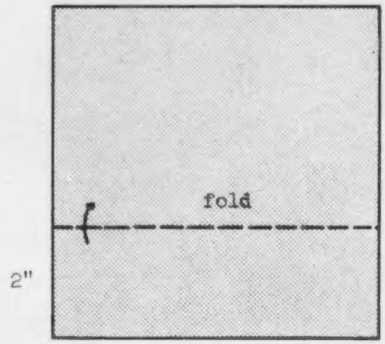
Children are to rename numbers, and, after computing the sums, fill the blanks.

Further Activities:

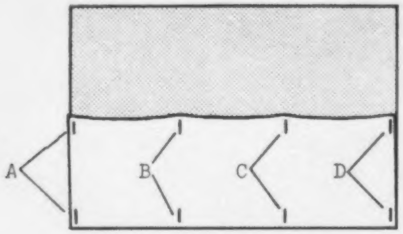
Have children use scratch paper and "Show-me" cards (description below) to show answers to problems written on the chalkboard.

## HOW TO MAKE "SHOW-ME" CARDS

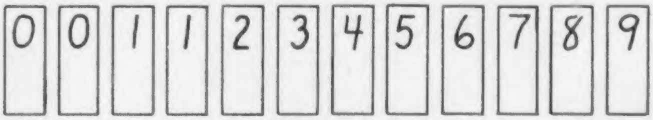
1. Use a piece of tagboard 6" × 6". Fold up 2" from the bottom.



2. Staple at A, B, C, and D to make 3 pockets, each almost 2" wide.



3. Cut a strip of tagboard 18" × 4" into 12 strips $1\frac{1}{2}$" × 4". With felt pen, write the numerals as follows:



4. Children should be taught early to lay out numeral cards in order on their desks and to replace them in order.

5. In the game, pupils compute the solution to a problem that you give orally or write on the chalkboard. Children place the numerals for the answer in the pockets, hold the cards against their chests with the answers concealed until the teacher says, "Show-me!" Then all turn cards and hold them, answers toward teacher, against their chests while you make a quick check to see who did and who did not make the correct response.

## Practice in Addition

| | |
|---|---|
| 74 + 15 = 89<br>70 + 4<br>10 + 5<br>80+9 | 12 + 45 = 57<br>10 + 2<br>40 + 5<br>50 + 7 |
| 36 + 61 = 97 | 63 + 24 = 87 |
| 57 + 12 = 69 | 81 + 16 = 97 |
| 58 + 30 = 88 | 75 + 22 = 97 |

- Use sticks or tickets as before, starting with a set of 46 members, having a set of 4 removed, and writing 46 - 4 = 42. Show this also as:

$$\begin{array}{r} 40 + 6 \\ - 4 \\ \hline 40 + 2 \end{array}$$

Next, replace the 4 sticks and have a child remove 34 sticks. Write:

46 - 34 = ____

Begin to rewrite the problem:

40 + 6

Ask what number is to be subtracted from 46. (34.)

> How can we show that 30 + 4 is the name of the number we want to subtract? If we put the minus symbol only in front of the 30, the problem will show only that we want to subtract 30 from 40.

If no child suggests parentheses around 30 + 4, explain that you will put parentheses around the expression which is the name of a number, and that the minus sign in front of the parentheses shows that you want to subtract both 30 and 4 from 46.

$$\begin{array}{r} 40 + 6 \\ - \underline{(30 + 4)} \end{array}$$

Emphasize by saying:

> We still have a plus sign between 30 and 4 because 30 + 4 is the name of the number we want to subtract.

Show the computation, asking as you work:

> Do we think 6 + 4 or 6 - 4? (6 - 4.)
> What do we think next? (40 - 30 = 10.)

Write:

$$\begin{array}{r} 40 + 6 \\ -\underline{(30 + 4)} \\ 10 + 2 \end{array}$$

Give similar examples, reminding children, if necessary, that the ones must be subtracted, as indicated by the minus sign in front of the name of the number.

Pupil's book, pages 125,126:

The first of these subtraction pages may require some supervision. The second should be completed independently.

Pupil's book, page 127:

Caution children to pay close attention to the plus or minus symbol.

## Subtraction

| | |
|---|---|
| 68 - 56 = 12<br>60 + 8<br>- (50 + 6)<br>10+2 | 45 - 24 = 21<br>40 + 5<br>-(20 + 4)<br>20 + 1 |
| 83 - 51 = 32<br>80 + 3<br>- (50 + 1)<br>30 + 2 | 31 - 21 = 10<br>30 + 1<br>- (20 + 1)<br>10 + 0 |
| 59 - 54 = 5 | 76 - 42 = 34 |
| 27 - 15 = 12 | 12 - 12 = 0 |

## Subtraction

| | |
|---|---|
| 99 - 53 = 46<br>90 + 9<br>- (50 + 3)<br>40 + 6 | 44 - 13 = 31<br>40 + 4<br>- (10 + 3)<br>30 + 1 |
| 86 - 43 = 43 | 57 - 34 = 23 |
| 78 - 27 = 51 | 65 - 42 = 23 |
| 75 - 22 = 53 | 87 - 17 = 70 |

## Practice with Addition and Subtraction

32 + 51 = 83

83 + 13 = 96

45 - 24 = 21

89 - 72 = 17

57 - 25 = 32

66 + 32 = 98

46 - 16 = 30

93 - 71 = 22

## IV-3. Addition and subtraction facts for sums of 11 through 14

**Objective:** To teach the basic addition and subtraction facts corresponding to sums of 11 through 14, using partitions.

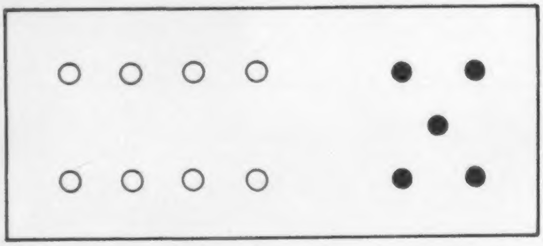
**Vocabulary:** (No new words.)

**Materials:** Objects for flannel board, yarn, felt numerals, sets of objects for children.

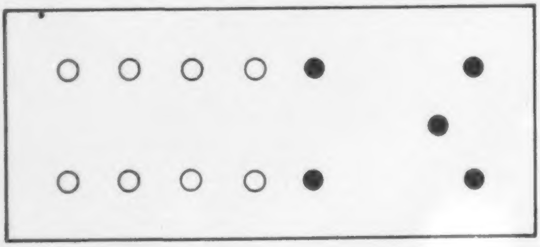
**Suggested Procedure:**

**Using Sets of 10 for Addition**

Display sets of 8 and 5 objects on the flannel board, such as:



Direct pupils' attention to the idea that forming a set of 10 may help us find how many objects would be in the new set if we were to join the set of 5 to the set of 8. Ask how many objects should be joined with the set of 8 to form a set of 10. (2.) Show this on the flannel board, with the objects now arranged in this way:



Write on the chalkboard

$8 + 5 = 10 + \_\_\_\_$

and have the children indicate how to complete the equation correctly.

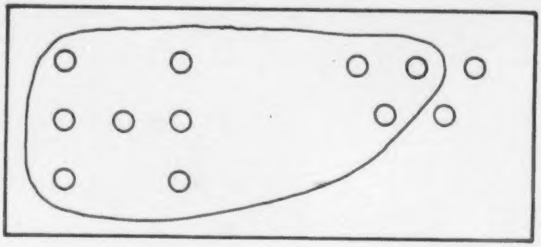
$$8 + 5 = 10 + \underline{\ 3\ }.$$

Help children to sense that since we know that $10 + 3 = 13$, we know that $8 + 5 = 13$. Write a final sentence on the chalkboard so that the following now appears:

$$8 + 5 = 10 + \underline{\ 3\ }$$

So, $\quad 8 + 5 = 13$

Use several similar examples. In each case use a piece of yarn or string to show the formation of a set of 10, as:



Along with this, write:

$$7 + 5 = 10 + \underline{\ 2\ }$$
$$7 + 5 = 12$$

Have children use sets of objects on their desks to find:

$9 + 4 =$ ___, $6 + 7 =$ ___, $8 + 6 =$ ___, etc.

Pupil's book, pages 128,129:

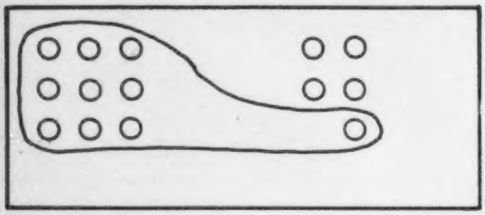
Children should draw a ring around a set of 10, then fill the blanks to complete the equations.

Pupil's book, page 130:

This page reviews partitions of 10, and prepares for forthcoming work, in which children are encouraged to use the associative property of addition to find a sum such as $9 + 4$ without using objects.

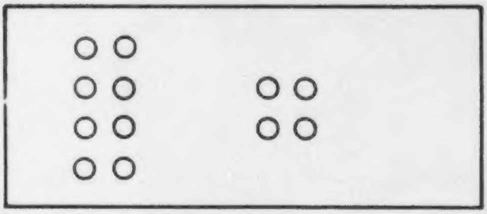
## Using Ten in Addition

Ring ten. Fill the blanks.



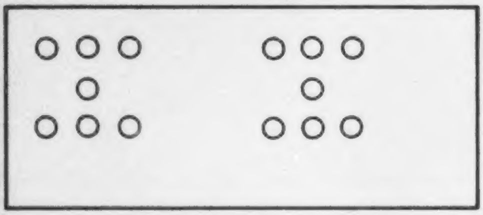
9 + 5 = 10 + 4

9 + 5 = 14



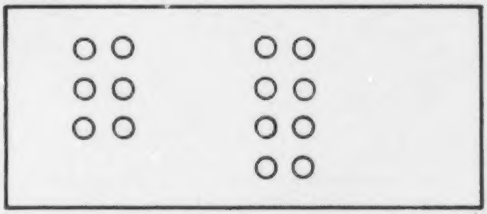
8 + 4 = 10 + 2

8 + 4 = 12



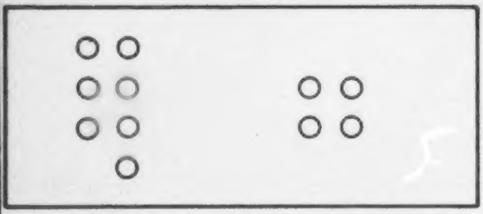
7 + 7 = 10 + 4

7 + 7 = 14



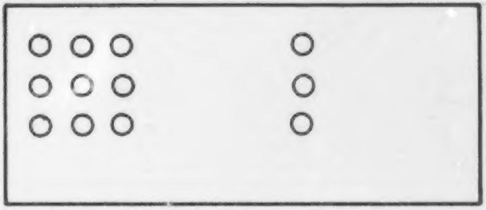
6 + 8 = 10 + 4

6 + 8 = 14



7 + 4 = 10 + 1

7 + 4 = 11

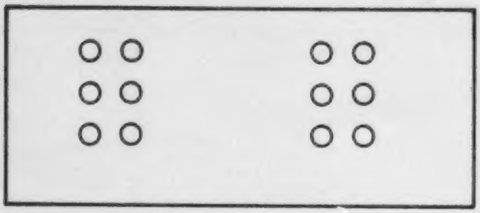


9 + 3 = 10 + 2

9 + 3 = 12

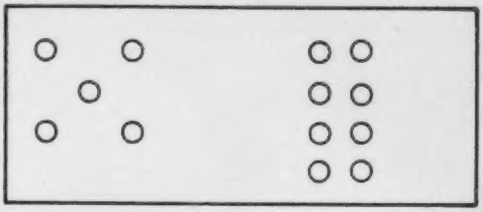
## Using Ten in Addition

Ring ten. Fill the blanks.



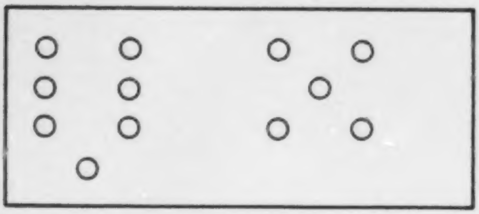
6 + 6 = 10 + 2

6 + 6 = 12



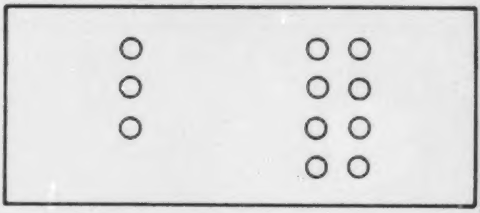
5 + 8 = 10 + 3

5 + 8 = 13



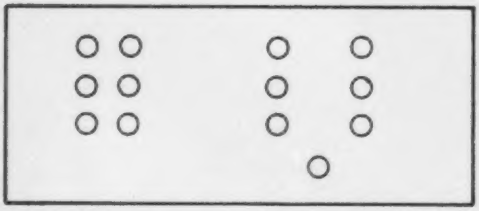
7 + 5 = 10 + 2

7 + 5 = 12



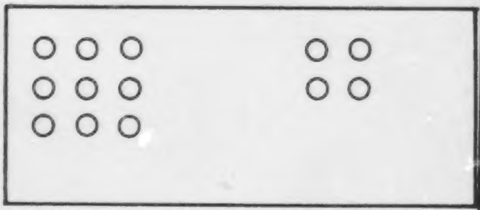
3 + 8 = 10 + 1

3 + 8 = 11



6 + 7 = 10 + 3

6 + 7 = 13



9 + 4 = 10 + 3

9 + 4 = 13

## Partitions of Ten

Fill the blanks.

| 10<br>is the sum of |
|---|
| 2 and 8 |
| 6 and 4 |
| 4 and 6 |
| 8 and 2 |
| 1 and 9 |
| 5 and 5 |
| 3 and 7 |
| 9 and 1 |
| 7 and 3 |

8 + 2 = 10

5 + 5 = 10

4 + 6 = 10

2 + 8 = 10

10 = 2 + 8

10 = 5 + 5

10 = 3 + 7

10 = 6 + 4

1 + 9 = 10

7 + 3 = 10

6 + 4 = 10

9 + 1 = 10

10 = 7 + 3

10 = 1 + 9

10 = 4 + 6

10 = 9 + 1

## Adding to Ten

10 + 4 = 14

10 + 3 = 13

10 + 1 = 11

10 + 6 = 16

10 + 9 = 19

10 + 7 = 17

10 + 8 = 18

10 + 5 = 15

10 + 2 = 12

- Arrange 11 objects on the flannel board as a set of 6 and a set of 5. Put felt numerals 6 and 5 under the sets.

Write:

$$6 + 5 = \_\_\_$$

> Ask yourself, "What subset of the set of 5 can I remove from the set of 5 and join to the set of 6 so that I will have a set of 10?" Think: Six plus what number equals ten? (4.) So we partition the set of 5 into a set of 4 and a set of 1.

Move 4 of the objects a little to one side and replace the 5 numeral with the numerals 4 and 1.

> Now we join the set of 4 to the set of 6.

Replace the 6 and 4 numerals with 10.

> And we join the set of 1 to the set of 10. How many members has the union? (11.)

Explain as you write:

$6 + 5 = 6 + (4 + 1)$ (4 + 1) is another name for 5.
$= (6 + 4) + 1$ (Associative property.)
$= 10 + 1$
$= 11$

Pupil's book, page 131:

> Children are encouraged to use their knowledge of the partitions of 10 in finding sums greater than 10 without manipulative materials or pictures.

## Using 10 in Addition

Fill the blanks.

9 + 4 = 10 + 3
9 + 4 = 13

7 + 5 = 10 + 2
7 + 5 = 12

5 + 6 = 10 + 1
5 + 6 = 11

7 + 4 = 10 + 1
7 + 4 = 11

6 + 8 = 10 + 4
6 + 8 = 14

7 + 6 = 10 + 3
7 + 6 = 13

8 + 4 = 10 + 2
8 + 4 = 12

3 + 8 = 10 + 1
3 + 8 = 11

4 + 9 = 10 + 3
4 + 9 = 13

5 + 8 = 10 + 3
5 + 8 = 13

6 + 7 = 10 + 3
6 + 7 = 13

5 + 9 = 10 + 4
5 + 9 = 14

4 + 7 = 10 + 1
4 + 7 = 11

3 + 9 = 10 + 2
3 + 9 = 12

- Using 10 in subtraction

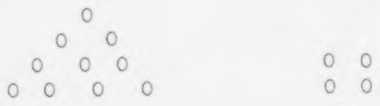
Note: Children may use several approaches to subtraction, three of which are suggested in this lesson. What is easiest for one child may be most difficult for another. Present all three methods and let each child decide which he will use.

Suppose a child is given the problem 14 - 6 = ____.

He may simply think of the inverse of a basic addition fact he knows: 8 + 6 = 14. Therefore, 14 - 6 = 8. If he worked with a set of 14 objects, he would simply remove a set of 6 objects and count the objects in the set remaining.

He may think of 14 as 10 + 4. He then subtracts 6 from the 10 and adds the resulting 4 to the 4 ones of 14. This may be illustrated with a set of 14 objects.

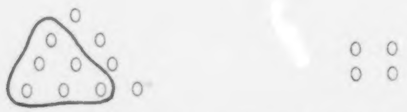
Place on the flannel board a set of 14 objects, arranged as shown:



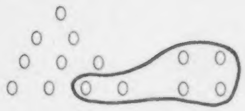
Have the number of the set identified, and write:

14 - 6 = ____

Ask a child to show how he would remove 6 objects from the set of 14. The child may remove a set of 6 from the set of 10, move the 4 objects over to the remainder set, and think 10 - 6 = 4 and 4 + 4 is 8.



The child may think of first subtracting the 4 ones from 14, and then subtracting 2 from 10. On the flannel board, with a set of objects arranged as before, he would first remove the set of 4, remove 2 more from the set of 10, and observe that there were 8 in the remaining set.



Give several problems and have children use manipulative materials to solve them. Suggest that materials be arranged on desks as sets of 10 and another set, as indicated by the problem. When manipulative materials have been put away, present problems and ask children to tell what they are thinking as they solve them.

Pupil's book, pages 132,133:

Children are to ring the set they think of removing. Children who need to use their knowledge of the partitions of 10 may continue to "find ten" for some time. Many children learn the "doubles" (6 + 6, 7 + 7) easily, and will find ways to use these in learning other facts. For instance, a child may think: 6 + 7 = 6 + 6 + 1, 6 + 6 = 12, so 6 + 7 = 13.

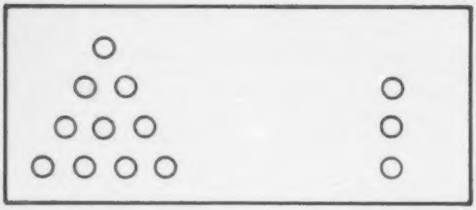
Pupil's book, pages 134-137:

These pages provide practice on basic facts for sums of 11 through 14. Additional practice may be given, using Doughnuts, flash cards, etc., while Chapter V is taught.

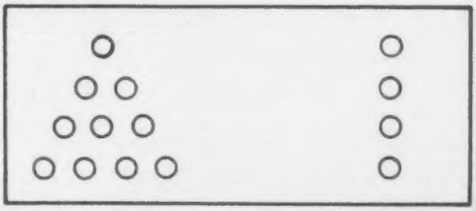
## Using 10 in Subtraction

Ring the set you think of removing.

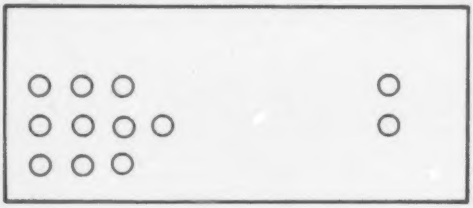
Fill the blanks.



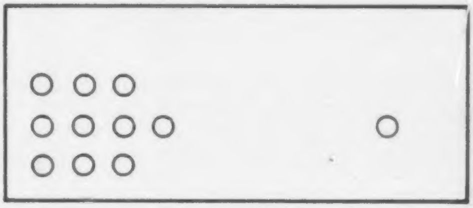
13 - 8 = 5



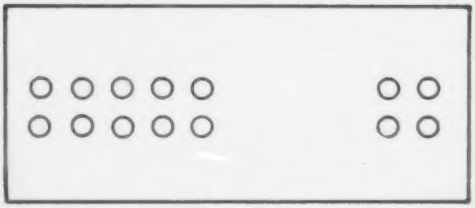
14 - 6 = 8



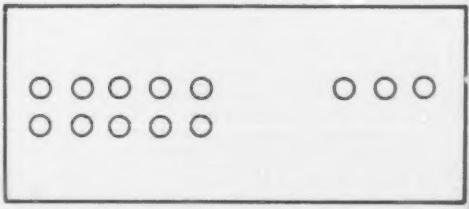
12 - 4 = 8



11 - 2 = 9



14 - 9 = 5

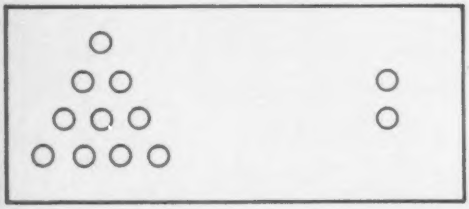


13 - 6 = 7

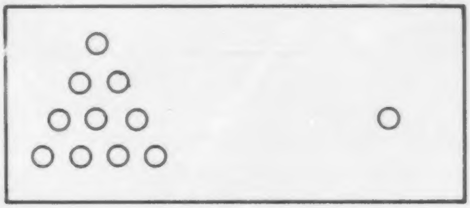
## Using 10 in Subtraction

Ring the set you think of removing.

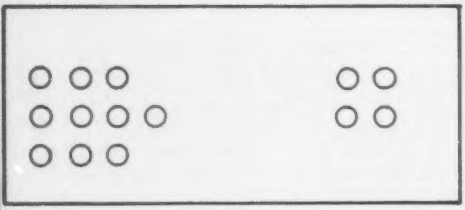
Fill the blanks.



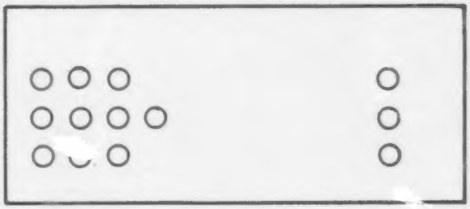
12 - 9 = 3



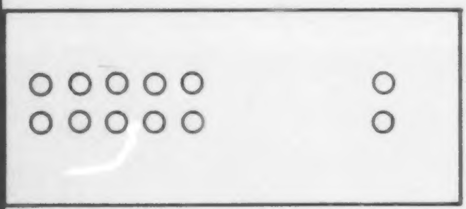
11 - 6 = 5



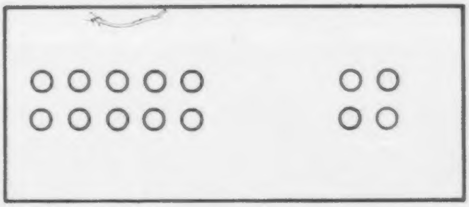
14 - 7 = 7



13 - 5 = 8



12 - 7 = 5



14 - 8 = 6

## Partitions, Addition, and Subtraction

11
is the sum of

3 and 8
7 and 4
5 and 6
2 and 9
8 and 3
4 and 7
6 and 5
9 and 2

5 + 5 = 10
5 + 6 = 11
5 + 7 = 12
6 + 6 = 12
6 + 5 = 11
10 - 5 = 5
11 - 5 = 6
10 - 6 = 4
11 - 6 = 5

12
is the sum of

8 and 4
5 and 7
9 and 3
4 and 8
7 and 5
3 and 9
6 and 6

10 - 8 = 2
12 - 8 = 4
10 - 4 = 6
12 - 4 = 8
10 - 9 = 1
12 - 9 = 3

10 - 3 = 7
11 - 3 = 8
10 - 7 = 3
12 - 7 = 5
10 - 5 = 5
12 - 3 = 9

12 - 6 = 6
11 - 7 = 4
12 - 5 = 7
4 + 8 = 12
7 + 4 = 11
3 + 8 = 11

Practice with Basic Facts

Fill the boxes.

| + | 3 | 2 |
|---|---|---|
| 5 | 8 | 7 |
| 7 | 10 | 9 |
| 9 | 12 | 11 |

| + | 4 | 3 |
|---|---|---|
| 8 | 12 | 11 |
| 6 | 10 | 9 |
| 4 | 8 | 7 |

| + | 5 | 4 |
|---|---|---|
| 7 | 12 | 11 |
| 5 | 10 | 9 |
| 6 | 11 | 10 |

4 + 6 = 10

3 + 8 = 11

8 + 4 = 12

2 + 10 = 12

6 + 5 = 11

7 + 5 = 12

9 + 2 = 11

7 + 5 = 12

4 + 8 = 12

Find the sums:

| | | | | | |
|---|---|---|---|---|---|
| 2 | 4 | 5 | 3 | 4 | 6 |
| 3 | 4 | 2 | 3 | 2 | 1 |
| 7 | 4 | 5 | 5 | 6 | 5 |
| 12 | 12 | 12 | 11 | 12 | 12 |

| | | | | | |
|---|---|---|---|---|---|
| 7 | 9 | 4 | 3 | 2 | 3 |
| 2 | 1 | 0 | 4 | 4 | 5 |
| 3 | 2 | 8 | 5 | 5 | 3 |
| 12 | 12 | 12 | 12 | 11 | 11 |

## Practice with Basic Facts

13
is the sum of

5 and 8
9 and 4
4 and 9
8 and 5
6 and 7
7 and 6

7 + 3 = 10
7 + 6 = 13
8 + 5 = 13
6 + 7 = 13
5 + 8 = 13
9 + 4 = 13
4 + 9 = 13
6 + 6 = 12

12 = 6 + 6
13 = 8 + 5
11 = 3 + 8
12 = 8 + 4
11 = 6 + 5
13 = 5 + 8
12 = 7 + 5
13 = 4 + 9

13 - 4 = 9
12 - 8 = 4
13 - 6 = 7
11 - 6 = 5

12 - 3 = 9
13 - 7 = 6
12 - 5 = 7
11 - 4 = 7

13 - 8 = 5
12 - 9 = 3
12 - 4 = 8
13 - 9 = 4

Write the sums:

| | | | | | | |
|---|---|---|---|---|---|---|
| 5 | 6 | 2 | 2 | 2 | 4 | 2 |
| 5 | 3 | 4 | 8 | 3 | 5 | 5 |
| 3 | 4 | 7 | 1 | 8 | 4 | 4 |
| 13 | 13 | 13 | 11 | 13 | 13 | 11 |

## Practice with Basic Facts

14
is the sum of

7 and 7

8 and 6

5 and 9

9 and 5

6 and 8

5 + 5 = 10

6 + 6 = 12

7 + 7 = 14

4 + 9 = 13

6 + 7 = 13

8 + 6 = 14

10 - 5 = 5

12 - 6 = 6

14 - 7 = 7

12 - 8 = 4

13 - 9 = 4

14 - 5 = 9

5 + 7 = 12

9 + 5 = 14

6 + 8 = 14

6 + 8 = 14

8 + 3 = 11

5 + 7 = 12

9 + 5 = 14

8 + 4 = 12

9 + 3 = 12

7 + 6 = 13

8 + 6 = 14

4 + 8 = 12

9 + 3 = 12

8 + 5 = 13

13 - 5 = 8

13 - 7 = 6

11 - 3 = 8

12 - 3 = 9

13 - 8 = 5

13 - 7 = 6

11 - 9 = 2

## IV-4. Addition and subtraction facts for sums of 15 through 18

Objective: To extend the study of addition and subtraction facts to those corresponding to sums through 18.

Vocabulary: (No new words.)

Materials: None.

Suggested Procedure:

Review the term basic facts and the way such facts are used in finding sums and differences of numbers when at least one of the numbers is greater than 10.

| | |
|---|---|
| 7 + 2 = 9 (basic fact) | 8 - 5 = 3 (basic fact) |
| 17 + 2 = 19 | 18 - 5 = 13 |
| 27 + 2 = 29, etc. | 28 - 5 = 23, etc. |

> How many basic facts do we know in which 6 is the first addend? For instance, there are: 6 + 5 = 11, 6 + 3 = 9.

Write three or four facts from children's suggestions, then suggest that there is a systematic way to write them, and make a chart on the chalkboard:

| + | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 6 | | | | | | | | | |

> What is 6 + 0? 6 + 1? etc.

When you reach 6 + 8, say:

> Now we will have a new fact. What is 6 + 9? (Write 15.) Let's go on. 6 + 10 = 16. Is this a basic fact? (No.)

Write:

$$\begin{array}{r} 6 \\ \underline{10} \\ 16 \end{array}$$

Discuss the idea that 6 + 0 is the fact needed in order to add in the one's place, and that 6 + 10 or 10 + 6 are simply ways of renaming 16.

Write:

$$\begin{array}{r} 6 \\ \underline{10 + 1} \end{array} \qquad \begin{array}{r} 6 \\ \underline{10 + 2} \end{array} \qquad \text{(Etc., to } 6 + 20\text{)}$$

Have children give the basic fact used in each problem. (6 + 1 = 7, 6 + 2 = 8, etc.)

> When you have learned 6 + 9 = 15, you will know all the basic addition facts that begin 6 + , and of course you will also know 9 + 6, 8 + 6, and the other facts which have 6 as the second addend.

Some children may wish to use their knowledge of the partitions of 10 in adding 6 and 9, and of course they should be allowed to do so as long as it is helpful to them. This is equally true of the other facts for sums to 18. However, many children will be encouraged to memorize the remainder of the basic facts simply because there are so few of them.

Make another chart for 7. Show that the only new facts are 7 + 8 = 15 and 7 + 9 = 16, and remind the children that when they know these, they also know that 8 + 7 = 15 and 9 + 7 = 16.

Repeat with 8 (8 + 8 = 16 and 8 + 9 = 17) and with 9 (9 + 9 = 18).

Write the basic addition facts which have 9 as first addend, and beside them show 10 plus each second addend, as shown below.

| | |
|---|---|
| 9 + 0 = 9 | 10 + 0 = 10 |
| 9 + 1 = 10 | 10 + 1 = 11 |
| 9 + 2 = 11 | 10 + 2 = 12 |
| 9 + 3 = 12 | 10 + 3 = 13 |
| etc. | etc. |

Children should observe that 9 plus a number is one less than 10 plus that number. 9 + 4 is 1 less than 10 + 4.

Pupil's book, page 138:

This page provides practice on basic addition and subtraction facts, and a review of comparison of numbers.

Pupil's book, page 139:

Children should at first show on the table only the sums requested. After checking to see that the use of the table is understood, have them complete the table.

Pupil's book, pages 140-141:

Children fill the blanks on 140. On the following page, they are to follow the dots by drawing a path in the order shown by their answers on page 140.

★ Pupil's book, page 142:

(Optional) for more able pupils. Problems review doing and undoing and may be written in the space provided.

Fill the blanks.

9 + 8 = 17

6 + 9 = 15

7 + 8 = 15

9 + 7 = 16

8 + 6 = 14

9 + 9 = 18

9 + 6 = 15

8 + 8 = 16

9 + 5 = 14

8 + 7 = 15

7 + 9 = 16

8 + 9 = 17

17 - 8 = 9

16 - 9 = 7

18 - 9 = 9

15 - 8 = 7

17 - 9 = 8

16 - 8 = 8

Put in < (is less than), > (is greater than), or =.

5 + 9 < 8 + 7

9 + 7 < 8 + 9

7 + 6 = 5 + 8

6 + 8 < 7 + 9

18 - 9 > 15 - 7

14 - 7 < 16 - 8

17 - 9 < 15 - 6

15 - 8 < 14 - 6

14 - 5 = 17 - 8

16 - 9 < 14 - 5

9 + 6 < 8 + 8

8 + 5 < 7 + 7

## An Addition Table

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | | |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | 14 |
| 6 | | | | | | | | | 14 | 15 |
| 7 | | | | | | | | 14 | 15 | 16 |
| 8 | | | | | | | 14 | 15 | 16 | 17 |
| 9 | | | | | | 14 | 15 | 16 | 17 | 18 |

Show the following on the table:

9 + 6　　9 + 7　　9 + 8　　6 + 9　　7 + 7

8 + 7　　7 + 9　　8 + 9　　9 + 5　　6 + 8

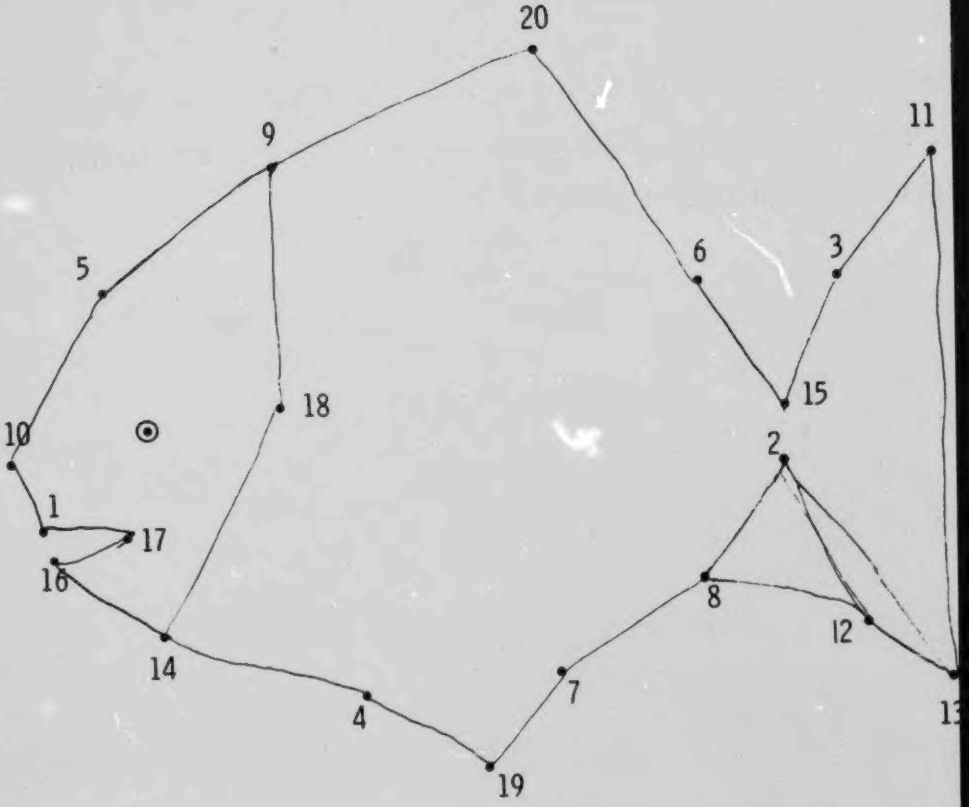
7 + 8　　8 + 8　　9 + 9　　8 + 6　　5 + 9

## For Fun!

Fill the blanks.

When you have the first blank filled, find the dot that shows your answer. Start there. Draw a path from that dot to the dot that shows your second answer, and so on.

1. 16 - 7 = 9
2. 9 + 9 = 18
3. 8 + 6 = 14
4. 13 - 9 = 4
5. 12 + 7 = 19
6. 14 - 7 = 7
7. 17 - 9 = 8
8. 10 - 8 = 2
9. 9 + 3 = 12
10. 8 + 5 = 13
11. 7 + 4 = 11
12. 11 - 8 = 3
13. 8 + 7 = 15
14. 15 - 9 = 6
15. 17 + 3 = 20
16. 14 - 5 = 9
17. 11 - 6 = 5
18. 6 + 4 = 10
19. 10 - 9 = 1
20. 9 + 8 = 17
21. 8 + 8 = 16
22. 9 + 5 = 14


20
9
11
6
3
5
15
18
10
2
1
17
16
14
8
12
1
7
4
19

What picture did you find?

## ★ Doing and Undoing

Fill the blanks.

36 + 42 = 78
78 - 36 = 42

23 + 41 = 64

67 - 44 = 23

26 + 42 = 68

47 + 12 = 59

95 - 54 = 41

78 - 13 = 65

74 - 40 = 34

## IV-5. Problem solving

**Objectives:** To develop skill in solving "two-step" problems in physical situations.

**Vocabulary:** (No new words.)

**Materials:** Colored pencils, 8 yellow, 8 of colors other than yellow. Flannel board and felt set materials including 2 red and 3 blue felt disks.

### Suggested Procedure:

Note: You should be constantly aware of the fact that for many story problems there are several ways of solving the problems and that one way may be just as correct as another way. As long as the child is able to explain how he reached the conclusion he should not be forced to use a way that may be given in "Suggested Procedure" to illustrate a possible development.

Children should be encouraged to relate the equation directly to the problem situation. However, after the equation is written, they may find it helpful to express the same relationship between the three numbers, using another equation. For example, for the problem

> Mother needed 35 candles for a
> birthday cake.
> She had 12 candles.
> How many more candles does she need?

A pupil first may write the equation

$$35 = 12 + \underline{\quad\quad}, \quad (\text{or } 12 + \underline{\quad\quad} = 35)$$

and then he may write

$$35 - 12 = \underline{\quad\quad}$$

## Solving Two-Step Story Problems

Write the following story on the chalkboard:

> Mother had 2 red hats.
> She had 3 blue hats.
> She bought a new white hat.
> How many hats does Mother have now?

Discuss what is to be found and the other information given in the story problem. The children should be encouraged to tell how this story is different from the story problems they have been solving. (There are more than two facts related to the unknown.)

> Ask a child to show on the flannel board, sets equivalent to Mother's red hats, her blue hats and her new white hat.
>
> Discuss how the problem might be solved. The children might say they could put the red and blue hats together to find how many hats Mother had before she got the white one.
>
> Have this action demonstrated by a child. Discussion should continue and lead the child to imagine joining the set just formed with the set of one white hat. The equation which may be used to help solve the problem should then be written.

$$2 + 3 + 1 = \_\_\_\_$$

Relating the equation to the set action, the children should be encouraged to think of the equation as $(2 + 3) + 1 = \_\_\_\_$ and to complete the equation.

> What is the answer to our problem?
> (Mother had 6 hats.)

Write the following story on the chalkboard:

> Kelly bought 2 new pencils.
> He already had 3 short pencils
> and 4 long ones.
> How many pencils does he have now?

Discuss the question asked and the information given. Write the equation which could be used to help solve the problem.

$$2 + 3 + 4 = \underline{\qquad}$$

Encourage the children to look carefully at the equation. Frame the two addends with your hands as you refer to them.

> We know that our answer will be the same whether we say $2 + 3 = 5$ and then add $5 + 4$, or whether we say $3 + 4 = 7$ and then add $2 + 7$. Either way the answer will be 9. We want to use the equation to help us solve this problem. (Indicate the story on the board.)

On the flannel board place sets of material which are equivalent to the sets of pencils in the story problem. Indicate the set which is equivalent to the set of pencils Kelly bought and the two sets which are equivalent to the set of pencils he already had. Determine which sets would most likely have been joined first. (The two sets of pencils he had.)

Join the set having 3 members with the set having 4 members. Enclose $3 + 4$ in parentheses. Write $2 + 7 = \underline{\quad}$ under the first equation and complete.

$$2 + (3 + 4) = \underline{\qquad}$$
$$2 + 7 = \underline{\ 9\ }$$

Erase the equation.

Discuss other possibilities. For example, perhaps Kelly had the 3 short pencils in one drawer and the 4 long ones in another drawer. When he brought the 2 new pencils home he put them in the drawer with the 4 long ones. Then what equation could we use that would help us solve the problem?

Encourage the children to show that in this case they might write:

$(2 + 4) + 3 = \_\_\_$

$6 + 3 = \underline{\ 9\ }$

Write the following story on the chalkboard:

Father had 3 golf balls.
He bought 6 golf balls.
He gave 4 golf balls to Mother.
How many golf balls did he have then?

Discuss how this story might be demonstrated with flannel board materials. There must be a set equivalent to the set of golf balls Father had and another set equivalent to the set of golf balls Father bought. Have the sets shown in positions slightly separated from each other.

Discuss what must be considered first. Did Father buy golf balls first or did he give golf balls to Mother first?

Relate buying the golf balls to the joining the two sets. Do not actually move the sets, just talk about joining them. The equation should then be written:

$$3 + 6 = 9$$

Encourage the children to tell the answer and then show by writing the appropriate equation how they know the answer is true.

Note: Some children may be ready to write $(3 + 6) - 4 = \_\_\_$ Do not discourage the use of this equation nor expect all children to understand.

Use similar procedures to discuss the following story problems:

1. Jack had 5 toy cars.
   He gave 2 cars to Pat.
   Jack's father gave another toy car
   to Jack.
   How many toy cars did Jack have then?

   ($5 - 2 = \underline{3}$, $3 + 1 = \underline{4}$)
   (Jack then had 4 toy cars.)

2. Sally got 3 dolls for her birthday.
She also got 4 stuffed toys.
She gave 2 stuffed toys to the baby.
How many stuffed toys does she have now?

(4 - 2 = 2 )
(She has 2 stuffed toys now.)

3. Susan picked 4 red apples.
She picked 5 greeen apples.
She gave 3 red apples to Bobby.
How many apples does Susan have now?

(4 + 5 = 9 , 9 - 3 = 6 )
(4 - 3 = 1 , 1 + 5 = 6 )
(Susan has 6 apples now.)

4. Miss Johns had 4 sheets of red paper
and 5 sheets of green paper.
She cut each sheet into 2 pieces.
How many pieces does she have now?

(4 + 5 = 9 , 9 + 9 = 18 )
(Miss Johns has 18 pieces of paper.)

5. Sue's mother baked 50 cookies.
Sue took 10 cookies to Mrs. Lee.
Sue's family ate 20 cookies.
How many cookies are there now?

(50 - 10 = 40, 40 - 20 = 20)
(There are 20 cookies now.)

6. Mrs. Bell bought a dozen eggs.
She broke 2 of the eggs.
She lent some eggs to Mrs. Fox.
How many eggs does Mrs. Bell have now?

No solution.

7. John had 6 kittens.
He gave 3 kittens to his friends.
He gave 1 kitten to the mailman.
How many kittens does John have now?

(6 - 3 = 3, 3 - 1 = 2)
(3 + 1 = 4, 6 - 4 = 2)
(John has 2 kittens.)

8. On Monday morning, Nan read 10 pages
from her book.
After lunch she read 5 pages.
In the evening she read 10 pages.
How many pages did she read on Monday?

(10 + 5 + 10 = 25)
(Nan read 25 pages on Monday.)

9. Ann had 18 cents.
She lost 8 cents.
Mother gave her 5 cents.
How many cents does Ann have now?

(18 - 8 = 10, 10 + 5 = 15)
(Ann has 15 cents now.)

10. At the seashore Betty found 8 pink shells
and 7 white shells.
She lost 5 of the shells.
How many shells did she have then?

(8 + 7 = 15, 15 - 5 = 10)
(8 - 5 = 3, 3 + 7 = 10)
(7 - 5 = 2, 2 + 8 = 10)
(She had 10 shells then.)

Pupil's book, pages 143,144: Children should be directed to complete these pages independently.

Problem Solving

Equations will vary.

Write the equations.

1. Jim had 40 marbles.
He gave 10 marbles to Bill
and 20 marbles to Jack.
How many marbles did Jim have then?
Jim had 10 marbles then.

2. Bob has 15 marbles.
Three marbles are red
and 7 marbles are blue.
How many marbles are not red or blue?
5 marbles are not red or blue.

3. Tom had 10 marbles.
Jerry gave 15 marbles to Tom.
Tom gave 5 marbles to Mike.
How many marbles did Tom have then?
Tom had 20 marbles then.

4. Jimmy had 20 marbles.
He gave 5 marbles to John.
Mother gave some marbles to Jimmy
and then he had 25 marbles.
How many marbles did Mother give to Jimmy?
Mother gave 10 marbles to Jimmy.

**Problem Solving**

Equations will vary.

Write the equations.

5. Father had 3 red books
and 4 brown ones.
He gave 2 of his brown books
to Grandfather.

____________________

____________________

How many red books and brown books
did Father have then?

Father had 5 red books and brown books.

6. Mother baked 20 chocolate cookies
and 30 white cookies.
The children ate 10 white cookies
and no chocolate cookies.
How many cookies did Mother have
then?

____________________

____________________

Mother had 40 cookies then.

7. Ten horses were in a field.
The farmer took 2 horses.

Eight cows came.
How many horses and cows are there
now?

____________________

____________________

There are 16 horses and cows now.

8. Kim had 2 dimes and
Mother gave him another dime.

He spent 8 cents.
How many cents does Kim have now?

____________________

____________________

Kim has 22 cents now.

### Preciseness in stating problems

Write the following problem on the chalkboard:

14 pencils are on a desk.
7 pencils are yellow.
How many pencils are not yellow?

Have the problem read. Discuss the question they are to answer. Encourage the children to give an answer.

Do you believe that your answer is right?

A child might show with the pencils how he thought out his response. Another child might write the equation $14 = 7 + \underline{7}$ or $14 - 7 = \underline{7}$.

Place 14 pencils on the desk, exactly 8 of which are yellow, the rest of a different color.

Are 7 of these pencils yellow?

The discussion will probably bring both "yes" and "no" responses. Ask a child to pick up 7 yellow pencils off the desk and then ask:

Are these 7 pencils yellow?

Have a child determine by counting that there are 7 yellow pencils in your hand. Point to the second sentence and ask:

Were 7 yellow pencils on the desk?
Were only 7 yellow pencils on the desk?
Does the problem tell us?
Could there have been more than 7 yellow pencils?
Let's see if we can change the wording of the problem so we can be sure that the number 7 is the answer.

Bring out that the second sentence should have "only" or "exactly" 7 yellow pencils in order to solve the problem with an exact number for an answer.

Write the following problem on the chalkboard and have it read aloud:

Mother has only 4 dresses.  
Two of the dresses are red.  
The rest of the dresses are blue.  
How many blue dresses does she have?

Discuss the problem and encourage the children to give the solution, and to show how they reached this conclusion. Have a child show the action with equivalent sets of felt shapes on the flannel board, e.g., "2 red disks and 2 blue disks Erase the word only in the first sentence of the story problem and put a set of 4 blue disks on the flannel board. Ask a child to read the first sentence, ("Mother has 4 dresses,") and to tell if the illustration on the flannel board shows that Mother had 4 dresses. She has 4 dresses but she may also have more than 4 dresses.

Will the answer to the question asked in  
the story problem be the same now?  
(No, because she may have more than 4 dresses.)

Read the following story problems to the children. Point out that the story problems presented to them assume some conditions because of the necessity of keeping the problem short and to the point. Determine for each problem what conditions are assumed to be true as described.

Tom made 4 home runs for his team.  
Harry made 4 home runs for the same team.  
How many home runs did Tom and Harry make  
during the game?

It is assumed that each of the boys made only 4 home runs during that game.

Jack had 10¢.  
He gave 6¢ to Lee.  
How many cents did he have then?

It is assumed that Jack had only 10¢ although he might have had more than that. It is assumed that Jack gave exactly 6¢ to Lee although he might have given more than 6¢ to Lee and still fulfilled the condition as stated in the problem. When we answer, "He had 4¢," then we are also assuming the conditions to be exactly as stated. Then our answer will be right exactly as it is stated. The answer will vary only as the conditions vary.

Discuss the following problems using set materials as necessary.

1. Sally had 4 hats.
   One hat had a bow.
   How many hats did not have bows?

Could more than one hat have a bow?
Our answer is right if only one hat had a bow.

2. Judy and Jane have 7 dolls in all.
   3 of the dolls are Jane's.
   How many dolls are Judy's?

Could more than 3 dolls belong to Jane?
Our answer is right if only 3 dolls belong to Jane.

3. Bob counted 6 cars and 8 trucks.
   How many cars and trucks did he count?

Could he have counted more than 6 cars or trucks?
Our answer is right for the cars and trucks we know about.

4. John read 4 pages in his book.
   He has 8 pages yet to read.
   How many pages are in his book?

Could there be some pages which John didn't intend to read?
Our answer is right if John intends to read every page in the book.

5. Sue had exactly 5 cookies.
   She gave only 1 cookie to David.
   How many cookies does Sue have now?

Is our answer likely to be incorrect?

Again, stress the fact that, while we do need to think about the statements made in story problems, we can most often assume that "only" or "exactly" is implied when stating something to be true , e.g., "Mother has 2 pairs of shoes," would tell us that we are going to talk about only 2 pairs of shoes, even though Mother might have other shoes not considered in the story.

## Chapter V

## LINEAR MEASUREMENT

### Background

In this chapter we discuss the measurement of line segments. Recall that a line segment is the set of points followed in passing along a straight path from a given point A to a given point B. Two line segments are congruent provided that they have the same size, so that one will fit exactly on the other.

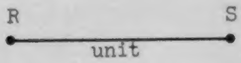
Long before the child comes to school he has experience in comparisons of order: his father is taller than he is; his sister is younger than he is; the new house is bigger than the old house; he woke up today before his mother did; this pail is heavier than that pail. He has also had experience with the notion of measure; he understands and makes such statements as, "My dad is 6 feet tall," "We get 3 quarts a day," "It takes me 15 minutes to get to school." Here we wish to extend the child's knowledge of linear measure and to deepen his intuitive understanding.

It may be of interest to note that our development parallels the historical one. The counting of separate objects (say, sheep) was a technique not applicable to measuring a region or curve (like a field and its boundary). Nevertheless, one could often make comparisons: this field is larger than that; this boundary is longer than that. Later, when fields bordered more closely on each other, actual measurement became necessary. When a unit of measure (e.g., that part of a rope between two knots) was agreed upon, it was possible to designate a piece of property as having a length of "50 units of rope" and having a width of "30 units of rope". With the increase in travel and communication it became obvious that "50 units of rope" did not always represent the same length. Hence, standard units were adopted.

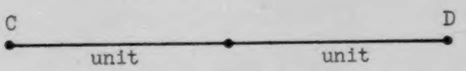
For convenience in measuring, rules or scales marked in these standard units were introduced.

### Measure, Length, Units

In measuring the line segments, we first select a particular line segment, say $\overline{RS}$, to serve as a unit.



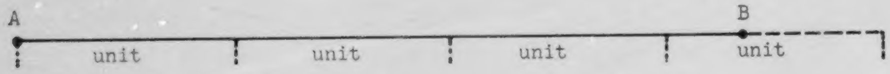
The length of $\overline{RS}$ itself is then 1 unit. To measure any given line segment $\overline{CD}$, we lay off the unit $\overline{RS}$ along it.



If the unit can be laid off exactly twice, as in the picture, we say that the measure of $\overline{CD}$ is 2, and that the length of $\overline{CD}$ is 2 units. If the unit could be laid off exactly three times, we would say that the measure of $\overline{CD}$ is 3, and that the length of $\overline{CD}$ is 3 units. The measure of a line segment is a number: the number of times the unit can be laid off on the line segment. When naming a length, we use both the measure and the unit.

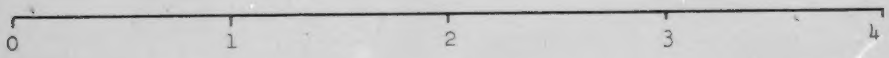
### Length to the nearest unit

More often than not, the unit will not fit exactly some number of times but there will be a part of a unit left over. In the picture the unit can be laid off along the segment $\overline{AB}$ 3 times, with a part of a unit left over, but it does not fit 4 times.



The length of $\overline{AB}$ is, then, greater than 3 units but less than 4 units. Moreover, in our example the length of $\overline{AB}$ is visibly nearer to 3 units than to 4 units. In this case, we say that the length of $\overline{AB}$ *to the nearest unit* is 3 units. This approximation is the best we can give without introducing fractional parts of a unit or shifting to a smaller unit.

A line segment on which a unit length has been laid off and marked some number of times, as below, is called a linear scale (or ruler).



*A word about terminology*. We do not add inches, any more than we add apples. All we add are numbers. If we have 3 apples and 2 apples, we have 5 apples altogether, because

$$3 + 2 = 5.$$

Likewise, if we have 3 yards of ribbon and 2 more yards of ribbon, we have 5 yards of ribbon altogether, again because

$$3 + 2 = 5.$$

## Standard Units and Systems of Measures

The acceptance of a standard unit for purposes of communication is soon followed by an appreciation of the convenience of having a variety of standard units. An inch is a suitable standard unit for measuring the edge of a sheet of paper, but hardly satisfactory for finding the length of the school corridor. While a yard is a satisfactory standard for measuring the school corridor, it would not be a sensible unit for finding the distance between Chicago and Philadelphia.

The standard units of linear measure--inch and foot--which we treat in this chapter, are units in the British-American System of Measures. No attempt has been made to introduce the metric units like meter and centimeter, though it would be well for the pupils to be aware that there are other widely used standard units.

## V-1. Comparing sizes

Objectives: To review the idea that two sets of discrete objects can be compared without counting.

To develop the idea that the size of a continuous object, such as a line segment, cannot be found by counting only.

To develop the idea that two line segments can be compared by laying a copy of one on the other.

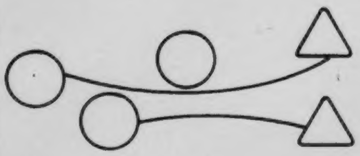
Vocabulary: (Review) line segment, fewer, more, as many as, comparing.

Materials: Cartons, straws, paint pans, brushes, long-handled paint brush, teacher's pointer, length of yarn or string, length of "straight" board, flannel board.

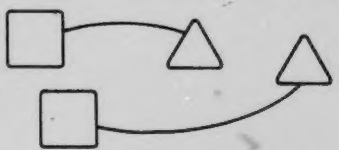
### Suggested Procedure:

### Comparing Sets of Objects

Use classroom objects, such as milk cartons and straws, easels and children, paint pans and brushes, etc., to review the idea of more than, fewer than, and as many as. On the flannel board show how members of sets of discrete objects can be paired to determine one-to-one correspondence.



fewer triangles than circles
more circles than triangles



as many squares as triangles
as many triangles as squares

## Pupil's book, pages 145-147: Comparing Sets of Objects

### Ideas

Two sets of discrete objects can be compared without counting.

One set will have more members, as many members as, or fewer members than another set.

### Examples 1-2

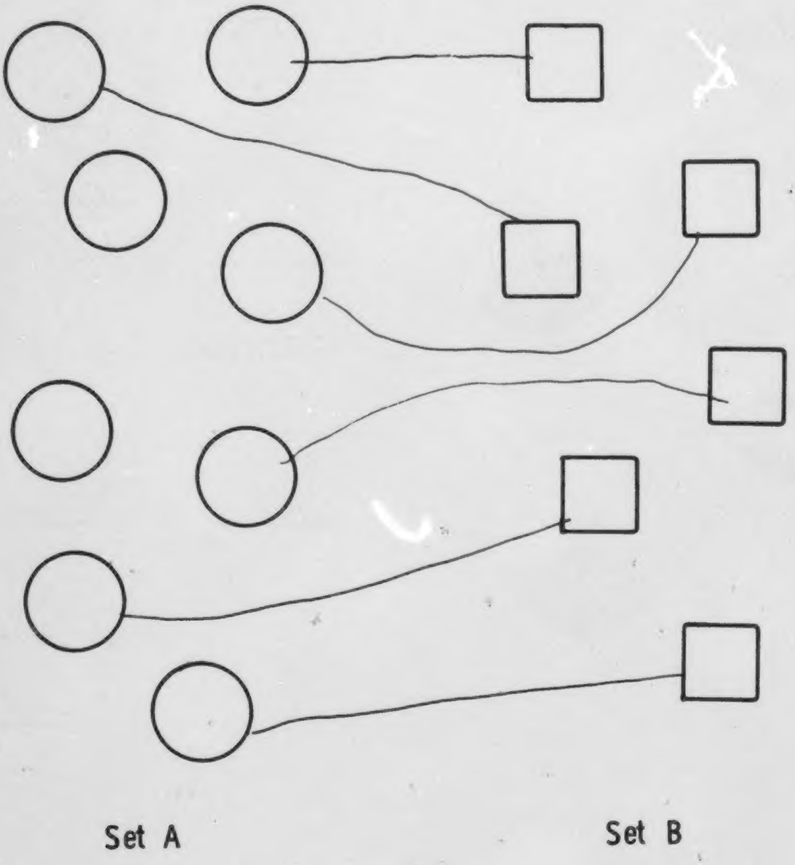
Describe the sets on each page. Read the instructions to the children and then ask them how they could compare the sets without counting. Call attention to the flannel board pattern and suggest to the children that they connect each member of one set to a member of the other set.

### Example 3

This page shows members of each set in varied positions. Emphasize that position does not influence the pairing. You may want to read the instructions for this page, noting that only one of three choices is to be made.

## Comparing Sets of Objects

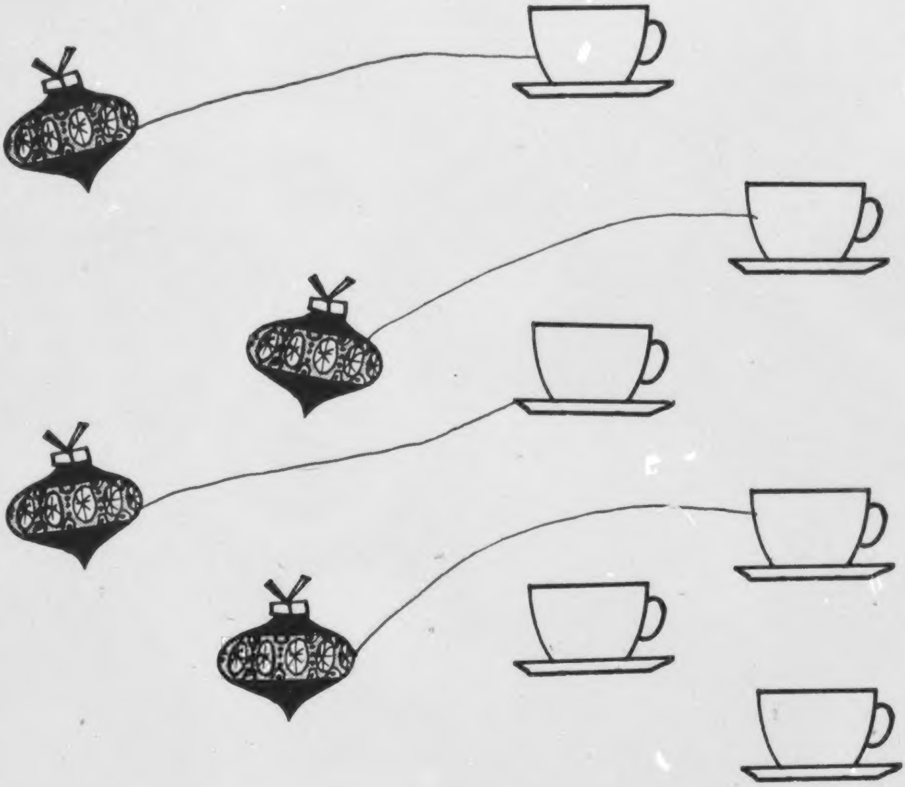
1. Tell without counting which set has more members.



Set A has more members.

## Comparing Sets of Objects

2. Tell without counting which set has fewer members.

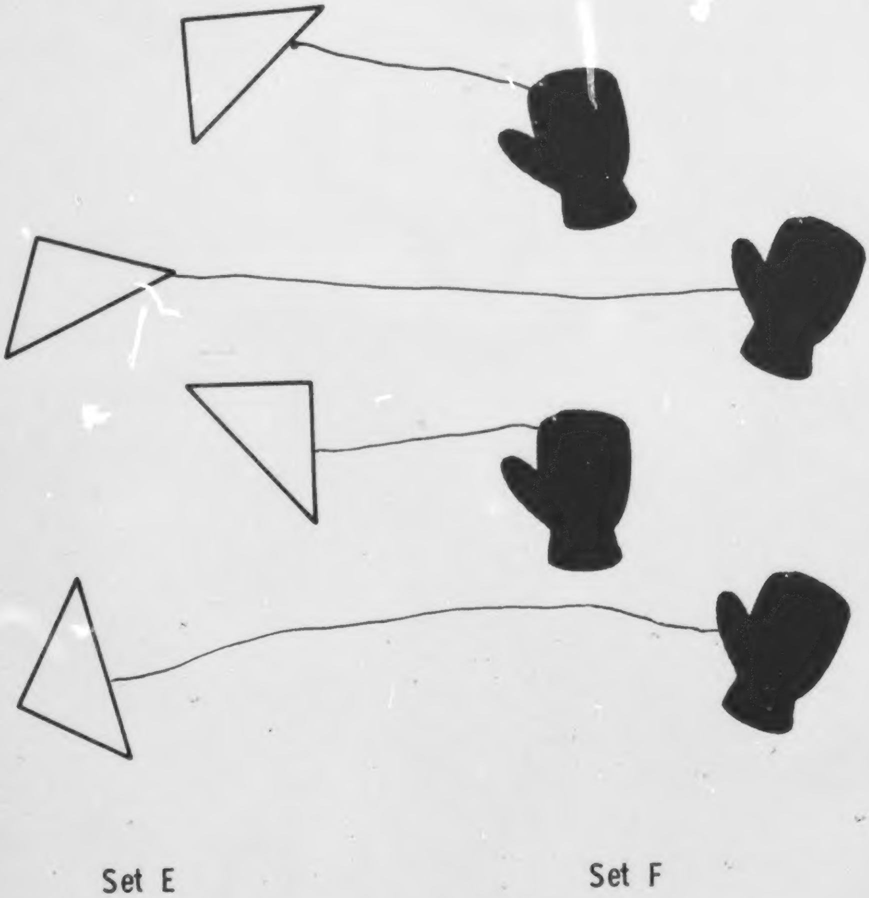


Set C

Set D

Set C has fewer members.

3. Without counting, compare these sets.



Put a ring around the correct words.

Set E has fewer members than / [✓ as many members as] / more members than Set F.

### ● Comparing Line Segments

Spend a little time in reviewing line segments as introduced in Chapter III. Then, on the chalkboard, use a "straight" piece of wood to draw two line segments of different lengths. Draw the line segments in different places on the chalkboard so the children cannot tell by observation which line segment is longer. Ask how the longer length might be found.

Direct the discussion so the children see that one picture of a line segment cannot be moved over to the other in order to compare them. Connecting the segments will not help. Ask what else could be done.

> We could use something on which we can mark one line segment and move it over to the other line segment.
>
> Does anyone have any idea what we could use? (String, yarn, the piece of wood, etc.)

Some children may have had an inch rule or yardstick introduced to them at school or home. Accept these answers but direct the children to utilize one of the more intuitive approaches above.

After determining which line segment is longer, give each child a piece of yarn or string, and direct the children's attention to page 148.

Pupil's book, page 148: Line segment

Ideas

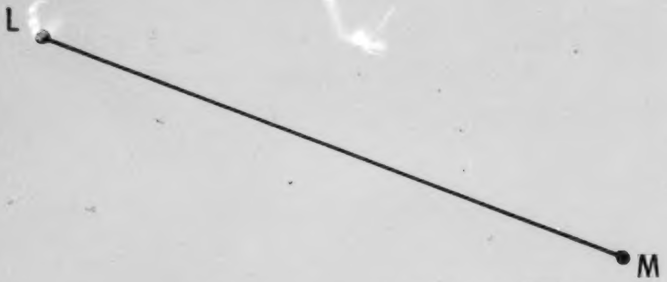
> A line segment is a straight curve with two endpoints. The line segment $\overline{XY}$ can be named as $\overline{YX}$.

Examples 1-2

> These examples serve as a brief review of the above ideas.

## Line Segments

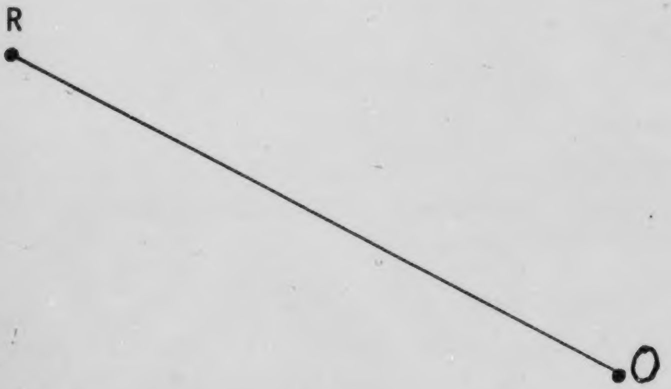
1.



Write names for the two endpoints of $\overline{LM}$. L M

Write another name for $\overline{LM}$. $\overline{ML}$.

2.



One point is named R.

Another point is marked.

Name this point O.

Then draw $\overline{OR}$.

## Pupil's book, pages 149-152: Comparing Line Segments

Ideas

Two line segments can be compared by laying a copy of one on the other.

A segment is shorter than, longer than, or the same length as another segment.

Example 1

Give each child a piece of string or yarn longer than 5 inches.

Call attention to the two line segments and to the question.

Read it aloud with the children.

Direct each child to place his piece of yarn along $\overline{AB}$ and to pick up the yarn at endpoints A and B. Then ask the children to place the yarn on $\overline{CD}$. Ask which line segment is longer.

Example 2

Use the same procedure for this pair.

Examples 3-5

These examples stress the comparison of segments. The words "shorter" and "longer" are read and used.

Example 6

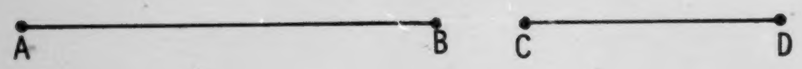
$\overline{AB}$ and $\overline{CD}$ are the same length. Encourage the children to read "is the same length as" and to discuss its meaning.

Examples 7-8

This page reviews the main ideas of the section.

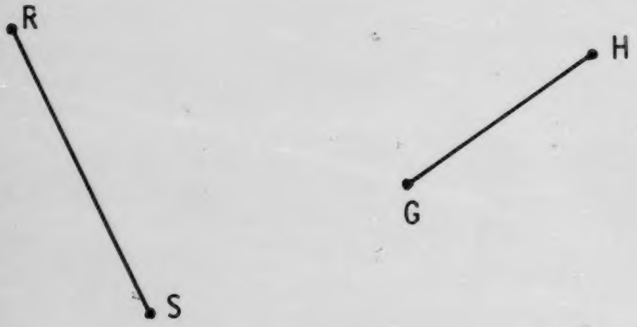
## Comparing Line Segments

1.



Which line segment is longer? $\overline{AB}$

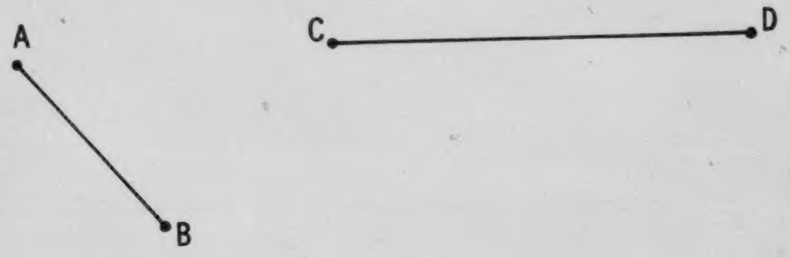
2.



Which line segment is shorter? $\overline{GH}$

## Comparing Line Segments

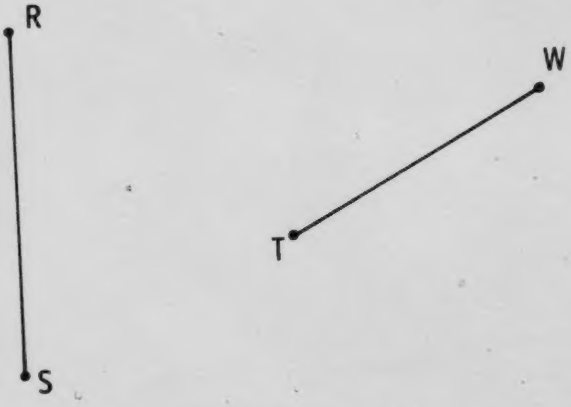
3.



Put a ring around the correct word.

[✓ shorter]
$\overline{AB}$ is than $\overline{CD}$.
longer

4.

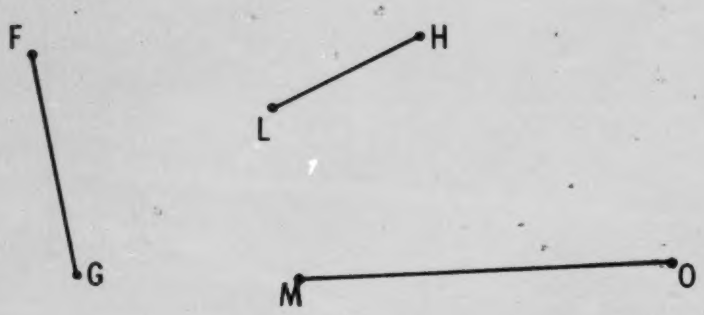


Put a ring around the correct word.

shorter
$\overline{RS}$ is than $\overline{TW}$.
[✓ longer]

## Comparing Line Segments

5.



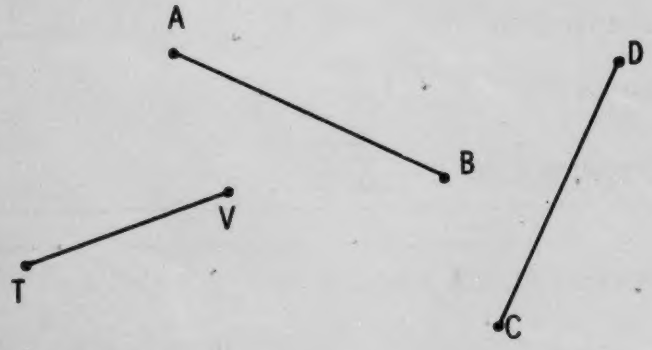
$\overline{FG}$ is shorter than $\overline{MO}$.

$\overline{GF}$ is longer than $\overline{HL}$.

$\overline{MO}$ is longer than $\overline{FG}$ and $\overline{HL}$.

$\overline{HL}$ is shorter than $\overline{FG}$ and $\overline{MO}$.

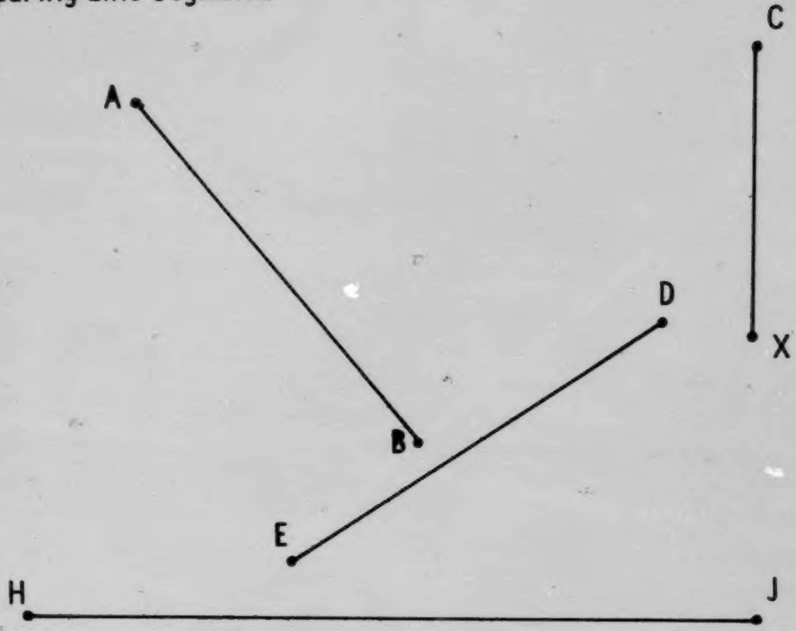
6.



Put a ring around the correct words.

is shorter than
$\overline{AB}$ [✓ is the same length as] $\overline{CD}$.
is longer than

## Comparing Line Segments



7. Write a name of a segment in each blank.

$\overline{CX}$ is shorter than $\overline{DE}$.

$\overline{HJ}$ is longer than $\overline{BA}$.

$\overline{AB}$ is the same length as $\overline{DE}$.

8. Write the correct word in each row.

$\overline{CX}$ is shorter than $\overline{AB}$.

$\overline{AB}$ is shorter than $\overline{HJ}$.

$\overline{ED}$ is longer than $\overline{CX}$.

## V-2. Congruence of line segments

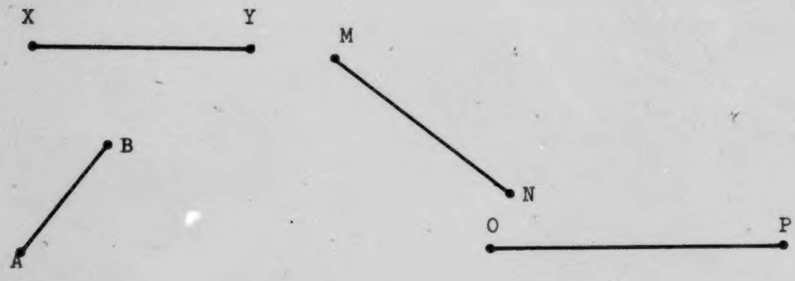
**Objective:** To develop an understanding of the meaning of congruent line segments.

**Vocabulary:** Straightedge, congruent, matching, model.

**Materials:** A large sheet of newsprint, individual sheets of 9" × 12" newsprint, pencils.

**Suggested Procedure:**

On the chalkboard, draw four line segments and label the endpoints as illustrated.



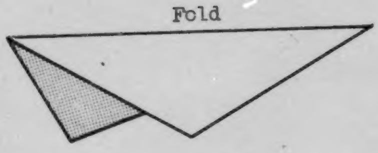
Ask whether all four seem to be the same size. It should be rather clear that $\overline{XY}$ is longer than $\overline{AB}$. Ask whether $\overline{XY}$ might be exactly as long as one of the other two.

> When two line segments are the same size we say that they are congruent.

Restate the question: ask whether $\overline{XY}$ is congruent to any of the other segments shown.

Discuss ways of finding out. Observe that the most direct way, if it could be done, would be to pick up $\overline{XY}$ and move it up against $\overline{MN}$ for comparison. But $\overline{XY}$ cannot be moved.

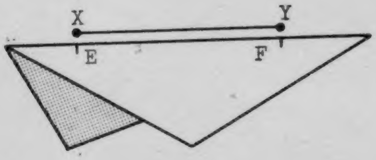
Fold a large piece of newsprint.



This edge (Indicate the fold.) is called a straightedge.

I can use this straightedge to make a copy of line segment $\overline{XY}$. I will place the fold on $\overline{XY}$ and mark the points that match X and Y. I will call them E and F.

Label them.



Look at line segment $\overline{EF}$ on the fold. Is it congruent to $\overline{XY}$? (Yes.)

Emphasize that our test for congruence is necessarily not exact: first, because the chalk marks are not exact; second, because our eye is imperfect.

Test $\overline{EF}$ against the other line segments drawn on the board. Discover if $\overline{EF}$ is the same size as $\overline{AB}$. (No.) $\overline{MN}$? (Yes.) $\overline{OP}$? (No.)

Is $\overline{EF}$ congruent to $\overline{MN}$? (Yes.)
We know $\overline{EF}$ is also congruent to $\overline{XY}$.
Is $\overline{MN}$ congruent to $\overline{XY}$? (Yes.)
Is $\overline{XY}$ congruent to $\overline{MN}$? (Yes.)

Recall to the children that we could not compare $\overline{XY}$ with $\overline{MN}$ directly, because we could not move either one. The way we established their congruence was to make a movable copy of one and match it to the other.

Distribute the smaller pieces of newsprint to the children and direct them to fold the paper to make a straightedge. Check to see that each child indicates the fold.

### Pupil's book, pages 153-156: Congruence of Line Segments

Ideas

Two line segments are congruent when one segment can be fitted exactly on the other.

Example 1

Direct the children to use their straightedge to make a copy of line segment $\overline{AB}$ by laying the fold along the segment and making marks on the fold which match endpoints $A$ and $B$. Call attention to the sentence in the middle of the page and ask children to use their copy to find out which segment is congruent to $\overline{AB}$.

Example 2

Direct the children to turn their folded paper over for new markings. The same procedure is involved as in Example 1.

Example 3

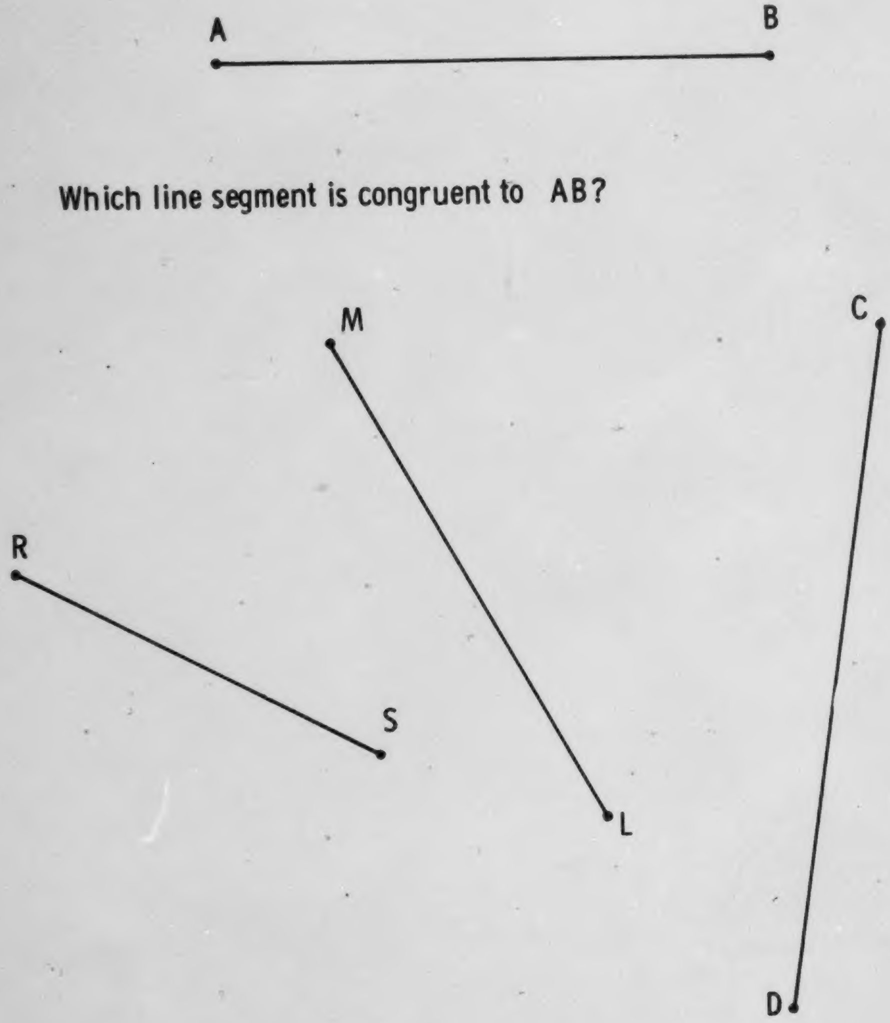
Here the children need to decide which two segments are congruent. Rather than to erase the markings on their straightedges, suggest to the children that they refold the paper in the opposite direction. Unless a child is having noticeable difficulty, let him explore the various combinations on his own and make his own discovery.

Example 4

This page is a review page. Several pairs of congruent line segments are to be found.

## Congruence and Line Segments

1.

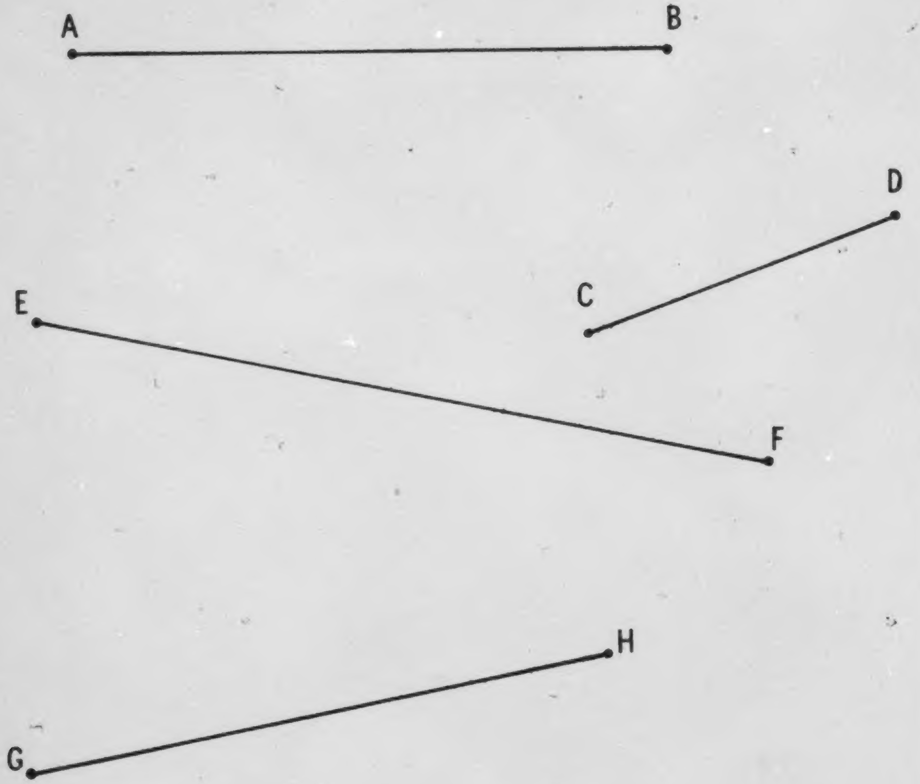


Which line segment is congruent to AB?

$\overline{ML}$ is congruent to $\overline{AB}$.

## Congruence of Line Segments

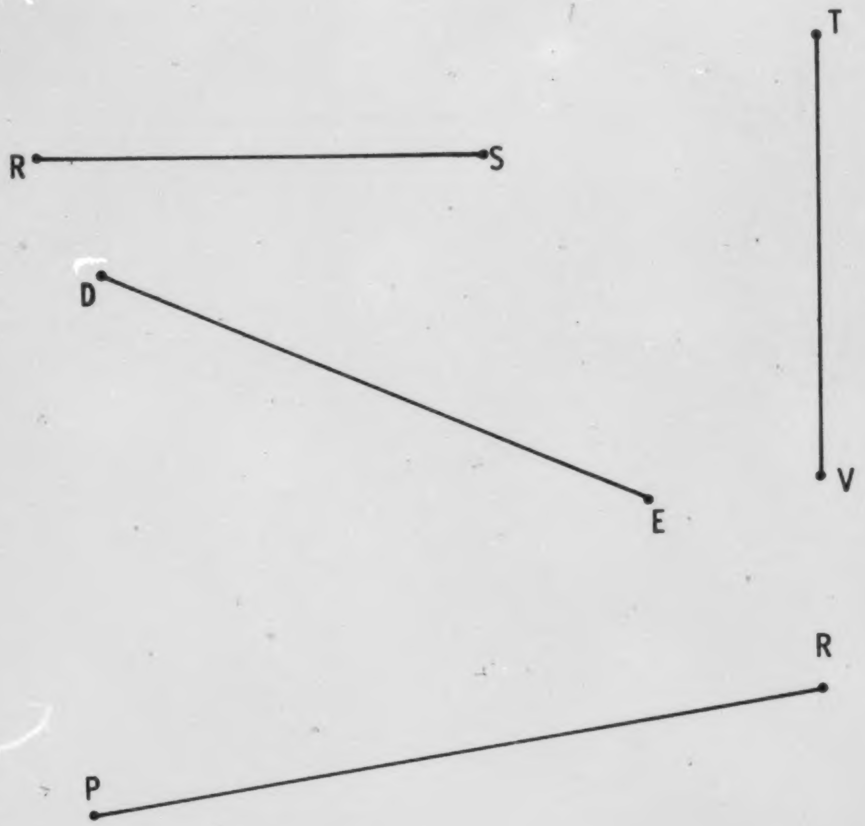
2.



$\overline{AB}$ is congruent to $\overline{GH}$.

## Congruence of Line Segments

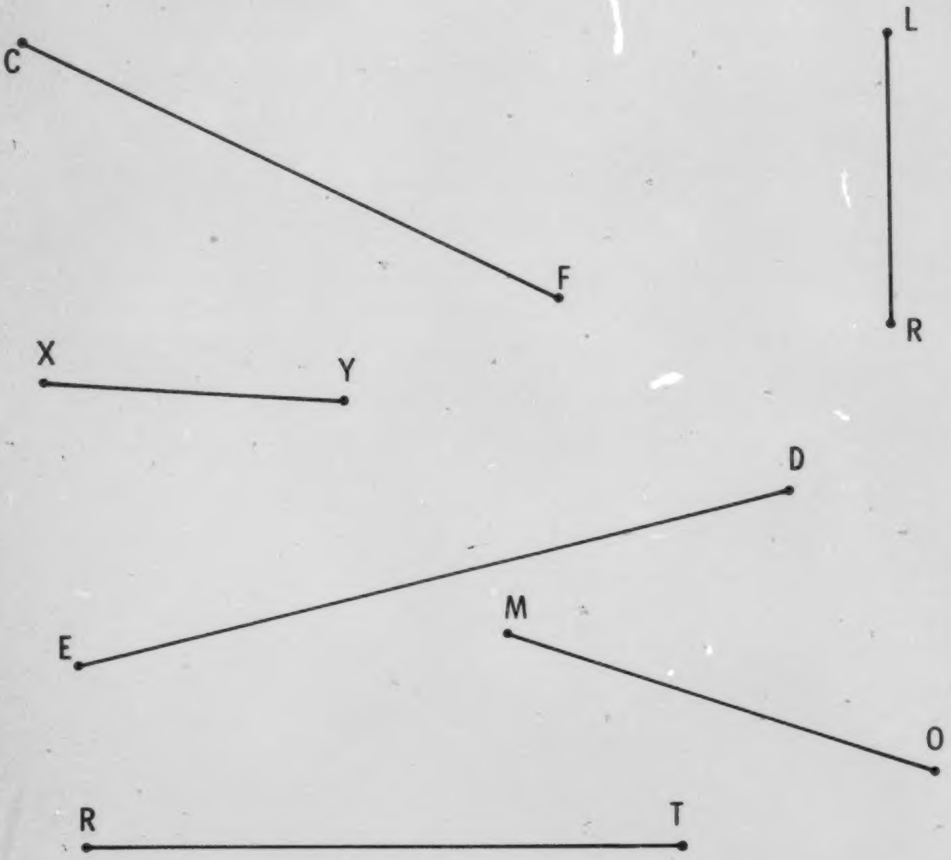
3.



$\overline{RS}$ is congruent to $\overline{TV}$.

## Congruence of Line Segments

**4.**


C
F
L
R
X
Y
D
E
M
O
R
T

$\overline{CF}$ is congruent to $\overline{RT}$.

$\overline{LR}$ is congruent to $\overline{XY}$.

## V-3. Units of length

**Objectives:** To develop, through the use of non-standard units, a recognition of the need for standard units.

To develop the understanding that one model of a line segment may be used repeatedly for finding length.

To develop the understanding that measurement does not usually "come out even" but between two whole numbers.

**Vocabulary:** Unit of length, hand-span.

**Materials:** String about 12 inches long, pencils of varying length.

**Suggested Procedure:**

Indicate a library table at the front of the room. Move your hand along the edge of the table which is nearest to the children.

> How long is the edge of this table?
> Can you tell just by counting? (No.)
>
> Does this edge of the table suggest a line segment? (Yes.)
>
> Does this pencil suggest a line segment? (Yes.)

State that the length of the table will be found by using the pencil to represent a line segment. Show that the pencil will be placed along the edge of the table to see how many times it fits. Note that the pencil fits more than ____ times but not quite ____ times.

Then write on a chart:

> This desk is between ____ and ____ pencils long.

State that the pencil has been used as a unit of length. Fill in the blanks.

Begin the demonstration that a hand-span can also be a unit of length by asking a child to go to the chalkboard and to spread his hand against the board so that his fingers are as far apart as possible. Mark a point on the board where his thumb is and another point where his little finger is. Ask the child to take his hand away, then connect the points to represent a line segment. State that the line segment across the hand between the tip of the thumb and the end of the little finger is called a hand-span.

Ask a child to use his hand-span to find the length of the table. Record this length as the second sentence on the chart:

This desk is between ____ and ____ hand-spans long.

Again, ask a child to find the length of the edge of the table. Give him the piece of string for the measurement. Record this length as the third sentence on the chart:

This desk is between ____ and ____ strings long.

You will then want to talk about the limitations of this procedure by introducing different lengths of string and pencils. Use your own hand-span. Record several of the results to compare them with those on the chart. This comparison can be discussed again, as shown, following the use of the work page.

Pupil's book, page 57: Units of length

Ideas

Some non-standard units of length are difficult to apply by different people in different situations. A model of a line segment can be used repeatedly for measurement.

Length does not usually "come out even" in terms of a unit, but can be described as "between" two numbers of a unit.

Read the sentences with the children. Be certain they understand that the first answers are to be found on the pupil page. The second set of answers refers to the actual desks of the children. Be sure varied lengths of pencils are being used by children.

After completion of this page, discuss what was found. Ask such questions as

a. Why were the lengths different when we used pencils?

b. What would happen if I had used my hand-span to measure your desks? Would there be more or fewer hand-spans for the length?

c. Do you think hand-spans and pencils are good units of length to use every time? Why or why not?

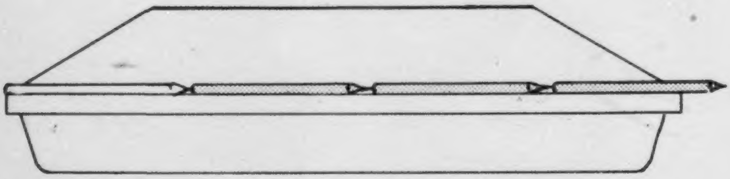
d. What units of length do you think would be more accurate?

e. Why do we say "between 3 and 4 pencils" long?

This page should lead to the use of a variety of units of measurement such as toothpicks, erasers, child's foot, or pointers to find the lengths of tables, the room, aquarium or chalkboard, and other classroom objects. Always emphasize that lengths of line segments are being found. For instance, if the length of the hall is to be found, the edge along the wall on the floor represents a line segment.

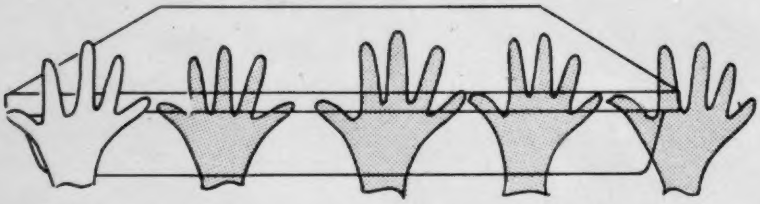
## Units of Length

1.



This desk is between 3 and 4 pencils long.

2.



This desk is between 4 and 5 hand-spans long.

3.

My desk is between ______ and ______ pencils long.

My desk is between ______ and ______ hand-spans long.

Answers will vary.

## V-4. Measure and length of line segments

**Objectives:** To develop ability to use a copy of a unit to find the length of a line segment.

To establish the relationship between the measure of a length, and length.

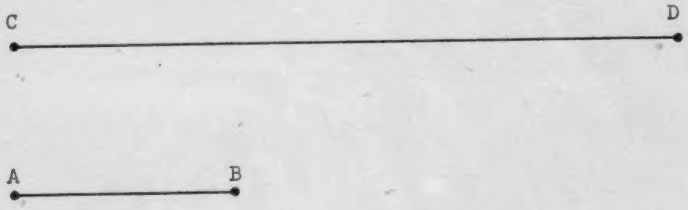
**Vocabulary:** Measure.

**Materials:** Paper, pencils.

**Suggested Procedure:**

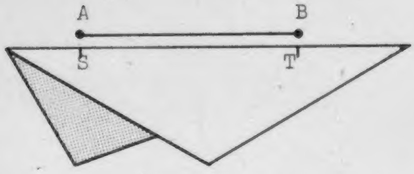
In preparation for class draw on the chalkboard a line segment 3 feet long and below it a second line segment 1 foot long. (Do not let the children see you use the standard unit.)

Label the endpoints of the longer line segment C and D; label the endpoints of the shorter line segment A and B.



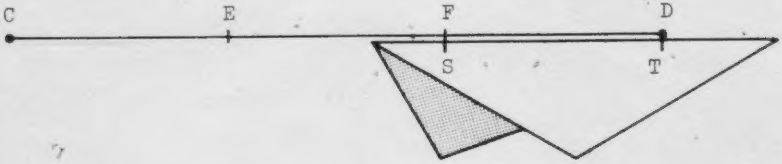
Explain that the length of line segment $\overline{CD}$ is to be found and that $\overline{AB}$ is the unit of length. Ask how the length might be found. (Make a copy of $\overline{AB}$.)

Recall how a straightedge was made on the fold of a piece of paper. Show how a segment on the fold can be made congruent to $\overline{AB}$. Label the endpoints S and T as shown.



Lay $\overline{ST}$ on $\overline{CD}$ with point S on point C. Mark where point T falls on line segment $\overline{CD}$. Name this point E. Place $\overline{ST}$ again on $\overline{CD}$ but now with point S on point E. Again mark where point T falls on $\overline{CD}$. Name this point F.

Once more place $\overline{ST}$ on $\overline{CD}$, this time with point S on point B. Ask where point T falls. (Point D.)



Now look at $\overline{CD}$ on the board. How many of our units of length are shown on $\overline{CD}$? (3.)

When we count, we know we think about numbers.

When we find out how many of a unit line segment are in a longer line segment, do we also count? (Yes.)

Are we finding a number? (Yes.)

Explain that the number found is the measure.

What is the measure of $\overline{CD}$? (3.)

Develop a chart with the following to filled in by the children in discussion.

The unit is ____.

The measure of $\overline{CD}$ is ____.

The length of $\overline{CD}$ is ____ units.

The measure of $\overline{ED}$ is ____.

The length of $\overline{ED}$ is ____ units.

Pupil's book, pages 158-159: Measure and Length

Ideas

The measure of a line segment is a number.

When naming the length of a line segment, the measure and the unit of length are both used.

Example 1

Relate this page to the chart work on the board. Ask the children what the unit is. ($\overline{RS}$.) Ask the children to make a straightedge to mark off the unit $\overline{RS}$, and to use this copy of $\overline{RS}$ to find the number of fittings of $\overline{RS}$ on $\overline{MP}$. Read the sentences and help the children determine the correct answer for the first two measures. (1 and 2.)

Example 2

This page differs only in that children are asked to write the length as 4 units ($\overline{KT}$ is the unit.)

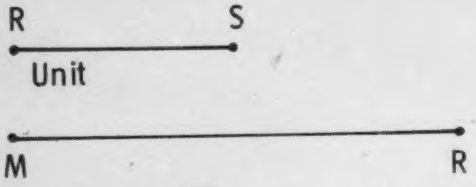
Example 3

Note with the children that $\overline{AB}$ is the unit and that $\overline{BC}$, $\overline{CD}$, and $\overline{DE}$ are congruent to unit $\overline{AB}$. Then help them to see that the length of any segment can be related to the unit by counting the number of times $\overline{AB}$ can be fitted into the segment.

This page can also be used for enrichment since the last few examples involve more difficult reasoning.

## Measure and Length

1.



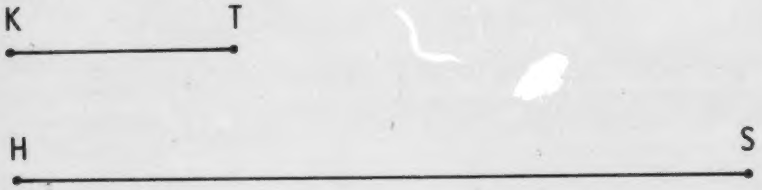
The unit is $\overline{RS}$.

The measure of $\overline{RS}$ is 1.

The length of $\overline{MR}$ is 2 units.

The measure of $\overline{MR}$ is 2.

2.



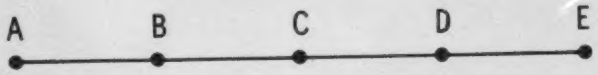
The unit is $\overline{KT}$.

The measure of $\overline{KT}$ is 1.

The length of $\overline{HS}$ is 3 units.

The measure of $\overline{HS}$ is 3.

3.



$\overline{AB}$ is the unit.

The length of $\overline{AC}$ is 2 units.

The measure of $\overline{AC}$ is 2.

The length of $\overline{AD}$ is 3 units.

The measure of $\overline{AD}$ is 3.

The measure of $\overline{BE}$ is 3.

The length of $\overline{BE}$ is 3 units.

The measure of $\overline{CE}$ is 2.

The length of $\overline{CE}$ is 2 units.

$\overline{BE}$ is congruent to $\overline{AD}$.

$\overline{CE}$ is congruent to $\overline{AC}$, and ______.

$\overline{ED}$ is congruent to $\overline{AB}$, $\overline{BC}$ and $\overline{CD}$.

## V-5. Length to the nearest unit

**Objective:** To introduce the idea of measuring to the nearest unit.

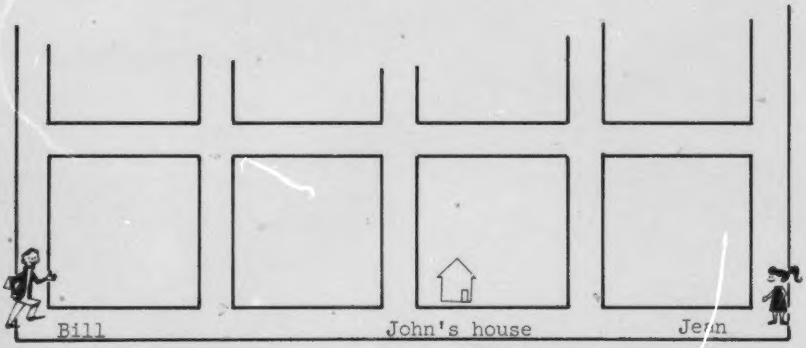
**Vocabulary:** (No new words.)

**Materials:** Large sheet of newsprint for work on the board.

Smaller sheets of paper for each child.

**Suggested Procedure:**

Draw on the board John's house as shown. Show Bill to be crossing the intersection to go to John's house. Ask if Bill is nearer to two blocks than to three blocks away. (2 blocks.) Show that Jean is walking toward John's house in the opposite direction. Ask if she is nearer to one than to two blocks away. (2 blocks.)



In preparation for class, write on the chalkboard:

The length of the table is greater than (11) pencils. (Numbers given here are examples only.)

The measure of the table is greater than (11).

The length of the table is less than __(12)__ pencils.

The measure of the table is less than __(12)__.

The length of the table to the nearest pencil unit is __(11)__ pencil units.

The measure of the table is nearer to __(11)__ than to __(12)__.

In another place on the board or on a chart write:

The length of the table is between ____ and ____ pencil units.

Suggest that a pencil again be used to find the length of the table used in an earlier lesson. Measure the edge of the table. Do not pick up the pencil the last time you lay it on the table. If the pencil will not balance, ask a child to hold it in place. Use chalk to mark the table at the point where the last full pencil unit is completed. This procedure is to avoid having to remeasure the table if the pencil is accidentally moved.

With the help of the children, complete the sentence above which asks for the length "between" the number of pencil units. Direct a child to read the first two sentences in the group of six sentences. Complete the sentences with the help of the children.

Follow the same procedure with the third and fourth sentences.

Have several children observe the pencil at the end of the table to determine the numbers to be entered in the fifth sentence.

Is the measure of the table nearer to __(11)__ or to __(12)__? (11.)

We say the measure to the nearest pencil unit is 11.

11 is nearer to the last pencil unit we used than 12 is.

Present on a chart several other examples of the idea of "nearer" until the children are at ease in making the distinction between using one or another number.

Now read the sixth sentence on the board. Ask:

> If the measure of the table is nearer to 11, what is the length of the table to the nearest unit? (11 pencil units.)

Remind the children that the length includes the measure and the name of the unit.

Pupil's book, page 160: Length to the nearest unit.

Ideas

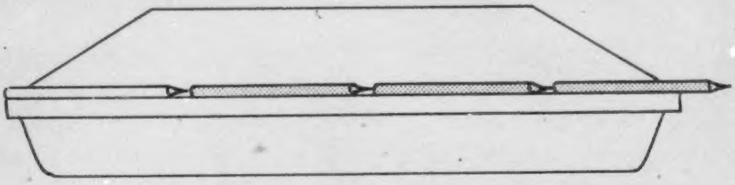
> Length can be measured more precisely by using the "nearest unit".

Example 1

> Read each group of two sentences together. Let the children fill in the blanks based on what they see on the pupil page. This page is not a measurement of their own desks.
>
> Following the class use of this page, let the children find the length of their own desks. Ask what the measure is when a length is given.
>
> If time permits, choose other appropriate objects.
>
> Do not force the distinction between measure and length, but in questions you ask call attention to the ideas:
>
> > Is the measure of your desk nearer to 4 or to 5? (5.)
> >
> > What is the length of your desk to the nearest pencil unit? (5 pencil units.)
>
> It is suggested the preceding experiences take at least one class period.

## Length to the Nearest Unit

1.



The length of the desk is more than 3 pencils.

The measure of the desk is greater than 3.

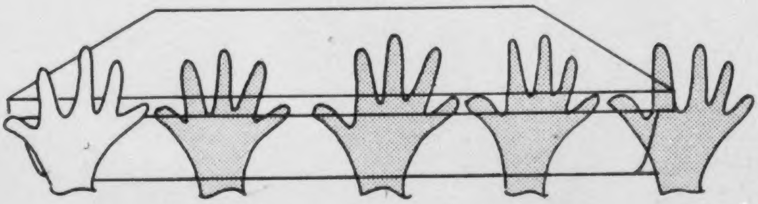
The length of the desk is between 3 and 4 pencils.

The measure of the desk is between 3 and 4.

The measure of the desk is nearer to 4 than to 3.

The length of the desk to the nearest pencil unit is 4 pencils.

2.



The length of the desk to the nearest unit is 4 hand-spans.

Introduce on the chalkboard a line segment which is 33 inches long. (The points could be marked before class and a "straight" piece of wood used for drawing the line segment when needed.) Label the points S and T. Beneath line segment $\overline{ST}$ draw a 10-inch line segment and label the endpoints G and H.

Fold the large piece of newsprint and ask a child to mark points on this straightedge to indicate a line segment that is congruent to line segment $\overline{GH}$. Have prepared a large reading chart with sentences similar to those of the previous day.

The length of $\overline{ST}$ is greater than ____ ____.

The measure of $\overline{ST}$ is greater than ____ ____,etc.

Different children should be asked to complete the measurements that are necessary to the sentences. Care should be taken to see that all the children have opportunities to read the sentences from the board. Several children might be asked to read the sentences together.

Pupil's book, pages 161-163 : Length to the Nearest Unit

Ideas

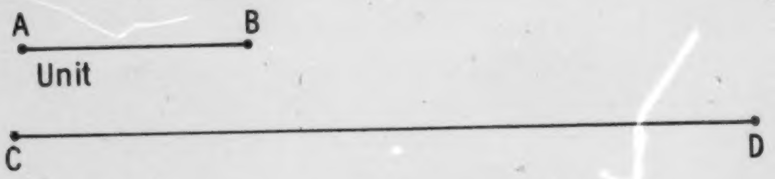
In some cases the length to the nearest unit is less than the actual length; in some cases greater.

Examples 3-10

Help children prepare a straightedge from pieces of paper distributed. Help them complete the first page, then assign the other pages independently. Ask in which direction each segment is to be measured.

## Length to the Nearest Unit

3.



The length of $\overline{CD}$ is greater than ___3___ units.

The measure of $\overline{CD}$ is greater than ___3___.

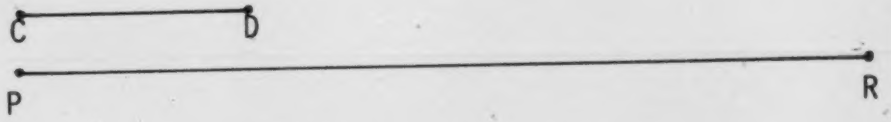
The measure of $\overline{CD}$ is less than ___4___.

The length of $\overline{CD}$ is less than ___4___ units.

The length of $\overline{CD}$ is nearer to ___3___ units than to ___4___ units.

The length of $\overline{CD}$ to the nearest unit is ___3___ units.

4.



The length of $\overline{RP}$ is greater than ___3___ units.

The measure of $\overline{RP}$ is greater than ___3___.

The length of $\overline{RP}$ is less than ___4___ units.

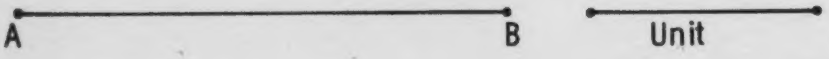
The measure of $\overline{RP}$ is less than ___4___.

The length of $\overline{RP}$ is nearer tc ___4___ than to ___3___ units.

The length of $\overline{RP}$ to the nearest unit is ___4___ units.

## Length to the Nearest Unit

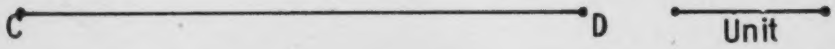
5.



The length of $\overline{AB}$ to the nearest unit is 2 units.

The measure of $\overline{AB}$ to the nearest unit is 2.

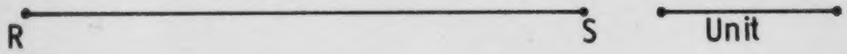
6.



The length of $\overline{CD}$ to the nearest unit is 4 units.

The measure of $\overline{CD}$ to the nearest unit is 4.

7.

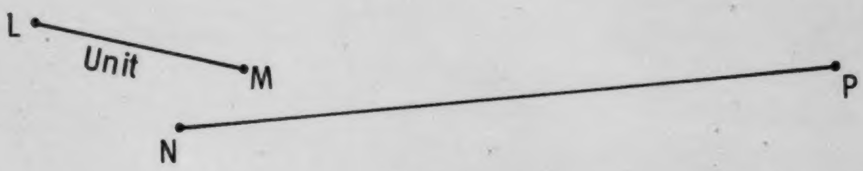


The length of $\overline{RS}$ to the nearest unit is 3 units.

5

## Length to the Nearest Unit

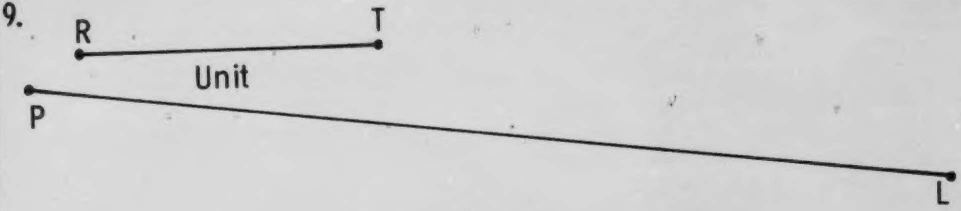
8.



The measure of $\overline{NP}$ to the nearest unit is 3.

The length of $\overline{NP}$ to the nearest unit is 3 units.

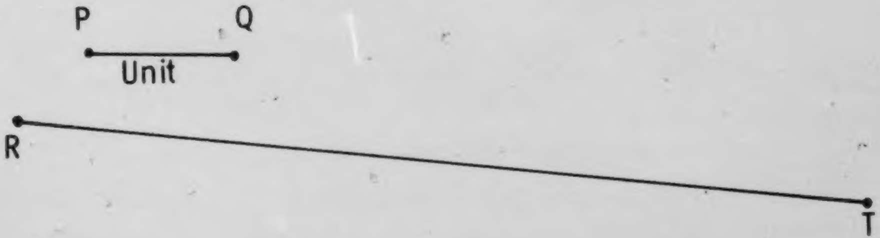
9.



The length of $\overline{LP}$ to the nearest unit is 3 units.

The measure of $\overline{LP}$ to the nearest unit is 3.

10.



The length of $\overline{TR}$ to the nearest unit is 6 units.

## V-6. Using a standard unit of length

**Objective:** To develop understanding of a standard unit.

**Vocabulary:** Standard unit, inch.

**Materials:** A $5\frac{1}{2}$ by $\frac{3}{4}$ inch tagboard straightedge for each child.

**Suggested Procedure:**

Ask the class what ball would be used if the children wanted to play kickball. Ask why they would not use a tennis ball (too small), baseball (too hard), or football (not the right shape).

Tell them that all baseball teams that play regularly with each other use a standard baseball. A standard ball is one that must be of a certain size, weight, and material.

Explain that there are units of measurement that are standard. Read the following sentences, asking the class to fill in the correct word.

> We drove 45 (miles, minutes) to Grandmother's house.
>
> We buy 3 (qts., gal., pts.) of milk every day.
>
> It was a warm day. The thermometer read 80 (degrees).
>
> I weigh 65 (pounds).
>
> It took me 3 (hours) to clean the kitchen.

Point out that miles, hours, quarts, gallons, and pounds are standard units. A mile is always the same distance whether it is in Iowa or in California.

Explain that long ago people used parts of the body as units of length. Ask how the class knows that hand-spans are not the same length. Ask the class if they think other parts of the body would be the same for different people.

Compare the feet of several children. Using pieces of string, let several children find the length from the tip of their nose, with face turned to the left, to the tip of fingers on their right hand outstretched to the right. Compare these lengths. (Many people still estimate a yard in this way.)

Then ask if pencils, hand-spans, and other parts of the body would be called standard units of length. Use these sentences to check understanding of nonstandard and standard units of length.

1. My desk is 5 hand-spans long. (No.)
2. Our door is 9 pencil units high. (No.)
3. Our flag is 2 yards long. (Yes.)
4. The corn is knee high. (No.)
5. I live two miles from school. (Yes.)

Provide each child with a tagboard straightedge. Point out that its edge is a good example of a line segment. On this edge, have the children mark a line segment congruent to the inch, shown in the Example 1 on pupil page 164. Label the endpoints 0 and 1 with 0 near the left end but not at the end.

Pupil's book, pages 164-165 : Using a Standard Unit of Length

Ideas

The inch is a standard unit of length.
Length of a segment can be obtained by beginning at either endpoint.

Examples 1-5, page 164

These examples are purposefully made greater than or less than exact measures to avoid the inaccuracies that can easily develop as children use these models.

Review what measure and length are to the nearest unit, now an inch.

Ask the children to lay their tagboard inch along $\overline{CD}$ as many times as it will fit.

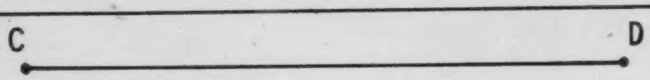
Examples 6-10, page 165

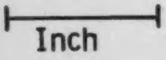
Some of the segments are named from right to left.

Since only one inch is noted on the straightedge, the measure can be noted easily from right to left. This procedure can be checked by using a colored pencil from left to right or by erasing the pencil marks.

## Using a Standard Unit of Length

1.

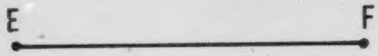




The measure of $\overline{CD}$ is 4.

$\overline{CD}$ is 4 inches long.

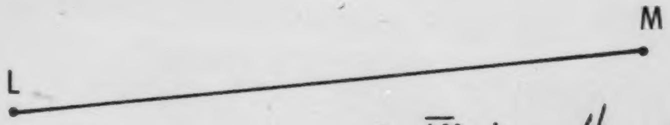
2.



The measure of $\overline{EF}$ is 2.

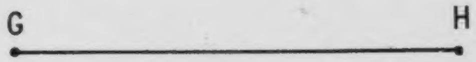
The length of $\overline{EF}$ is 2 inches.

3.



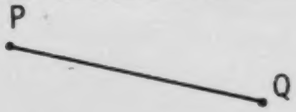
$\overline{LM}$ is 4 inches long.

4.



Length of $\overline{GH}$ is 3 inches.

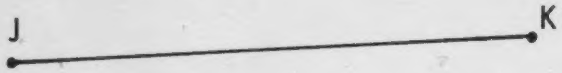
5.



Length of $\overline{PQ}$ is 2 inches.

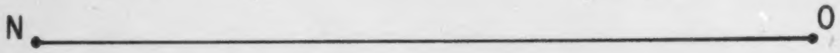
## Using a Standard Unit of Length

6.



$\overline{KJ}$ is 3 inches long.

7.



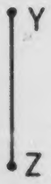
The measure of $\overline{ON}$ is 5.

The measure of $\overline{NO}$ is 5.

8.

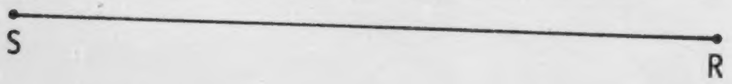
The length of $\overline{WX}$ is 1 inch.

9.



$\overline{YZ}$ is 1 inches long.

10.



The length of $\overline{RS}$ is 5 inches.

The length of $\overline{SR}$ is 5 inches

## V-7. Linear scale

Objective: To construct and use a linear scale.

Vocabulary: Linear scale, ruler, foot.

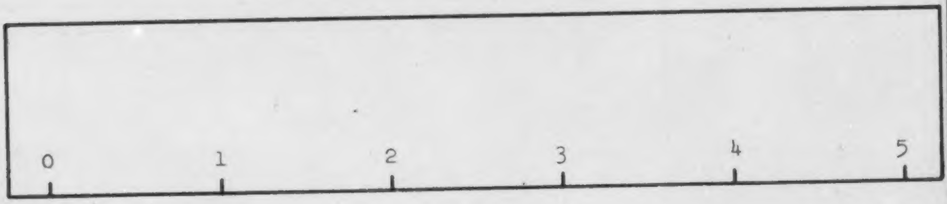
Materials: Straightedge for the teacher and individual ones for the children. (These have been used previous to this lesson.) Cardboard strips one foot in length.

Suggested Procedure:

Distribute the straightedges used in the previous lesson. Ask the children to turn to page 164 to check that the measurement on their straightedge is an inch. Mark a new congruent line segment beginning at the point marked 1.

> How many units have been marked off altogether? (Two.)
>
> What is the measure of this new length? (2.)

Direct the children to show this idea by labeling the new point as 2. Continue to mark congruent line segments of the inch unit until a total of five are marked.



Compare the straightedge with a number line. Point out that this part of a number line is called a linear scale.

Ask the children if they could use the line segment between 2 and 3 to measure one inch. Help them to understand that each of the lengths between any two consecutive numerals is congruent to the inch and therefore measures the same length.

Continue by letting the children measure their own pencils, a piece of paper, their erasers, etc., and report results to the class.

Some children may measure by placing the endpoint of the object to be measured exactly at the endpoint of the linear scale as is done with a ruler. Be sure they understand that the endpoint of the object to be measured must match exactly the point of the scale labeled 0.

Pupil's book, page 166: Linear Scale

Ideas

A linear scale is a device for measuring line segments.

In finding the length of a line segment the point labeled 0 on a scale should be placed at an endpoint of the line segment.

Example 1

The scale can be used to find the length of the line segments.

The lengths given are intended to be whole numbers of inches. If inaccuracies appear, and they will with this age group, emphasize that each length they report is to the nearest inch.

As a follow-up, colored strips of paper of varying lengths can be pasted on cardboard. The children can measure all of the strips, then turn the cardboard over to check their measurements.

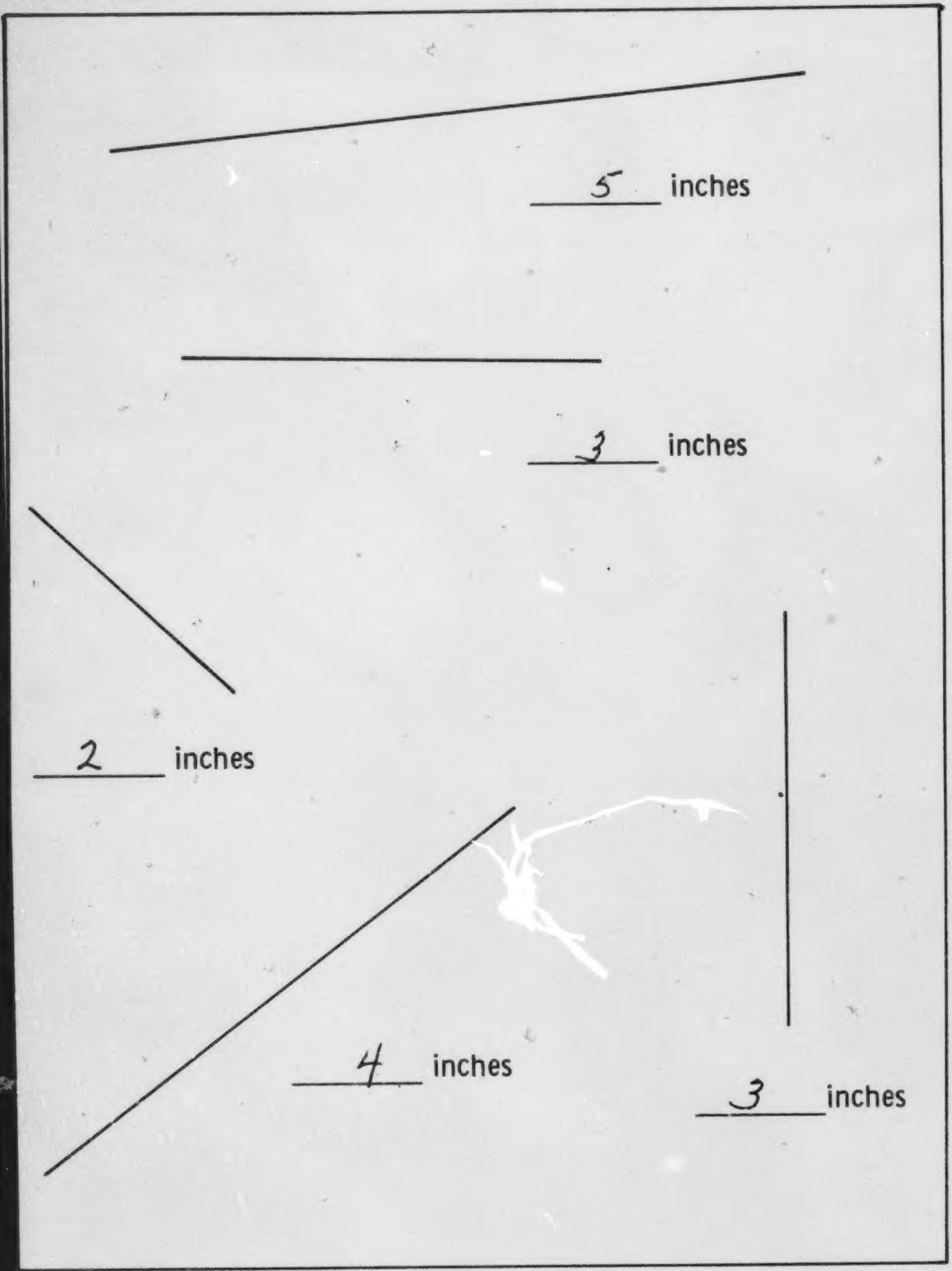
Use a foot length cardboard strip to measure various items in the classroom but do not tell the children it is a foot measure. After the strip has been used for measuring several items, discuss with the children that its length is another standard unit. We call this unit the foot. Compile a list of items in the room to be measured with the foot unit.

An example of the types of items are listed: countertop, window, bookcase, chalktray, jump rope, etc. (Be sure the children know what segment is to be measured on each item.)

Let the children work together in groups of four or five, recording the lengths, to the nearest foot, of each of the items on the chart. (The results among groups can be compared on a chart.) Be certain to measure the objects by starting at the endpoint of the object and the end of the foot unit. Care must be taken to begin the second foot at the point where the first foot ends. For long lengths children can use two or more foot lengths, end to end, more easily than they can mark the endpoints.

A further observation and use of the foot is that it can be marked off into twelve one-inch units and then used to measure items that are less than a foot in length. Children should observe that it is often not possible to list the 0 on this foot measure because of lack of room. Show a standard foot rule to illustrate this problem, preferably one with inch markings only.

## Linear Scale



5 inches

3 inches

2 inches

4 inches

3 inches

## The thermometer and negative numbers

Objective: To use a linear scale in measuring temperature and to utilize the idea of "below zero" to introduce negative numbers.

Vocabulary: (Review) hot, cold, warm, cool, temperature.
(New) thermometer, degree (°), liquid, negative.

Materials: Pictures of things that suggest heat or cold (snowy landscapes, beach scenes, fall landscapes, clothing for hot or cold weather, stoves, refrigerators, cold drinks, hot drinks, etc.); (desirable: hot plate or source of warm water, ice containers for water). Thermometer that can be used outdoors or inside; clinical thermometer; large demonstration thermometer (commercial or teacher made) with movable ribbon and including a special mark at $32^{\circ}$; large dittoed pictures of thermometers.

Suggested Procedure:

Scientific matters are closely linked with this unit, but because we do not know what the science sequence has been, we will discuss the thermometer purely in terms of linear measurement. The teacher should bring to bear in the discussion of temperature whatever she thinks would be desirable: the notion of a liquid, the concept of freezing, different freezing points for different liquids, etc.

Ask children what they might do to tell someone who is not there, a grandparent or friend in another part of the country, for instance--how hot or cold it is here. Someone will probably suggest using a thermometer. Children may be interested to know that the word itself means measure (meter) of heat (thermos).

Explain that the thermometer we are talking about is the type used for measuring the temperature of the air in the room or outdoors. There are other special thermometers for the oven, for taking a person's temperature, etc.

Discuss freezing levels of various liquids. Explain that both mercury and alcohol are used in household thermometers because of the ease with which they expand and contract, and because ordinary air temperatures usually are lower than their boiling points and higher than their freezing points.

Use the demonstration thermometer. Move the ribbon up and down and ask the children to tell whether the temperature shown would be hotter or colder. Recall with the class that when we measure line segments, we need a unit of measure. Explain that the standard unit of measure used for temperature is the degree ($^\circ$). Write the word and the symbol on the chalkboard.

Explain that we read a thermometer to the nearest unit by finding the mark closest to the level of the liquid in the tube. On most thermometers there is a mark on the scale for every two degrees; however, numbers are usually not printed for each mark but only for every ten degrees (every five marks). Check the temperature of your classroom and of the air outside and have children show how the demonstration model would read.

Tell the children that most temperatures--indoors or out; winter or summer; in America, Africa, or anywhere--fall between $0^\circ$ and $130^\circ$ shown on the thermometer. Point out the special mark at $32^\circ$. Explain that this is the temperature at which water turns to ice. A home freezer usually stores food at $0^\circ$.

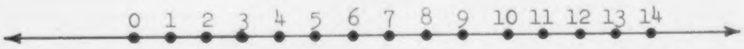
Have children observe that the thermometer looks like a number line in an up-and-down position. If children are not familiar with expressions like "ten below zero", say that when a temperature is colder than zero, it is called "below zero". To show this temperature a minus sign is used. Point to the mark just below zero on the thermometer.

> How much colder than zero would this be,
> (2 degrees.)

Write $^{-}2$ on the board. Point to the next mark, and repeat. Continue and note the markings at $^{-}10$, $^{-}20$, and $^{-}30$ as you get to them.

Ask how much colder $10^\circ$ below $0^\circ$ is than $0^\circ$. ($10^\circ$.) Ask how much colder $15^\circ$ below is than $10^\circ$ below. ($5^\circ$.) When the principle seems to be understood, ask how much colder $5^\circ$ below is than $5^\circ$ above. ($10^\circ$.) Ask how much colder $10^\circ$ below is than $32^\circ$ above. If children have difficulty in seeing this, have them count on the scale (by twos) from $^{-}10^\circ$ to $32^\circ$.

Point to a drawing of the number line as shown below. Remind children that the arrows at the ends of the drawing mean that the line goes on in both directions without end.



> We have shown points to the right of 0, and we know there are many more points to the right of these on this number line. We can also mark points to the left of 0. (Do so.)
>
> If 0 is 1 less than 1, what do you think is true about the first marked point to the left of 0? (It is 1 less than zero.)
>
> To show numbers less than 0 on the number line, we write a minus sign, a little higher than usual, like this: $^{-}1$. We call this "negative one". The thermometer can help us learn some things about negative numbers.

Write $^{-}1$ on the number line, and show $^{-}2$, $^{-}3$, and $^{-}4$. Point to the picture of the number line.

> Is 13 less than 14? (Yes.)
> That is why 13 is to the left of 14 on this

number line. On this number line the numbers become less as we move to the left.

As we go to the right on this number line, the numbers become greater and greater.

Point to ¯6. Ask whether ¯6 is greater than or less than ¯3. (Less.) Ask whether ¯1 is greater than or less than ¯5. (Greater.) Review inequality symbols (< and >) and write:

$6 > 3$          2   ¯4
$1 < 5$          ¯6   ¯3

Ask a child to compare 2 with ¯4. ( 2 > ¯4.) Continue this practice at your discretion.

Return to the thermometer. Ask whether 40 is greater than or less than 30. (Greater.)

As we go down this number line, do the numbers become greater or less? (Less.)
Is ¯20 greater than or less than ¯10? (Less.)
Is ¯2 greater than or less than 2? (Less.)
As we go up the number line, do the numbers become greater or smaller? (Greater.)
Is 0 greater than or less than ¯2? (Greater.)
Is 16 greater than or less than ¯16? (Greater.)
Is ¯16 greater than or less than 16? (Less.)

Pupil's book, page 167: Writing temperatures

Ideas

The thermometer is used to measure different temperatures. Numbers below zero are written with the symbol (¯) and are called negative numbers.

Page 167

Children are to write, below each thermometer, the temperature it shows. Questions should be discussed in class.

## Pupil's book, page 168: Using Negative Numbers

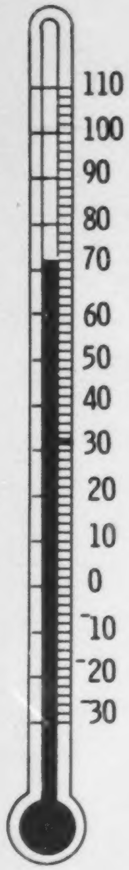
### Ideas

Numbers, including negative numbers can be compared. Negative numbers are less than whole numbers.

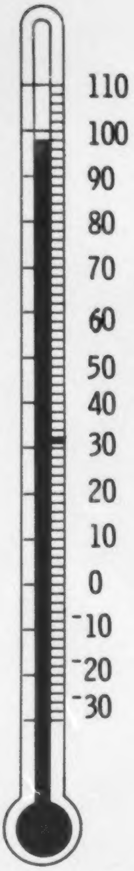
### Page 168

This page is optional. It will demand an understanding of order of negative numbers. ($^{-}64$ is greater than $^{-}65$, $^{-}475$ is less than $^{-}369$, etc.)

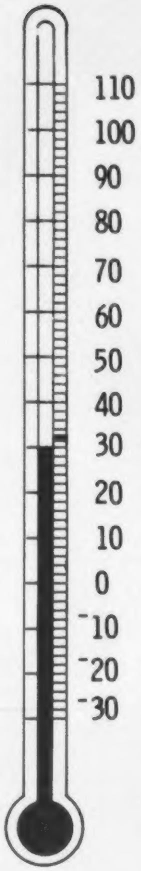
## Writing Temperatures


110
100
90
80
70
60
50
40
30
20
10
0
¯10
¯20
¯30

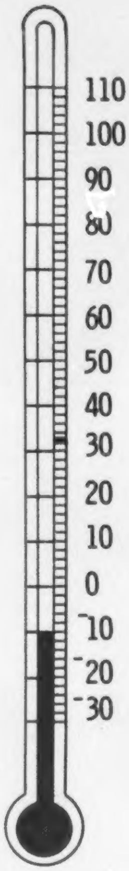
72°


110
100
90
80
70
60
50
40
30
20
10
0
¯10
¯20
¯30

98°


110
100
90
80
70
60
50
40
30
20
10
0
¯10
¯20
¯30

30°


110
100
90
80
70
60
50
40
30
20
10
0
¯10
¯20
¯30

¯10°

Which temperature is warmest? 98°

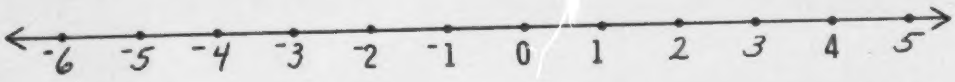
Which temperatures are below freezing? 30° ¯10°

Which temperature is coldest? ¯10°

Which temperature is nearest to the temperature of our room? 72° probably

## Using Negative Numbers

Write the missing numbers.



Write < or >.

35 > 22

2 > 0

2 > -1

-3 < 3

-6 < 2

-22 > -35

15 > -213

1 > 0

46 > -29

-3 < 0

82 > -82

-212 < 0

51 > -54

14 < 16

7 < 8

-1 < 0

5 > 3

-5 < 3

4 > -42

-78 < 20

-65 < -64

475 > -569

15 > -32

82 < 83

-423 > -424

-72 < 79

98 > 67

17 < 35

42 > 13

5 > -5

6 > -2

51 > -49

-498 < 2

-21 > -22

-757 < -38

-68 < -64

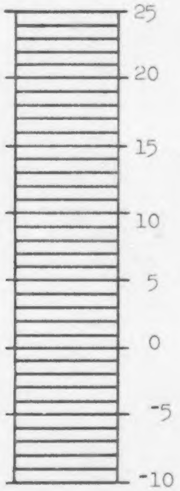
-83 < 82

6 > -6

-724 < 72

Further Activities:

The thermometer scale makes an interesting method of keeping score in team games. Children are separated into teams (boys vs. girls, for instance) and sent to the chalkboard in pairs. They may be given any sort of problem to solve. The child who first gets the correct answer scores two points for his team. If a child gets the correct answer but is slower than his rival, he scores one point for his team. If a child gets an incorrect answer, 2 points are subtracted from the team's score. Using a cardboard thermometer, as shown below, keep score for one team with a pin on the right of the "tube", and for the other team with a pin on the left. At first, negative scores are likely to be shown (one team is "below zero".) and children soon learn to tell how far ahead or behind their team is. This game may continue for some time, perhaps as a way of using occasional short periods of time--until one team reaches a given score.



## Chapter VI

## COMPUTING SUMS AND DIFFERENCES

### Background

Section VI-1 reviews the decimal place value system for writing three-digit numerals, using tabulations to show the equivalence of the "hundreds, tens, and ones" form, the expanded form, and the usual form, as in the example:

| | | |
|---|---|---|
| 3 hundreds, 2 tens, and 6 ones | 300 + 20 + 6 | 326 |

The equivalence of

3 hundreds and 2 tens

with

32 tens

is also brought out.

Section VI-2 begins with adding and subtracting tens, writing in both horizontal form, as in the example

$$40 + 50 = 90$$

and also in vertical form

$$\begin{array}{r} 40 \\ \underline{50} \\ 90. \end{array}$$

It then proceeds to sums like $42 + 84 = 126$, where both tens and ones are involved. To try to keep children aware of the meaning of the process, only the expanded form

$$\begin{array}{l} \;\;40 + 2 \\ \;\;\underline{80 + 6} \\ 120 + 6 = 126 \end{array}$$

is used in this Section; and there is no renaming of ones as tens ("carrying") or of tens as ones ("borrowing"). We avoid the terms "carrying" and "borrowing" as not being descriptive of the processes involved.

Section VI-3 introduces a vertical algorithm for computing sums:

$$\begin{array}{r} 42 \\ 86 \\ \hline 8 \\ 120 \\ \hline 128 \end{array} \quad \text{or} \quad \begin{array}{r} 42 \\ 86 \\ \hline 120 \\ 8 \\ \hline 128 \end{array}$$

Then addition in which ones are renamed as tens is presented, in Section VI-4, first in the expanded vertical form, as in the example

$$\begin{array}{l} 20 + 8 \\ \phantom{20 + }3 \\ \hline 20 + 11 = 31. \end{array}$$

Here it is expected that, from manipulations with bundles of sticks, strips of paper squares, etc., children will learn to do the renaming step mentally, as above. However, at the outset and for children who have difficulty with this, the computation may be written out in more detail:

$$20 + 11 = 20 + 10 + 1 = 30 + 1 = 31.$$

The vertical algorithm for this is also presented:

$$\begin{array}{r} 28 \\ 3 \\ \hline 20 \\ 11 \\ \hline 31 \end{array} \qquad \begin{array}{r} 28 \\ 3 \\ \hline 11 \\ 20 \\ \hline 31 \end{array}$$

Undoubtedly many children may prefer this brief vertical form; but a child who would rather continue with the expanded form should be allowed to do so.

In Section VI-5 the goal is the following sort of column computation form:

$$\begin{array}{r} 32 \\ 46 \\ \underline{17} \\ 15 \\ \underline{80} \\ 95 \end{array}$$

This is led up to by computation in the expanded form:

$$\begin{array}{l} 30 + 2 \\ 40 + 6 \\ \underline{10 + 7} \\ 80 + 15 = 95. \end{array}$$

Section VI-6 approaches subtraction, where renaming of tens as ones ("borrowing") is involved, in much the same way as Section VI-4 approached the similar addition situation.

Section VI-7 concerns the use of the letter "n" to represent an unknown number in an equation or word problem. If we know, for instance, that $n$ is a number for which

$$13 - n = 9,$$

we can reason that

$$\begin{aligned} n &= 13 - 9 \\ &= 4. \end{aligned}$$

This is sometimes called "solving for the unknown number $n$". Similarly, if $n$ is a number for which

$$n - 7 = 8,$$

then

$$\begin{aligned} n &= 8 + 7 \\ &= 15. \end{aligned}$$

Or, if $n$ is a number for which

$$14 + n = 17,$$

we may solve for $n$, getting

$$\begin{aligned} n &= 17 - 14 \\ &= 3. \end{aligned}$$

An equation like

$$n = 13 + 14$$

is already in the solved form for $n$. In such a case there is really no point in using "n" at all, because there is no "unknown". All that is required here is to rename the known number $13 + 14$ in the usual way:

$$13 + 14 = 27.$$

However, in a word problem the children are encouraged to begin by naming the number to be found "n", without stopping to reflect whether the equation that results is going to be in the solved form for $n$ or not. For instance, consider the following.

Problem .

Tom had 13 marbles.
Sam gave him 14 more.
How many marbles did Tom have after that?

Solution.

Let $n$ be the number of marbles Tom had after that. Then

$$n = 13 + 14 \qquad \begin{array}{r} 13 \\ \underline{14} \\ 27 \end{array}$$
$$n = 27$$

Section VI-8, on the application to monetary units, is self-explanatory.

## VI-1. Place value

**Objective:** To review the decimal place value system through the hundreds place.

**Vocabulary:** (No new words.)

**Materials:** Sets of small objects for counting, such as toothpicks, beans, buttons, ceramic tiles.

**Suggested Procedure:**

Use 100 counting objects and ask a child to partition the set of objects into subsets with 10 members each until there are 10 sets.

Draw these charts on the board:

| Numeral |
|---|
| 100 |
| 146 |
| 95 |
| 210 |

| hundreds | tens | ones |
|---|---|---|
| 1 | 0 | 0 |
| 1 | 4 | 6 |
|  | 9 | 5 |
| 2 | 1 | 0 |

| tens | ones |
|---|---|
| 10 | 0 |
| 14 | 6 |
| 9 | 5 |
| 21 | 0 |

Discuss with the children the method of recording numbers in the chart and ask in what ways 10 tens may be recorded. (See above.) Using objects, as before, have sets of ten counted to show many different numbers, recording as shown in other entries on the chart above.

**Pupil's book, page 169:**

Children are to complete chart, showing a numeral, the same number as hundreds, tens, and ones, and the number as tens and ones.

**Further Activities:**

Mark manila envelopes each with a separate letter of the alphabet. In each envelope place a set of graph paper blocks of 100 squares, strips of 10 squares, and strips of less than 10 squares which the children are to count as hundreds, tens, and ones.

Give each child a paper with a chart form as shown below. Then give each child an envelope and ask him to count the material and record in the proper row the hundreds, tens, and ones and both numeral forms, as illustrated in row A. When they have completed work with one envelope, they replace the contents, get another envelope and work with this as they have worked with other sets.

| | hundreds | tens | ones | numerals | |
|---|---|---|---|---|---|
| A | 3 | 1 | 6 | 300 + 10 + 6 | 316 |
| B | | | | | |
| C | | | | | |
| D | | | | | |
| E | | | | | |
| F | | | | | |
| | | | | | |

## Renaming Numbers

| | | Hundreds | Tens | Ones | | Tens | Ones |
|---|---|---|---|---|---|---|---|
| 165 | is | 1 | 6 | 5 | or | 16 | 5 |
| 210 | is | 2 | 1 | 0 | or | 21 | 0 |
| 198 | is | 1 | 9 | 8 | or | 19 | 8 |
| 147 | is | 1 | 4 | 7 | or | 14 | 7 |
| 235 | is | 2 | 3 | 5 | or | 23 | 5 |
| 73 | is | 0 | 7 | 3 | or | 7 | 3 |
| 321 | is | 3 | 2 | 1 | or | 32 | 1 |
| 150 | is | 1 | 5 | 0 | or | 15 | 0 |
| 223 | is | 2 | 2 | 3 | or | 22 | 3 |
| 286 | is | 2 | 8 | 6 | or | 28 | 6 |
| 182 | is | 1 | 8 | 2 | or | 18 | 2 |
| 365 | is | 3 | 6 | 5 | or | 36 | 5 |

- Renaming numbers

Write: $300 + 40 + 5 = \_\_\_\_$

Ask what number completes the equation. (345.) If children are hesitant, remind them that this is 3 hundreds, 4 tens, and 5 ones. Give other examples, and then ask children to rename numbers such as 152 as sums of hundreds, tens, and ones. (100 + 50 + 2.)

Ask children to complete equations such as $180 + 5 = \_\_\_$. (185.) Also have them express numbers such as 139 as the sum of 2 numbers. (130 + 9.)

Pupil's book, page 170

Explain to children that three names for each number are to be shown: the usual numeral for the number, the number as a sum of hundreds, tens, and ones, and the number as a sum of tens and ones.

Renaming Numbers

Fill the blanks.

145 = 100 + 40 + 5
213 = 200 + 10 + 3
168 = 100 + 60 + 8
196 = 100 + 90 + 6
457 = 400 + 50 + 7
394 = 300 + 90 + 4
140 = 100 + 40 + 0
180 = 100 + 80 + 0
253 = 200 + 50 + 3
124 = 100 + 20 + 4
279 = 200 + 70 + 9
177 = 100 + 70 + 7
159 = 100 + 50 + 9
210 = 200 + 10 + 0

145 = 140 + 5
213 = 210 + 3
168 = 160 + 8
196 = 190 + 6
457 = 450 + 7
394 = 390 + 4
140 = 140 + 0
180 = 180 + 0
253 = 250 + 3
124 = 120 + 4
279 = 270 + 9
177 = 170 + 7
159 = 150 + 9
210 = 210 + 0

## VI-2. Adding and subtracting: tens and ones

Objective: To add and subtract tens and then, without regrouping, tens and ones.

Vocabulary: (No new words.)

Materials: Sticks or tickets in bundles or strips of ten.

Suggested Procedure:

Review the basic facts whose sums are greater than 10, using Drill Doughnuts, flashcards, or chalkboard charts such as the following:

| + | 4 | 7 | 9 | 3 | 6 | 5 | 8 |
|---|---|---|---|---|---|---|---|
| 7 | | | | | | | |

Have a child use sticks or tickets and join 4 tens to 5 tens. Ask for and write the equation:

$$40 + 50 = \underline{90}$$

Ask what basic fact was used. (4 + 5 = 9.) Ask someone to join 8 tens to 5 tens.

> How many tens will be in the union of these two sets? (13.)
> How do we write 13 tens? (130.)

Ask for and write the equation:

$$50 + 80 = 130$$

Also, write this vertically:

$$\begin{array}{r} 50 \\ \underline{80} \\ 130 \end{array}$$

What basic fact is used? (5 + 8 = 13.)

Give other examples of this sort, in which basic facts with sums greater than 10 are used.

Ask a child to select a starting set of 6 tens and remove 4 tens. Ask for and write the equation:

$$60 - 40 = 20.$$

Show this in vertical form, and ask what basic fact is used.

$$\begin{array}{r} 60 \\ -\ 40 \\ \hline 20 \end{array} \quad (6 - 4 = 2.)$$

Ask a child to use a starting set of 12 tens, and remove 8 tens. Write the equation:

$$120 - 80 = 40.$$

Discuss the number 120 in terms of hundreds, tens, and ones. As you write the problem in vertical form, emphasize the fact that you will write the numeral 8 directly below the numeral 2, keeping the numerals in tens and ones places directly below each other:

$$\begin{array}{r} 120 \\ -\ 80 \\ \hline 40 \end{array}$$

Show also that the numeral 4 is written directly under the numerals for the tens in the problem itself.

Give children opportunities to write and solve such problems, either on the chalkboard or on paper. Remind them, as needed, to keep ones and tens columns straight. Dictate problems in two forms: "15 tens - 8 tens = how many tens?" and 160 - 90 = ___.

Pupil's book, pages 171-172:

These pages may require class discussion. Children should notice that the same problem is expressed in three different forms.

Pupil's book, page 173:

Children are to rewrite each problem vertically, observing carefully the plus and minus symbols.

**Addition**

4 tens and 8 tens are 12 tens.

40 + 80 = 120

$$\begin{array}{r}40\\ \underline{80}\\ 120\end{array}$$

---

7 tens and 6 tens are 13 tens.

70 + 60 = 130

$$\begin{array}{r}70\\ \underline{60}\\ 130\end{array}$$

---

3 tens and 9 tens are 12 tens.

30 + 90 = 120

$$\begin{array}{r}30\\ \underline{90}\\ 120\end{array}$$

---

5 tens and 8 tens are 13 tens.

50 + 80 = 130

$$\begin{array}{r}50\\ \underline{80}\\ 130\end{array}$$

---

9 tens and 7 tens are 16 tens.

90 + 70 = 160

$$\begin{array}{r}90\\ \underline{70}\\ 160\end{array}$$

---

4 tens and 6 tens are 10 tens.

40 + 60 = 100

$$\begin{array}{r}40\\ \underline{60}\\ 100\end{array}$$

---

8 tens and 9 tens are 17 tens.

80 + 90 = 170

$$\begin{array}{r}80\\ \underline{90}\\ 170\end{array}$$

## Subtraction

Start with 15 tens. Remove 7 tens.

You have 8 tens left.

150 - 70 = 80

$$\begin{array}{r} 150 \\ -\ 70 \\ \hline 80 \end{array}$$

Start with 16 tens. Remove 8 tens.

You have 8 tens left.

160 - 80 = 80

$$\begin{array}{r} 160 \\ -\ 80 \\ \hline 80 \end{array}$$

Start with 14 tens. Remove 8 tens.

You have 6 tens left.

140 - 80 = 60

$$\begin{array}{r} 140 \\ -\ 80 \\ \hline 60 \end{array}$$

Start with 12 tens. Remove 5 tens.

You have 7 tens left.

120 - 50 = 70

$$\begin{array}{r} 120 \\ -\ 50 \\ \hline 70 \end{array}$$

Start with 13 tens. Remove 9 tens.

You have 4 tens left.

130 - 90 = 40

$$\begin{array}{r} 130 \\ -\ 90 \\ \hline 40 \end{array}$$

Start with 11 tens. Remove 5 tens.

You have 6 tens left.

110 - 50 = 60

$$\begin{array}{r} 110 \\ -\ 50 \\ \hline 60 \end{array}$$

## Addition and Subtraction

Write each problem another way. Fill the blanks.

30 + 80 = 110

$$\begin{array}{r} 30 \\ \underline{80} \\ 110 \end{array}$$

130 - 70 = 60

$$\begin{array}{r} 130 \\ \underline{-70} \\ 60 \end{array}$$

50 + 50 = 100

70 + 80 = 150

160 - 70 = 90

150 - 90 = 60

140 - 50 = 90

60 + 60 = 120

80 + 40 = 120

100 - 70 = 30

110 - 40 = 70

160 - 80 = 80

- Using sticks or tickets as before, have a child join a set of 84 sticks to a set of 42 sticks.

  How many tens are in the union? (12.)

  How many ones are there? (6.)

  How can we write 12 tens and 6 ones? (126.)

Have children rename 42 and 84, using the expanded form, and show:

$$\begin{array}{r} 40 + 2 \\ \underline{80 + 4} \\ 120 + 6 = 126 \end{array}$$

Give other examples of this type, and let children write and solve problems, either at the chalkboard or using scratch paper and "Show-me" cards.

Note: Although children are not to be required to add the ones first, and then the tens, you may wish to set a subtle example by always asking first: "What is the sum of the ones?" Then, "What is 40 + 80?"

Write: 135 - 61 = _____

Ask how this might be rewritten. Children may wish to write: 100 + 30 + 5. Suggest that they think of the ones first, and write:

$$\begin{array}{r} 5 \\ \underline{1} \end{array}$$

Ask how they will finish rewriting 61. (60 + 1.) Suggest that they think of 135 as a number of tens and ones.

How would we write 13 tens? (130.)

Finish rewriting:

$$\begin{array}{r} 130 + 5 \\ \underline{-\ (60 + 1)} \end{array}$$

Recall with the children the use of parentheses, and ask them to subtract the ones (5 - 1 = 4) and the tens (130 - 60 = 70). Give other examples of this kind as before.

Pupil's book, pages 174-175:

Children are to rewrite the problems, solve, and fill the blanks.

## Addition

34 + 83 = 117

30 + 4
80 + 3
110 + 7

76 + 82 = 158

70 + 6
80 + 2
150 + 8

54 + 92 = 146

80 + 47 = 127

75 + 63 = 138

91 + 84 = 175

82 + 57 = 139

78 + 30 = 108

## Subtraction

176 - 85 = 91

170 + 6
- (80 + 5)
90 + 1

144 - 73 = 71

140 + 4
- (70 + 3)
70 + 1

109 - 63 = 46

100 + 9
- (60 + 3)
40 + 6

167 - 86 = 81

160 + 7
- (80 + 6)
80 + 1

128 - 74 = 54

185 - 92 = 93

169 - 94 = 75

113 - 40 = 73

## VI-3. A vertical algorithm for addition

Objective: To introduce a vertical algorithm for addition involving 2-digit numerals.

Vocabulary: (No new words.)

Materials: None.

Suggested Procedure:

Write: $46 + 23 = \_\_$

Review with the children the method they have been using to add.

> First we rename 46 as 40 + 6 and 23 as 20 + 3. We add the ones, add the tens, and then add the sums of the tens and ones.

Show:

$$\begin{array}{l} 40 + 6 \\ \underline{20 + 3} \\ 60 + 9 = 69. \end{array}$$

We can do this another way, just by thinking of renaming the numbers without actually writing 40 + 6 and 20 + 3.

Write:

$$\begin{array}{r} 46 \\ \underline{23} \end{array}$$

> What is the sum of the ones? (6 + 3 = 9.) We write it under the numerals in the ones place.
>
> What is 40 + 20? (60.)
>
> Instead of writing 60 + 9 across, we can write 60 here, under the 9. We have to be careful to keep ones under ones, so the 0 of 60 goes under the 9. Now we add again.

Show:

$$\begin{array}{r} 46 \\ \underline{23} \\ 9 \\ \underline{60} \\ 69 \end{array}$$

Write 69 in the blank to complete the original equation.

Give other examples and have children write and solve problems using the new algorithm.

Some children may have difficulty in writing the sum of the tens in such a way as to keep the ones column straight. You may wish to suggest adding the tens first, particularly in such problems as:

$$\begin{array}{r} 82 \\ \underline{65} \\ 140 \\ \underline{7} \\ 147 \end{array}$$

Pupil's book, page 176:

Discuss the first two problems with the group before allowing the children to complete the page independently.

## Addition

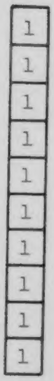
| | | |
|---|---|---|
| 12<br>45<br>7<br>50<br>57 | 34<br>92<br>8<br>120<br>128 | 36<br>61<br>7<br>90<br>97 |
| 63<br>24<br>7<br>80<br>87 | 47<br>22<br>9<br>60<br>69 | 64<br>43<br>7<br>100<br>107 |
| 59<br>70<br>9<br>120<br>129 | 75<br>21<br>6<br>90<br>96 | 21<br>66<br>7<br>80<br>87 |
| 85<br>32<br>7<br>110<br>117 | 53<br>83<br>6<br>130<br>136 | 92<br>65<br>7<br>150<br>157 |

## VI-4. Computing sums involving renaming ones as tens

Objective: To compute sums in which 2-digit numerals and renaming of ones as tens are involved, using both expanded and vertical forms.

Vocabulary: (No new words.)

Materials: Sticks in bundles of ten and single sticks, theater tickets in tens and ones, or a set of paper strips for each child. (Cut hundreds-square paper into strips. Write 1 in each square:



Cut two strips apart to make 20 ones. Give each child a sheet of hundreds-square paper, as well as a set of 10 tens-strips and 20 ones.)

Suggested Procedure:

Write: $28 + 1 = \_\_\_\_.$

Have a child show that 2 tens and 8 ones and one more is 2 tens and 9 ones. Show this in expanded form:

$$\begin{array}{r} 20 + 8 \\ \underline{\phantom{20 + }1} \\ 20 + 9 = 29 \end{array}$$

Write: 28 + 2 = ___.

Have a child show that the 8 single sticks and 2 single sticks may be bundled to make another ten. Show this in expanded form:

$$\begin{array}{r} 20 + 8 \\ \underline{\phantom{20 + }2} \\ 20 + 10 \end{array}$$

Ask children for the sum of 20 and 10, and write 30 in the blank to complete the equation 28 + 2 = $\underline{30}$.

Write: 28 + 3 = ___

Again have a child show that 8 single sticks and 2 single sticks may be used to make a bundle of 10, and that one stick will be left over.

Write:

$$\begin{array}{r} 20 + 8 \\ \underline{\phantom{20 + }3} \\ 20 + 11 \end{array}$$

We know that we can rename 11 as 10 + 1. What is 20 + 10 + 1? (30 + 1 or 31.)

Would it be possible for us to find the sum of 28 and 3 without rewriting 28 as 20 + 8? Let's try it this way.

$$\begin{array}{r} 28 \\ \underline{3} \end{array}$$

We add the ones. 8 + 3 = 11. We write 11 as shown below:

$$\begin{array}{r} 28 \\ \underline{3} \\ 11 \end{array}$$

We can see by the position of the numerals that 11 is 1 ten and 1 one. The ten is in tens place, under the numeral for tens in the problem, and the one is in ones place. Now we add the tens. Two tens and zero tens is two tens. How do we write 2 tens? (20.) We write 20 under the 11, with the 2 in tens place and the 0 in ones place.

$$\begin{array}{r}28\\ \underline{\phantom{0}3}\\ 11\\ \underline{20}\end{array}$$

Now we find the sum of 11 and 20.

$$\begin{array}{r}28\\ \underline{\phantom{0}3}\\ 11\\ \underline{20}\\ 31\end{array}$$

Have children solve a few more problems of this type, in which a number less than 10 is to be added to a number greater than 10, and in which regrouping is necessary. Then proceed to a problem such as

$$26 + 16 = \underline{\quad\quad}.$$

Have children use theater tickets or paper strips to see that 6 ones and 6 ones make one ten and 2 ones, and that there are, besides, 3 tens, for a total of 4 tens and 2 ones.

On the chalkboard show expanded notation:

$$\begin{array}{l}20 + 6\\ \underline{10 + 6}\\ 30 + 12\\ 30 + 10 + 2 = 42.\end{array}$$

Next, show how the vertical algorithm may be used.

$$\begin{array}{r}26\\ \underline{16}\\ 12\\ \underline{30}\\ 42\end{array}$$

Give children opportunities to use both forms, and allow them to choose the form they find best for them. A few children may discover that they do not need to write "partial sums" of 12 and 30, but will proceed to "thinking" the 10 or writing a small 1 above the tens numerals in the problem. This need not be discouraged, but it is expected that most children will continue to use either the expanded form or the vertical algorithm presented above for some time.

Note: Children may, of course, prefer to begin by adding the tens.

$$\begin{array}{r} 26 \\ \underline{16} \\ 30 \\ \underline{12} \\ 42. \end{array}$$

Pupil's book, pages 177-180:

Problems on these pages gradually become more difficult. If children have difficulty with the first two pages, give opportunities to use paper strips as shown in Further Activities.

Further Activities: Children place paper strips of 10 squares and single squares on hundreds-square paper to show the numbers to be added. For instance, for the sum of 37 and 46, a child would place first 3 and then 4 tens strips vertically on the paper. He would next place squares for ones in the column next to the tens, and see that he had to start a new column for 3 ones.

## Addition

55 + 8 = 63

You may do this:

$$\begin{array}{l} 50 + \phantom{1}5 \\ \phantom{50 + 1}8 \\ \hline 50 + 13 \\ 50 + 10 + 3 = 63 \end{array}$$

You may do this:

$$\begin{array}{r} 55 \\ 8 \\ \hline 13 \\ 50 \\ \hline 63 \end{array} \quad \text{or} \quad \begin{array}{r} 55 \\ 8 \\ \hline 50 \\ 13 \\ \hline 63 \end{array}$$

Use the way you like best.

42 + 8 = 50

26 + 7 = 33

45 + 9 = 54

57 + 4 = 61

Addition

72 + 19 = 91

You may do this:

$$\begin{array}{l} 70 + 2 \\ \underline{10 + 9} \\ 80 + 11 = 91 \end{array}$$

You may do this:

$$\begin{array}{r} 72 \\ \underline{19} \\ 11 \\ \underline{80} \\ 91 \end{array} \qquad \text{or} \qquad \begin{array}{r} 72 \\ \underline{19} \\ 80 \\ \underline{11} \\ 91 \end{array}$$

Use the way you like best.

| | |
|---|---|
| 46 + 47 = 93 | 24 + 56 = 80 |
| 73 + 17 = 90 | 39 + 61 = 100 |

## Addition

35 + 48 = 83

$$\begin{array}{r} 30 + 5 \\ 40 + 8 \\ \hline 70 + 13 \end{array}$$

or

$$\begin{array}{r} 35 \\ 48 \\ \hline 13 \\ 70 \\ \hline 83 \end{array}$$

Use the way you like best.

| | |
|---|---|
| 29 + 46 = 75 | 67 + 49 = 116 |
| 58 + 25 = 83 | 13 + 48 = 61 |
| 64 + 38 = 102 | 37 + 45 = 82 |

## Addition

28 + 89 = 117

76 + 57 = 133

63 + 27 = 90

38 + 56 = 94

18 + 88 = 106

45 + 65 = 110

92 + 16 = 108

82 + 49 = 131

## VI-5. Column computation

**Objective:** To use column computation for sums involving three 2-digit numerals.

**Vocabulary:** (No new terms.)

**Materials:** None.

**Suggested Procedure:**

Write on the chalkboard:

$$\begin{array}{r} 2 \\ 6 \\ \underline{7} \end{array}$$

Recall with the children that when they add "from the top" they are thinking, "First $2 + 6 = 8$, and then $8 + 7 = 15$". Also recall that this is shown, using parentheses, thus:

$$(2 + 6) + 7 = 8 + 7 = 15.$$

Next, ask if the same sum results when we add "from the bottom". (Yes.) Ask how we think of this. (First $7 + 6 = 13$, then $13 + 2 = 15$.) Ask how this is shown using parentheses. ($(7 + 6) + 2 = 13 + 2 = 15$) Ask whether we used different addition facts in adding from the top and adding from the bottom. (Yes.) Emphasize that this is why adding from the top and adding from the bottom are good checks on each other.

Next, write on the board: $30 + 40 + 10 = \_\_\_$.

Rewrite as:

$$\begin{array}{r} 30 \\ 40 \\ \underline{10} \end{array}$$

Ask how we think of this in adding from the top. (First $30 + 40 = 70$ and then $70 + 10 = 80$.) Ask if we get the same sum when we add from the bottom. (Yes.) Ask how we think of this. (First $10 + 40 = 50$, and then $50 + 30 = 80$.)

Next write $32 + 46 + 17 = \_\_\_$.

Ask how to rename each number as a sum of tens and ones:

$$\begin{array}{l} 30 + 2 \\ 40 + 6 \\ \underline{10 + 7} \end{array}$$

Ask how this suggests that we might add 34, 46, and 17. (First, add the 2, 6, and 7, getting 15; next, add the 30, 40, and 10, getting 80; finally add the 80 and the 15.)

$$\begin{array}{l} 30 + 2 \\ 40 + 6 \\ \underline{10 + 7} \\ 80 + 15 = 95 \end{array}$$

Say that we may also write all this as follows:

$$\begin{array}{r} 32 \\ 46 \\ \underline{17} \\ 15 \\ \underline{80} \\ 95 \end{array}$$

Extend these ideas to sums of three numbers where hundreds, as well as tens and ones are involved. For example:

$$52 + 87 + 39 = \_\_\_$$

$$\begin{array}{lcrcr} 50 + 2 & & 52 & & 52 \\ 80 + 7 & \text{or} & 87 & \text{or} & 87 \\ \underline{30 + 9 \qquad\quad} & & \underline{39} & & \underline{39} \\ 160 + 18 = 178 & & 18 & & 160 \\ & & \underline{160} & & \underline{18} \\ & & 178 & & 178 \end{array}$$

Have children work at chalkboard or use scratch paper and "Show - me" cards to work problems you dictate. Emphasize the fact that keeping columns straight helps in preventing errors, and remind them that a good way to check work is first to "add down", then to "add up".

Pupil's book, pages 181-182:

Children are to rewrite problems, using either expanded or vertical form, compute the sums, and fill the blanks to complete the equations.

## Addition

Use the way you like best.

| | |
|---|---|
| 28 + 42 + 53 = 123 | 37 + 62 + 36 = 135 |
| 14 + 39 + 54 = 107 | 71 + 28 + 55 = 154 |
| 36 + 79 + 42 = 157 | 30 + 47 + 59 = 136 |

## Addition

44 + 57 + 38 = 139

69 + 95 + 33 = 197

15 + 90 + 27 = 132

41 + 14 + 98 = 153

33 + 52 + 45 = 130

26 + 74 + 98 = 198

## VI-6. Subtraction, renaming tens as ones

**Objective:** To introduce subtraction computations involving 2-digit numerals and the renaming of tens as ones.

**Vocabulary:** (No new words.)

**Materials:** Sticks in bundles of 10 and single sticks, or theater tickets.

**Suggested Procedure:**

Discuss with the children the fact that in joining sticks arranged in bundles of ten and single sticks, they sometimes found it possible to make another bundle of ten from the single sticks, because they had ten or ten more ones.

Ask a child to use a set of 43 sticks (4 tens and 3 ones) and remove 6 sticks. If he hesitates, after removing the 3 single sticks, suggest that if he could bundle 10 sticks together in order to make another 10, he should be able to take a bundle of 10 apart and remove some sticks from it.

Write: $43 - 6 = 37$

We will try to see how we could have found the number that would be in the remaining set if we had not really had sticks to work with.

Write:

$$\begin{array}{r} 40 + 3 \\ \underline{\quad 6} \end{array}$$

In all our addition and subtraction, we have been using basic facts. Do you know a basic fact $3 - 6$? (No.) But you do know a basic fact $13 - 6$. If we can rename 43 as some number + 13, we will be able to think $13 - 6 = 7$.

If we think of one of the tens in 40 as changed into ones, how many tens do we have left? (3 tens.) So we can rename 40 as 30 + 13.

$$\begin{array}{r} 30 + 13 \\ \underline{\phantom{30 + 1}6} \\ 30 + 7 = 37 \end{array}$$

- Write: 59 - 6 = ___.

Let's write this problem vertically, like this:

$$\begin{array}{r} 59 \\ \underline{\phantom{5}6} \end{array}$$

Look first at the numbers in the ones place. Do you know a fact that begins 9 - 6 = ___? (Yes, 9 - 6 = 3.) Then we will write the problem like this, as we have often done:

$$\begin{array}{r} 59 \\ \underline{\phantom{5}6} \end{array} \qquad \begin{array}{r} 50 + 9 \\ \underline{\phantom{50 + }6} \end{array}$$

Have children finish the computation and write 53 in the blank above to complete the equation.

Use sticks or tickets and give many examples involving the necessity of taking apart a bundle of ten in order to remove sticks. Each time, have a child show how the problem would be written, renaming a number in order to have 10 more ones in the ones place. Include, however, some examples in which it is not necessary to change a ten to ones, and have children explain why.

Pupil's book, pages 183-184

Discuss all examples before children begin to solve problems. Ask which problems involve renaming, with ten more ones, and which do not. If they are able to determine whether or not they know a basic fact to use in ones place without writing the problem in the form

$$\begin{array}{r} 59 \\ \underline{\phantom{0}3} \end{array}$$

as an intermediate step, they should be permitted to rename at once.

Subtraction

Think what fact you will use. Then rename.

Fill the blanks.

42 - 7 = 35

$$\begin{array}{r} 42 \\ -7 \\ \hline \end{array} \qquad \begin{array}{r} 30 + 12 \\ -\ \ 7 \\ \hline 30 + 5 \end{array}$$

78 - 3 = 75

$$\begin{array}{r} 78 \\ -3 \\ \hline \end{array} \qquad \begin{array}{r} 70 + 8 \\ -\ 3 \\ \hline 70 + 5 \end{array}$$

21 - 9 = 12

73 - 8 = 65

86 - 6 = 80

35 - 8 = 27

54 - 5 = 49

45 - 3 = 42

## Subtraction

Think what fact you will use. Then rename.

Fill the blanks.

88 - 9 = 79

26 - 8 = 18

67 - 6 = 61

74 - 6 = 68

46 - 7 = 39

95 - 7 = 88

32 - 5 = 27

27 - 9 = 18

- **Subtracting numbers greater than 10, with regrouping**

Use theater tickets or sticks. Display a set consisting of 5 tens bundles and 6 single sticks. Have a child remove 34 sticks. Ask for and write the equation, and rewrite in vertical form.

$$56 - 34 = \underline{\quad} \qquad \begin{array}{r} 50 + 6 \\ \underline{- (30 + 4)} \end{array}$$

Review subtraction of a number greater than 10 in examples in which renaming is not needed, including use of parentheses. Show a set consisting of 4 tens bundles and 3 single sticks. Ask a child to remove 17 sticks. Remind him that he can take a bundle apart if necessary. Ask for and write the equation and rewrite in vertical form:

$$43 - 17 = \underline{\quad} \qquad \begin{array}{r} 43 \\ \underline{- 17} \end{array}$$

Discuss each step as you show renaming 43 as 30 + 13 and 17 as 10 + 7.

$$\begin{array}{r} 43 \\ \underline{- 17} \end{array} \qquad \begin{array}{r} 30 + 13 \\ \underline{- (10 + 7)} \end{array}$$

Emphasize the fact that children do not know a number to complete $3 - 7 = \underline{\quad\quad}$. Hence they should rename 43 so that they will be able to say $13 - 7 = \underline{6}$. Give many more examples, using sticks or tickets as needed. Include examples in which renaming is not needed, as well as examples which do require renaming. Some examples should be of the type: $50 - 25 = \underline{\quad}$. Children should practice with "Show-me" cards or at the chalkboard before using their books. If more able pupils can use the conventional algorithm, they should not be required to continue the expanded form, but do not encourage them to use it before they are ready or ask to do so.

Note: Some children may wish to rename the number to be subtracted so that it, too, will have more ones. They should observe that this defeats their purpose.

Pupil's book, page 185-186:

Children should write the problem vertically, rename as needed and write the difference in the blank.

Further Activities:

You may wish to review doing and undoing and to show children how to check their answers to subtraction problems by adding the difference to the number subtracted. Also, review the relationship between partitioning and subtracting $7 = 5 + 2$. Therefore, $7 - 5 = 2$ and $7 - 2 = 5$. Suggest that the child use scratch paper and subtract the difference he found from the number from which he first subtracted: $75 - 28 = 47$. $75 - 47 = 28$. This method of checking, of course, provides further practice in subtracting.

Caution: It is not advisable to insist that children show their work in checking their answers, for many children learn quickly simply to copy the appropriate numerals and avoid the computation involved.

## Subtraction

Rename in a way that is helpful. Fill the blank in the equation.

95 - 39 = 56

32 - 16 = 16

56 - 28 = 28

77 - 39 = 38

48 - 46 = 2

80 - 53 = 27

63 - 49 = 14

27 - 13 = 14

## Subtraction

Rename in a way that is helpful. Fill the blank.

96 - 38 = 58

82 - 45 = 37

60 - 26 = 34

77 - 58 = 19

93 - 27 = 66

82 - 24 = 58

71 - 58 = 13

86 - 79 = 7

## VI-7. Using "n" in equations and problems

**Objective:** To use the letter "n" to stand for, and help find, a "missing number" in an equation.

**Vocabulary:** n

**Materials:** (None.)

**Suggested Procedure:**

Write: $5 + 3 = 8.$

Ask what would undo adding 3 to 5. (Subtracting 3 from 8.)

Write: $n + 12 = 19.$

Explain that in the past you have often given problems like ___ + 3 = 8 and children have just thought of subtracting 3 from 8 to find the missing addend. Now they are going to write equations to show what they are thinking.

> The letter "n" in the equation $n + 12 = 19$ stands for a number, so we may think, "The number n plus 12 equals 19". How can we find out what number n is? (Subtract 12 from 19.) Let's write "n equals" and show what we are thinking:
>
> $$n = 19 - 12.$$
>
> This equation names the number n as the difference 19 - 12. In what simpler way can you name n? (n = 7.)

Give more examples and write the equations; for example:

$n + 13 = 24$　　$n + 68 = 83$

$n = 24 - 13$　　$n = 83 - 68$

n = ___　　n = ___

The second equation in each problem tells what number n will fit in the first equation. The third equation names this number in the usual way. Explain that children will often need to write the problem in still another form in order to do the work. If $n = 83 - 68$, for instance, they will write at the side:

$$\begin{array}{r} 83 \\ -\ 68 \\ \hline 15 \end{array} \qquad \begin{array}{r} 70 + 13 \\ -\ (60 + \ 8) \\ \hline 10 + \ 5 \end{array}$$

They can then write

$$n = 15.$$

Write:

$$n - 32 = 45.$$

Recall with the children that to undo subtracting 32 in this problem, they must add 32 to 45. Below the equation $n - 32 = 45$, write:

$$\begin{array}{l} n = 45 + 32 \\ n = 77 \end{array} \qquad \begin{array}{r} 40 + 5 \\ 30 + 2 \\ \hline 70 + 7 \end{array}$$

Give more examples of this type. Include some equations such as $68 - n = 40$ and $46 + n = 75$. Review the fact that $46 + n = n + 46$. Therefore, to solve $46 + n = 75$, one first uses the commutative property, then "undoing", so

$$\begin{array}{l} 46 + n = 75 \\ n + 46 = 75 \\ n = 75 - 46 \\ n = 29 \end{array}$$

If children have difficulty with $68 - n = 40$, explain that they may think of the relationship between partitions and subtraction, just as in $10 - n = 3$.

Pupil's book, pages 187-188:

These pages provide practice. Children are to complete the second equation, do the work at the side, then give the usual name for the number represented by $n$.

Pupil's book, page 189: (Optional)

This page, for more able pupils, has problems of increasing difficulty. It will be helpful to present first some problems like the following, which require more than one operation:

4 + 2 + __ = 13     __ - 6 = 2 + 3     10 - (3 + __) = 4

**Doing and Undoing**

**Find n.**

**Show your work here.**

n + 55 = 81

n = 81 − 55

n = 26

n - 36 = 49

n = 49 + 36

n = 85

n - 21 = 39

n = 39 + 21

n = 60

n + 49 = 92

n = 92 - 49

n = 43

**Doing and Undoing**

Find n.

Show your work here .

| |
|---|
| 88 - n = 59<br>n = 88-59<br>n = 29 |
| 17 + n = 85<br>n = 85-17<br>n = 68 |
| n - 38 = 75<br>n = 75+38<br>n = 113 |
| 78 - n = 35<br>n = 78-35<br>n = 43 |

## *Two Step Problems

Find n.

| | |
|---|---|
| n - 8 = 7 + 6<br>n = 21 | 7 + 8 + n = 31<br>n = 16 |
| 75 - n = 21 + 16<br>n = 38 | (52 + n) - 13 = 61<br>n = 22 |
| 26 + 49 + n = 129<br>n = 54 | n - 10 = 35 + 28 + 52<br>n = 125 |

- Write the following story on the chalkboard:

John had some toy cars.
He got 3 toy cars for his birthday.
Then he had 9 toy cars.
How many toy cars did John have before his birthday?

Encourage the children to discuss what they are asked to find. Try to draw from the children the fact that they now have a symbol "n" which they may use to name the number they are seeking. Ask one child to write the equation and complete the solution for n

$(n + 3 = 9,\ \ 9 - 3 = n$ or $n = 9 - 3,\ \ n = 6.)$

Ask another child to give the answer to the story problem. (John had 6 cars before his birthday.)

- Tell the children that you are going to read a story problem and then ask someone to write the equation. Direct them to listen carefully.

David had 16 marbles.
He gave some marbles to Don, and
then he had 7 marbles.
How many marbles did David give to Don?

$(16 - n = 7,\ \ n = 16 - 7,\ \ n = 9.)$
(David gave 9 marbles to Don.)

- Write $7 + 9 = n$ and $n + 7 = 9$ on the chalkboard. Read the following story problem and determine with the children which equation is related to the problem and why.

John had some pencils on his desk.
He had 7 pencils in his desk.
If John had 9 pencils in all, how many pencils were on his desk?

($n + 7 = 9$ is the right equation. The symbol "n" tells the number of pencils on John's desk; we do not know how many were there. 7 is the number of pencils in his desk and 9 is how many pencils he had altogether. The equation 7 + 9 = n is not right because in the story the 7 pencils were a subset of the 9 pencils so you can't add 7 + 9 to find the number for n in this problem.)

(Two pencils were on John's desk.)

- Write 14 + n = 23 and 23 + 14 = n on the chalkboard. Direct the children to listen as you read the following story problem.

Susan had 14 cents.
Mother gave her some cents and then she had 23 cents.
How many cents did Mother give to Susan?

Discuss which equation is related to the story problem and why.

The equation 14 + n = 23 is the right one, because the 14 is the number of cents Susan had to begin with, n is the number of cents Mother gave to her, and 23 is the number of cents Susan had altogether.)

(14 + n = 23, n = 23 - 14, n = 9.)

(Mother gave 9 cents to Susan.)

Ask the children to turn to Pupil's book, pages 190-191: Read the stories over together and discuss how they are similar to the ones just completed. Direct the children to complete the pages independently.

Solving Problems

Write the equation.

Use n for the number you do not know.

1. Sam had some balls.
He got 2 more balls.
Now he has 6 balls.
How many balls did Sam have at first?

$n + 2 = 6$

$n = 6 - 2$

$n = 4$

Sam had 4 balls at first.

2. Mary had 7 books.
Sam took 2 books.
How many books did Mary have then?

Equations will vary

Mary had 5 books then.

3. Jimmy had 3 cookies.
Mother gave him some cookies.
Then he had 8 cookies.
How many cookies did Mother give to Jimmy?

Mother gave 5 cookies to Jimmy.

## Solving Problems

Which equation is related to the story?
Cross out the one that does not belong.

1. Billy had 6 apples.
He gave 2 apples to Jane.
How many apples does Billy have now?

$2 + n = 6$
~~$6 + 2 = n$~~

Billy has 4 apples now.

2. Beth had some toys.
She gave 3 toys to her little brother.
Then she had 4 toys.
How many toys did she have at first?

$n - 3 = 4$
~~$n + 3 = 4$~~

Beth had 7 toys at first.

3. John had 8 cents.
He found some cents.
Then he had 10 cents.
How many cents did he find?

$8 + n = 10$
~~$8 + 10 = n$~~

John found 2 cents.

## VI-8. Monetary units and place value

**Objective:** To practice converting between the monetary units cents, dimes, and dollars, as a concrete example of place value computations where converting between ones, tens, and hundreds is involved.

**Vocabulary:** Dollar sign, decimal point. (Review) dollar, dime, cent, penny.

**Materials:** Dollar bill, 10 dimes, 10 pennies.

**Suggested Procedure:**

Show a dollar bill, a dime, and a penny. Review the fact that 10 cents are worth 1 dime. Place 10 pennies beside the dime. Put down 9 more dimes in a column under the first. Have children count by tens to find out how many cents 10 dimes are worth. (100.) Ask how many cents one dollar is worth. (100.) Ask how many dimes one dollar is worth. (10.)

Note: Do not say, or write, "One dime equals ten cents." This would mean that one dime is the same as 10 cents; but one dime is not the same as 10 cents, for ten cents (as pennies) can not be used in a candy vending machine that operates only on dimes. It should always be emphasized that we may say "is worth" or "has the value of", but not "equals" when we are comparing coins of different denominations.

Write: $1.11. Ask what amount of money is shown. (One dollar and eleven cents.) Explain that the dollar sign together with the numeral indicates an amount of money. Discuss the fact that our money base (our standard measure of money) is the dollar, and we use a decimal point after the number of dollars. We say "and", as we read it. Anything that comes after the decimal point in a money expression means an amount of money less than a dollar. Therefore, if we want to show an amount of money that is less than a dollar, we write a dollar sign, a decimal point, and then the numeral for the number of cents.

> How could we write ninety-nine cents, using a dollar sign? ($.99.)
>
> Ninety cents? ($.90.) If we had ninety cents worth of dimes, how many dimes would we have? (9.)
>
> How would we write thirty-five cents, using a decimal point? ($.35.)
>
> If we have thirty-five cents as 3 dimes and 5 pennies, which numeral shows dimes? (3.) Notice that the number of dimes comes right after the decimal point. Which numeral shows pennies? (5.) The number of pennies comes after the number of dimes.
>
> How would we write one cent, using a decimal point? (If a child suggests writing $.1, remind him that the numeral next to the decimal point means the number of dimes, or ten cent pieces, and you are speaking of no dimes and 1 cent. Therefore, he had to use a zero to show the number of dimes and then a 1 for the cent: $.01.)

Have children write various amounts of money on the chalkboard, including amounts such as $4.59, $.62, $.07, $.0.01. Write various amounts and have children read them.

Hold up the dollar bill and ask how to write that amount of money. Children may wish to write only $1, and you may explain that while this is sometimes done on price tags or advertisements, it is usually written to show one dollar and no more: $1.00. Stress the importance of the decimal point by writing $632 and $6.32. Discuss the meanings of the two expressions as written. Show that $632.00 is another way of writing the first amount. Write 39¢. Explain that the cents sign is often used to show amounts of less than a dollar. Have several amounts of money written in either way.

Pupil's book, pages 192-193:

Have the children look at the picture showing toys, read the prices, and talk about which toy costs most, which least, etc. With an advanced group, you might want to talk about how many dimes one would have to save to buy a certain toy, including some that cost more than a dollar. Mention the fact that often we say "four forty-nine" instead of four dollars and forty-nine cents" or "A dollar eighty-nine" instead of "one dollar and eighty-nine cents" when we are talking about how much things cost. Review number words with the children. If necessary, write the words and numerals for multiples of ten and for the "teen" numbers on the chalkboard:

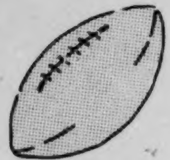
| | | | |
|---|---|---|---|
| twenty | 20 | thirteen | 13 |
| thirty | 30 | fourteen | 14 |
| forty | 40 | fifteen | 15 |
| etc. | | etc. | |

Have children write the numerals showing the amounts of money given in words on page 193 . (These amounts are given under the pictures on page 192 .)

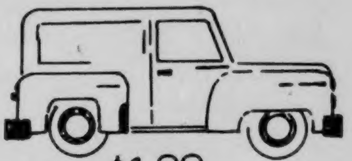
Pupil's book, pages 194-196:

Read the problems aloud. The first page may be used as a class activity. Remind children that it is numbers that are added or subtracted, but the answer to the problem, if a money expression, must show dollar sign and decimal point or cents sign. Children are to write the equations they need at the left and use the space at the right for computation.

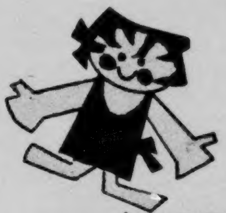
# TOYS



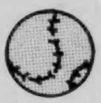
$4.49



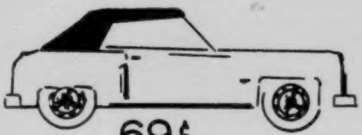
$1.89



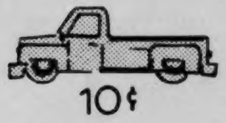
$9.95



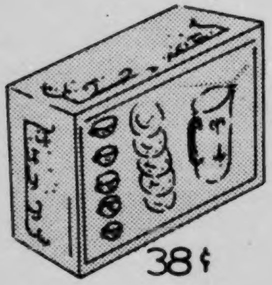
79¢



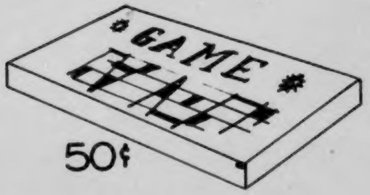
69¢



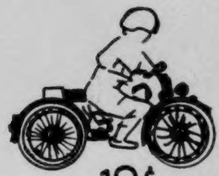
10¢



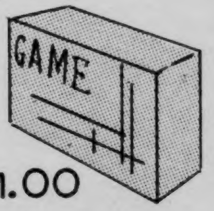
38¢



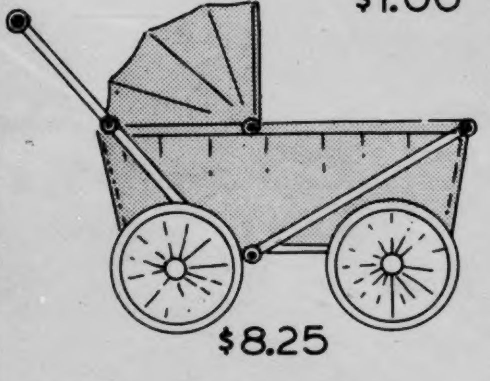
50¢



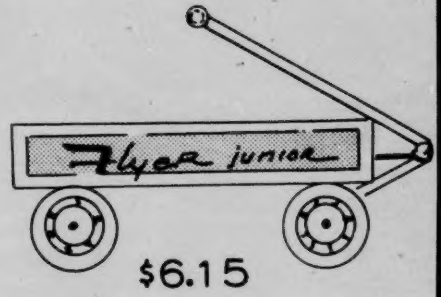
19¢



$1.00



$8.25



$6.15

## Money

What amounts of money are shown?

Write each, using numerals and money signs.

sixty-nine cents 69¢ or $.69

nineteen cents 19¢ or $.19

six dollars and fifteen cents $6.15

thirty-eight cents 38¢ or $.38

ten cents 10¢ or $.10

four dollars and forty-nine cents $ 4.49

one dollar and eighty-nine cents $1.89

fifty cents 50¢ or $.50

eight dollars and twenty-five cents $8.25

seventy-nine cents 79¢ or $.79

one dollar $ 1.00

Steve had saved $.55. He wanted to buy a ball that cost $.89. How much more money did he need?

Steve needed $\$$.34 more.

Kathy bought doll dishes for 49¢ and a cookie set for 79¢. How much did both things together cost?

Both things together cost $\$$1,28.

When Larry went to the toy store, he had 50¢. He bought a horn. The man at the store gave him 11¢ change. How much did the horn cost?

The horn cost 39¢.

Scott took a toy truck and a 25¢ car to the check-out counter. The truck did not have a price mark on it. The girl at the counter said, "Sixty-two cents, please." What was the price of the truck?

The price of the truck was 37¢.

Betty had a stamp album. Before Christmas she had 82 stamps in it. Her brother gave her 25 stamps for Christmas. How many stamps did she have then?

Betty had 107 stamps.

Jim's airplane needed paint. He bought some paint for 19¢. How much change did he get from a quarter?

He got 6¢ change.

David bought a boat for 49¢. Jack bought a boat for 98¢. How much more did Jack's boat cost than David's?

Jack's boat cost 49¢ more than David's.

Aunt Sally bought some toys for her sister's new baby. She paid 35¢ for a rattle, 25¢ for a duck, and 98¢ for a stuffed bear. How much did she spend for the toys?

She spent $1.58 for the toys.

Dick had saved $1.75. After he spent 98¢ for a baseball and 49¢ for a kite, how much money did he have?

He had 28¢.

# CHAPTER VII

# CONGRUENCE OF ANGLES AND TRIANGLES

## Background

In this chapter we continue the study of sets of points that was initiated in Chapter III.

The idea of congruence was introduced in Chapter V, but only for very primitive geometric figures--namely, line segments. We now apply the idea to more involved geometric figures such as triangles and rectangles. Instead of working directly with the triangles and rectangles themselves, we emphasize the corresponding triangular and rectangular regions as being easier for the child to deal with.

Definitions of the technical terms are as provided in the text. A more detailed exposition appears in the introduction to Chapter III of this book.

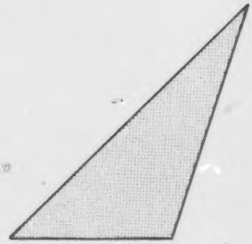
## VII-1. Congruence of triangular regions

Objectives: To introduce the general idea of congruence.

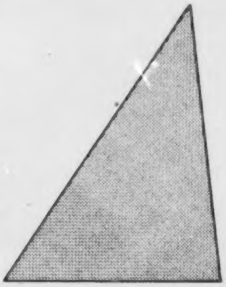
To introduce the distinction between a triangle and the region it encloses.

Vocabulary: Congruence, congruent; (Review) triangle, side, vertex (vertices) of a triangle, triangular region.

Materials: Tracing paper and oaktag straightedge for each child; triangular regions of construction paper as follows: congruent red and green regions (say with sides 8", 10", and 12"), and a "smaller" and differently shaped yellow region (say with sides 6", 9" and 12").



Yellow



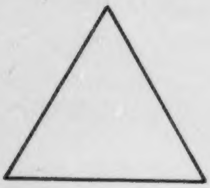
Red, Green

Suggested Procedure:

Before class, draw black crayon borders on each region, one side of paper only. (Draw the borders before cutting the paper!)

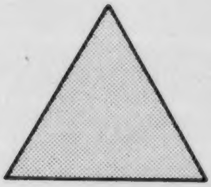
Recall with the children that a triangle is a polygon made up of three line segments, called the sides of the triangle. Indicate the one drawn on the chalkboard. Distinguish for the children between the triangle itself (made up of line segments) and the inside of the triangle (the part enclosed by the triangle).

Illustrate by shading the inside of the triangle. The two together--the triangle itself plus the inside--form what we call a triangular region.



triangle

inside



triangular region

Show the red paper as an example of a triangular region. Run your finger around the edges to show the sides of the triangle; point to the black crayon border as a way of forcefully calling the sides to the children's attention. Show the vertices of the triangle. Run your hand over the paper to display the inside of the triangle. Then pin the paper to the bulletin board, at one side.

Remind the children that they have worked with congruence of line segments. Now we will discuss congruence of other geometric figures as well.

Two geometric figures are said to be congruent if they have the same size and shape. (In the case of line segments we had to consider size--length--only, since all line segments have the same "shape".) A test of congruence is whether one figure can be fitted exactly onto the other. In practice, the objects may not be conveniently movable; then tests for congruence are made by making a movable copy of one and checking it against the other.

Bring out the yellow triangular region and pin it at the other side of the bulletin board, away from the red one. Ask whether the red and yellow regions are congruent. Have the children state reasons for answering no. ("The red one is bigger." "The yellow one has a sharper corner." "The yellow one bends out." Etc.)

Now display the green paper and ask which of the other two regions the children think this one might be congruent with. Verify by unpinning the red paper and fitting the two together; since the regions match exactly, they are congruent. Emphasize that in this matching, the vertices of the green triangle match the vertices of the red. Show how each side of the green triangle exactly matches a side of the red; a green-red pair of matching sides is an example of a pair of congruent line segments, familiar (hopefully) from the discussion in Chapter 5. Run your finger around the edges to show that the green triangle is congruent with the red. Observe that the inside of the green triangle is congruent with the inside of the red. Finally, recapitulate that the green triangular region is congruent with the red triangular region.

Pupil's book, pages 197-199:

Ideas

> Two figures are congruent if they have the same size and shape. They need not be shown in the same position.

Pupil's book, page 197:

The children should do these by observation.

Pupil's book, page 198:

Some instruction in tracing may be in order. The recommended way is to mark and label the vertices; then put in the sides, not by tracing but by using the straightedge; then compare the result with the original as a check.

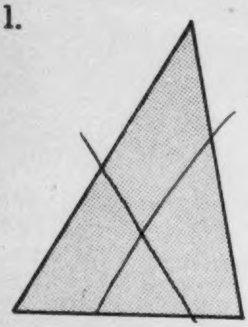
Pupil's book, page 199:

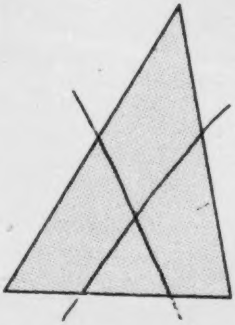
The children should do these by observation. They will have to rotate the figures in their imagination.

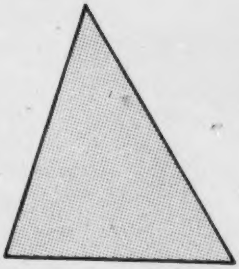
## Congruence of Triangular Regions

In each row mark the two regions that are congruent.

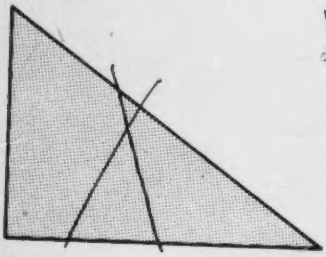
1.

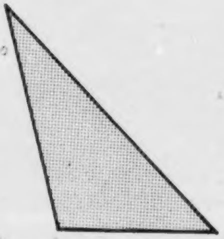


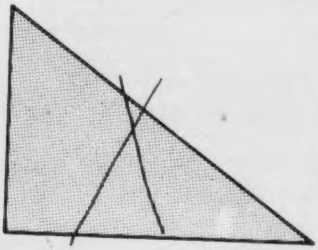




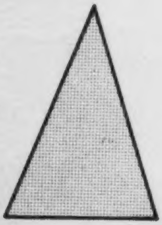
2.

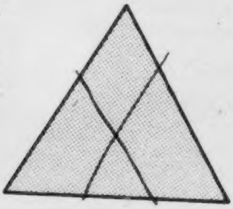


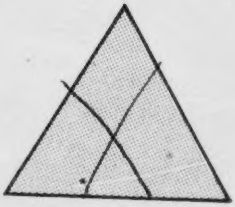




3.



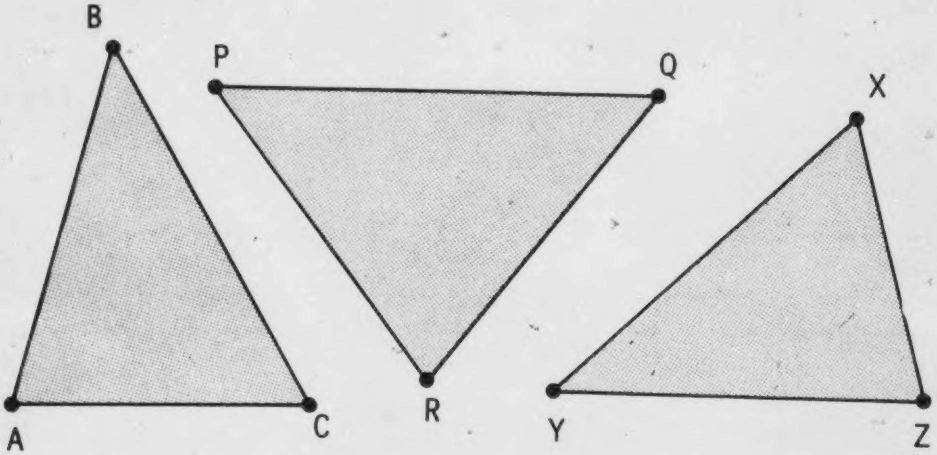




## Congruence of Triangular Regions

4. Make a tracing of $\triangle ABC$.

Mark the points A, B, C on the tracing.



Line segment $\overline{AB}$ is congruent to $\overline{YZ}$ and to $\overline{QR}$.

Line segment $\overline{BC}$ is congruent to $\overline{XY}$.

Line segment $\overline{CA}$ is congruent to $\overline{XZ}$.

$\triangle ABC$ is congruent to $\triangle XYZ$.

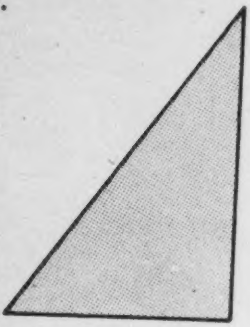
The inside of $\triangle ABC$ is congruent to the inside of $\triangle XYZ$.

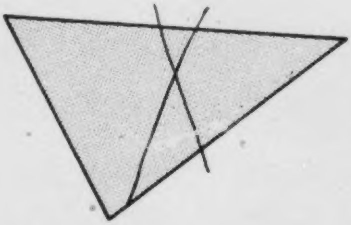
The triangular region ABC is congruent
to the triangular region XYZ.

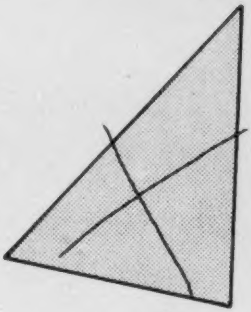
## Congruence of Triangular Regions

In each row mark the two regions that are congruent.

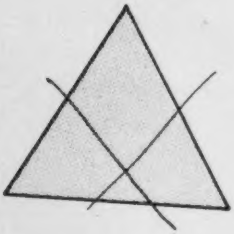
5.

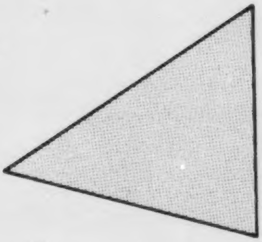


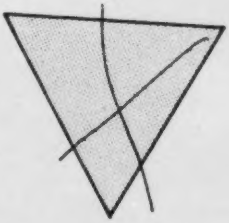




6.







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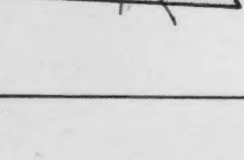

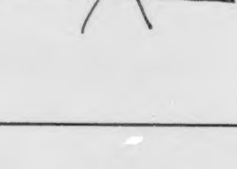

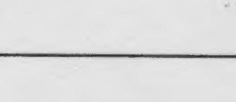

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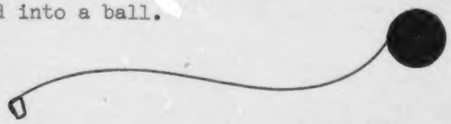
## VII-2. Rays

Objective: To introduce the idea of a ray.

Vocabulary: Ray, (Review) line segment, line.

Materials: Unmarked oaktag straightedge (one for each child), masking tape, a piece of string longer than the room. Mark one end of the string with masking tape, and roll the other end into a ball.



### Suggested Procedure:

Review line segments and lines from Chapter III. Recall that a line segment is a straight curve between two points. Review the [illegible] $\overline{AB}$. Recall that a line is obtained (in the imagination) by extending a line segment infinitely far in both direction. Remind the children of the experiment in unrolling a piece of string to demonstrate a succession of longer and longer line segments.

Pupil's book, page 200

### Ideas

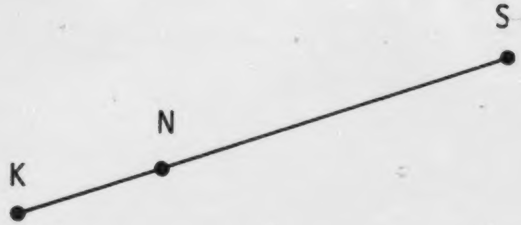
Any two points on a line segment can be used to name the line segment.

Any two points on a line can be used to name the line.

Line segments are subsets of lines.

## Review

1. Line segment $\overline{KS}$ is shown below.

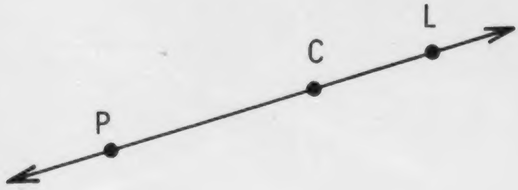


Write names for two other line segments. $\overline{KN}$ $\overline{NS}$

Is $\overline{KN}$ part of $\overline{KS}$? [✓ Yes] No

Is $\overline{KN}$ part of $\overline{NS}$? Yes [✓ No]

2. Line $\overleftrightarrow{CL}$ is shown below.



Write four more names for this line. $\overleftrightarrow{CP}$ $\overleftrightarrow{LC}$ $\overleftrightarrow{PL}$ $\overleftrightarrow{LP}$

Is line segment $\overline{CL}$ part of line $\overleftrightarrow{CL}$? [✓ Yes] No

Is $\overline{PC}$ part of $\overleftrightarrow{CL}$? [✓ Yes] No

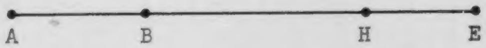
Ask two children to come to the front of the room. Have one child stand against the wall and hold the taped end of the string. Give the ball of string to the other child. Mark a point near the ball with masking tape, and have the children pull the string tightly to show a line segment between the two marked points. Now ask the second child to move away, unrolling the string as he goes. Mark another point on the string and observe that the original line segment has been extended to a longer one. Have the children imagine repeating the procedure again and again without end. The result would be a ray. The point marked at the first child is the endpoint of the ray. A ray has one endpoint.

Objects found outside may suggest rays. Have the children choose a point on the boundary of the volleyball court or at the tip of a pole, and imagine a set of points that goes on and on in one direction from that endpoint. Think also of the beam of a flashlight and of the rays of the sun.

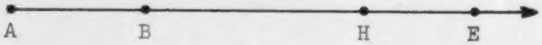
Draw a line segment on the board; label it $\overline{AB}$. Ask the children to imagine a longer line segment that contains $\overline{AB}$ but still has A as one of its endpoints. Draw such a line segment; label the new endpoint H.

Emphasize that the point B, an endpoint of the original line segment, is not an endpoint of the new one (although it is a point on it).

Extend $\overline{AH}$ to form a new line segment $\overline{AE}$.



Have the children agree that the process of extending line segments in one direction can, in our imagination, be repeated again and again without end. The result will be a ray. A ray has just one endpoint. Explain that we show a ray with the help of one arrow.



Ask how many rays there can be having A as endpoint and passing through the point B. When the children agree that there can be only one (the one shown), observe that it is also the only ray having A as endpoint and passing through H, or through E, etc. Introduce the symbol $\overrightarrow{AB}$, and agree that other equally good names for the ray shown are $\overrightarrow{AH}$ or $\overrightarrow{AE}$.

Have the children name all the line segments whose endpoints are marked. (There are six: $\overline{AB}$, $\overline{AH}$, $\overline{AE}$, $\overline{BH}$, $\overline{BE}$, and $\overline{HE}$.)

Emphasize that $\overrightarrow{BA}$ is an entirely different ray from $\overrightarrow{AB}$. Draw $\overrightarrow{BA}$, using chalk of a different color.

Pupil's book, pages 201-203

Ideas

A ray is an extension of a segment in one direction.

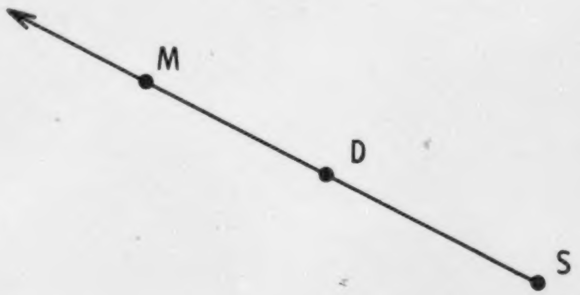
A ray has one endpoint.

The endpoint of a ray is named first.

Line segments are subsets of rays.

## Rays

1. Ray $\overrightarrow{SM}$ is shown below.



Write another name for $\overrightarrow{SM}$. $\overrightarrow{SD}$

Name the endpoint of $\overrightarrow{SM}$. S

Is the endpoint named first? [✓ Yes] No

2. How many endpoints does a line segment have? None 1 [✓ 2]

How many endpoints does a line have? [✓ None] 1 2

How many endpoints does a ray have? None [✓ 1] 2

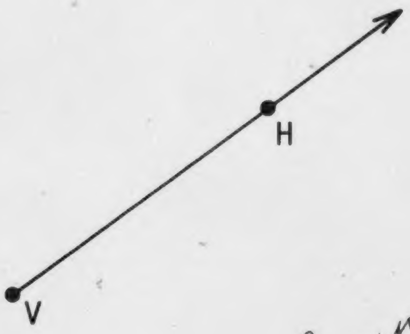
## Rays

3. Here is ray $\overrightarrow{VH}$.

Draw two more rays with V as endpoint.

Mark another point F on one ray you drew.

Mark another point L on the other ray you drew.

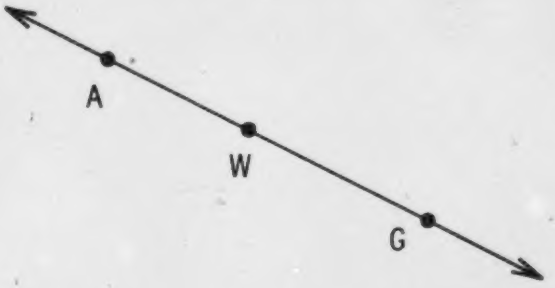


Rays will vary.

Name the rays you drew. $\overrightarrow{VF}$ $\overrightarrow{VL}$

## Rays

4. Here is line $\overleftrightarrow{AG}$.

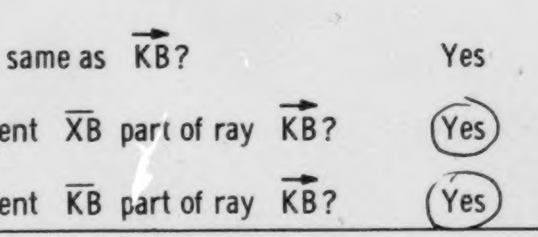


Is ray $\overrightarrow{GA}$ the same as ray $\overrightarrow{AG}$? Yes [✓ No]

Is $\overrightarrow{AW}$ the same as $\overrightarrow{AG}$? [✓ Yes] No

Is line segment $\overline{AW}$ part of ray $\overrightarrow{AG}$? [✓ Yes] No

5. Here is ray $\overrightarrow{KB}$.



Is $\overrightarrow{XB}$ the same as $\overrightarrow{KB}$? Yes [✓ No]

Is line segment $\overline{XB}$ part of ray $\overrightarrow{KB}$? [✓ Yes] No

Is line segment $\overline{KB}$ part of ray $\overrightarrow{KB}$? [✓ Yes] No

## VII - 3. Angles

**Objective:** To introduce the idea of an angle.

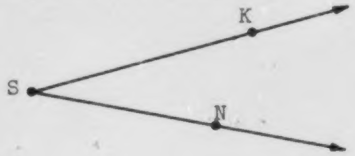
**Vocabulary:** Angle, vertex of an angle (vertices).

**Materials:** Unmarked oaktag straightedge (one for each child).

**Suggested Procedure:**

Two rays with an endpoint in common form an *angle*. Explicitly, the angle is defined as the *union* of the two rays. We shall exclude the case in which the two rays lie on the same line; this is a technicality, however, that need not be belabored.

Draw an angle on the chalkboard, as shown.



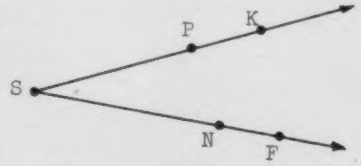
Call attention to how the figure is formed. Show ray $\overrightarrow{SK}$, with endpoint S. Show ray $\overrightarrow{SN}$, with endpoint S. Emphasize that these rays have the same endpoint, S. Introduce the word *angle*.

Draw several other angles, label them, and have the children read off the names of the rays forming them and the names of their common endpoints.

Introduce the word *vertex* (plural: *vertices*); the vertex of an angle is the common endpoint of the two rays forming the angle. The vertex of the angle shown above is S. Go back over the angles drawn on the board, asking the children to identify the vertices.

Show the children the symbol $\angle$ KSN. Another name for the angle shown above is $\angle$NSK. Emphasize that when we name an angle, we always put the name of the vertex in the middle.

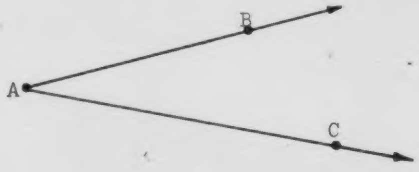
Mark two additional points on the angle, P and F as shown.



Ask the children to read new names for the angle. (There are six: KSF, FSK, PSF, FSP, PSN, NSP.)

Many objects on the playground and in the classroom will suggest angles; the corners of the volleyball court, the braces of a pole, the top of the slide, the hands of the clock. In each case, have the child identify the vertex.

The concept that an angle is the union of two rays, rather than of two line segments, may be difficult and need not be belabored. On the contrary, it is important to emphasize that two line segments with an endpoint in common do determine an angle, since they form a corner: line segments $\overline{AB}$ and $\overline{AC}$ form a corner of angle $\angle$ BAC.



Pupil's book, pages 204-206

Ideas--page 204

An angle is formed by two rays with a common endpoint. The common endpoint of the rays is called the vertex of the angle.

Ideas--page 205

In naming an angle, the name of the vertex always goes in the middle.

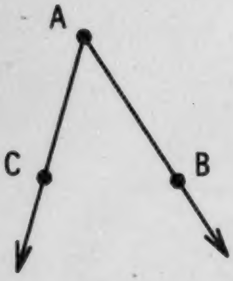
Ideas--page 206

Example 3. Since rays have many names, the angles formed by them have many names.

Example 4. Two line segments with an endpoint in common form the corner of an angle.

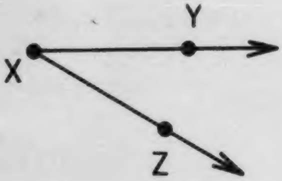
## Angles

1. In each angle, name the vertex and the rays.



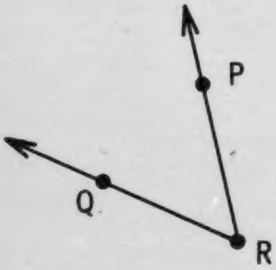
vertex A

rays $\overrightarrow{AC}$ $\overrightarrow{AB}$



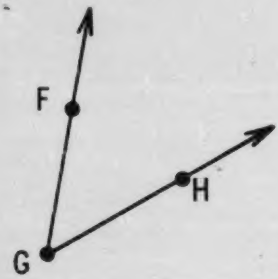
vertex X

rays $\overrightarrow{XY}$ $\overrightarrow{XZ}$



vertex R

rays $\overrightarrow{RP}$ $\overrightarrow{RQ}$

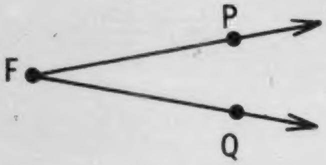


vertex G

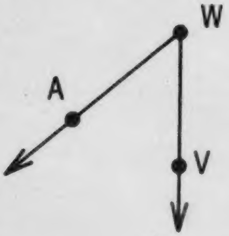
rays $\overrightarrow{GF}$ $\overrightarrow{GH}$

## Angles

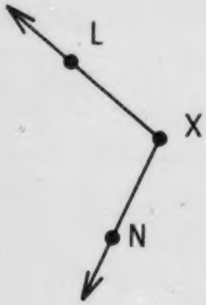
2. Write two names for each angle.



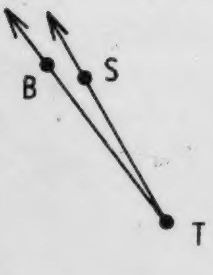
∠PFQ ∠QFP



∠AWV ∠VWA



∠LXN ∠NXL

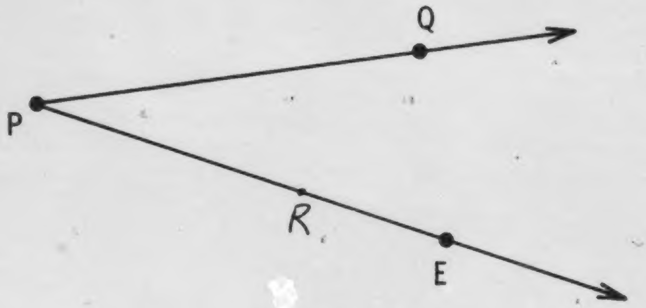


∠BTS ∠STB

Angles

3. Name the vertex of the angle. P

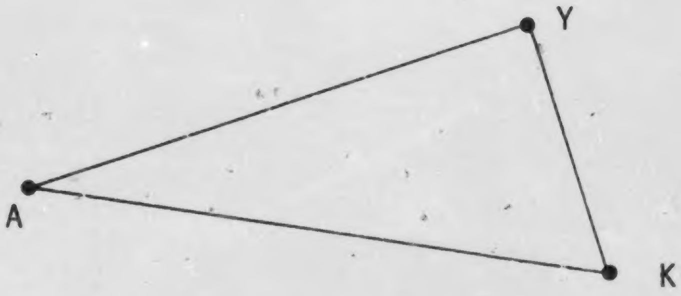
Write two names for the angle. ∠QPE ∠EPQ



Mark another point R on $\overrightarrow{PE}$.

Write two more names for the angle. ∠RPQ ∠QPR

4. Draw triangle △AYK.



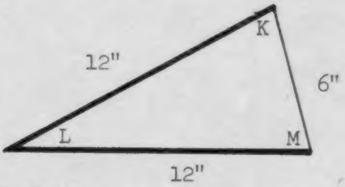
Name three angles whose corners are shown. ∠AYK ∠YKA ∠KAY

## VII-4. Congruence of Angles

**Objective:** To realize that two angles are congruent if they match at the corners, and that an angle of one triangle can be congruent to an angle of another even when the triangles themselves are not congruent.

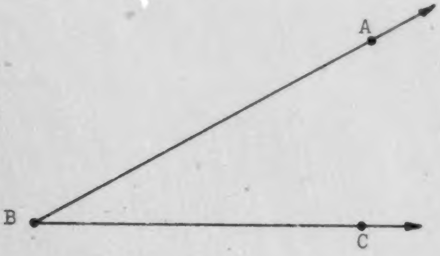
**Vocabulary:** (No new words.)

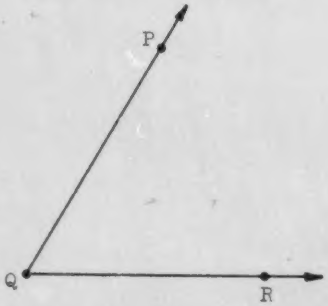
**Materials:** Straightedge for use at the chalkboard. Triangular piece of tagboard in the approximate dimensions shown; border $\overline{LK}$ and $\overline{LM}$ in black to emphasize $\angle KLM$. Tracing paper and oaktag straightedge for each child.



**Suggested Procedure:**

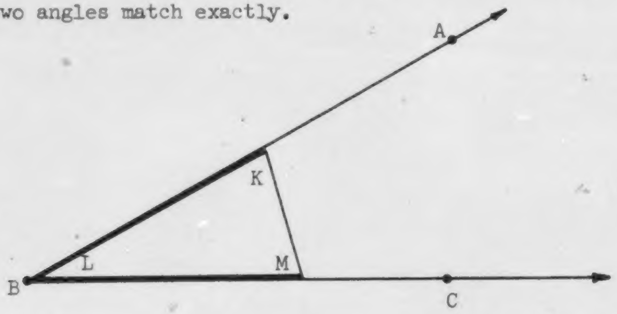
Before class, draw two angles on the chalkboard as shown. Obtain $\angle ABC$ from $\angle KLM$ by tracing. Make $\angle PQR$ visibly larger than $\angle ABC$.



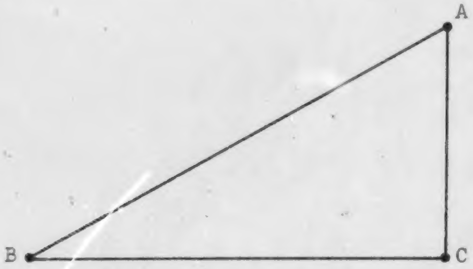


Ask the children whether $\angle ABC$ is congruent to $\angle PQR$. This means that they would fit exactly if we could move one onto the other. It should be clear to the children that they are not congruent. Ask the children to tell why. (The corner at Q is wider than the corner at B, etc.)

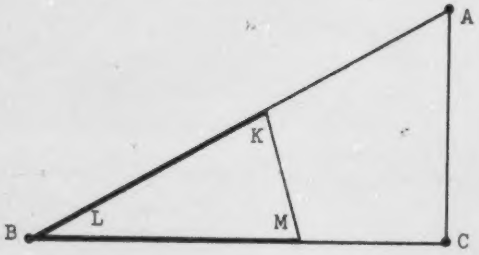
Now hold up the piece of tagboard and ask whether $\angle KLM$ appears to be congruent to either of the angles shown: $\angle ABC$ and $\angle PQR$. Explain that when dealing with congruence of angles, it is the corners that are important. Hold $\angle KLM$ up against $\angle ABC$ to show how they fit. Point out that ray $\overrightarrow{LK}$ is now the same as ray $\overrightarrow{BA}$, and that ray $\overrightarrow{LM}$ is now the same as ray $\overrightarrow{BC}$; therefore the two angles match exactly.



Return $\triangle KLM$ temporarily to the table. On the chalkboard, erase the arrows at A and C, and draw $\overline{AC}$, thus emphasizing $\triangle ABC$. Show how the sides of the triangle form the corners of three angles. The vertices of the angles are the vertices of the triangle. The angles of $\triangle ABC$ are $\angle ABC$, with vertex at B; $\angle BCA$, with vertex at C; and $\angle CAB$, with vertex at A.



Once more, hold the tagboard triangle KLM up against $\triangle ABC$ to show that $\angle KLM$ is congruent to $\angle ABC$. Observe that $\triangle KLM$ is not congruent with $\triangle ABC$.



This shows that an angle of one triangle can be congruent to an angle of another triangle even when the triangles themselves are not congruent.

Pupil's book, pages 207-209

Ideas

Two angles are congruent if they match at the corners.

Page 208

On this page the tracings must be rotated to fit.

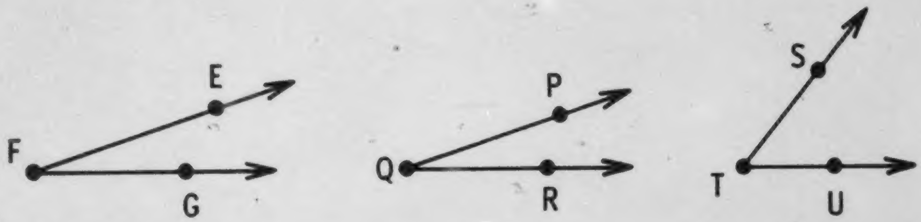
Ideas--page 209

An angle of one triangle can be congruent to an angle of another even when the triangles themselves are not congruent.

## Congruence of Angles

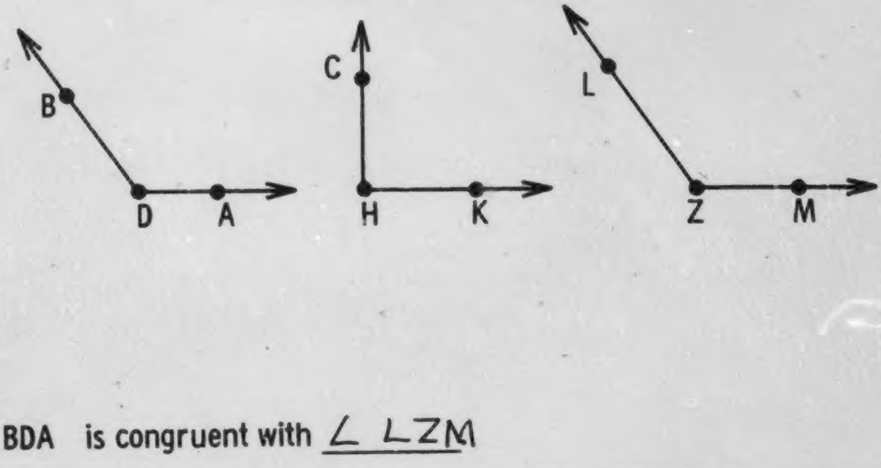
Find out by tracing.

1.



∠EFG is congruent with ∠PQR.

2.

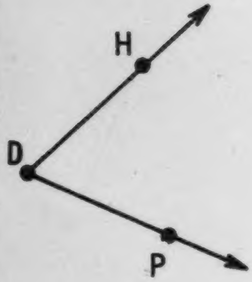


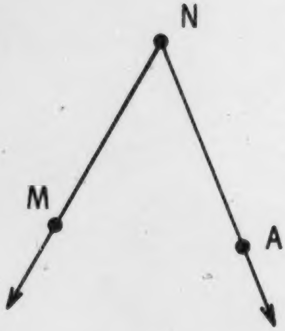
∠BDA is congruent with ∠ LZM

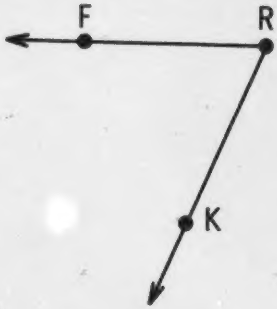
## Congruence of Angles

Find out by tracing.

3.

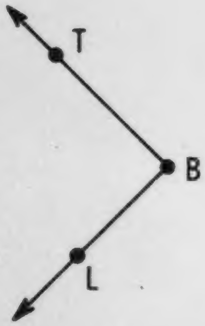


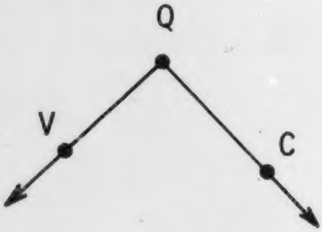


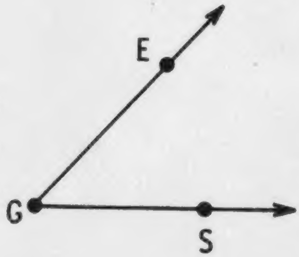


∠ HDP is congruent with ∠ FRK.

4.



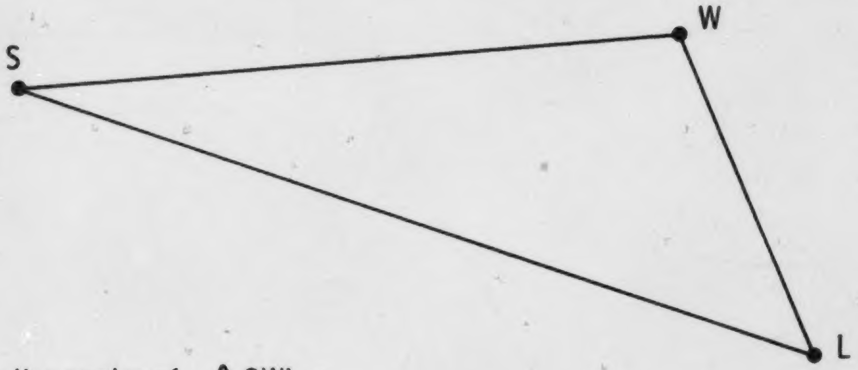




∠ TBL is congruent with ∠ VQC

## Congruence of Angles

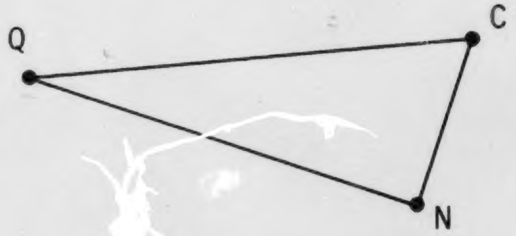
5.



Name the angles of $\triangle SWL$.

∠WSL ∠SLW ∠LWS

6.



∠CQN is congruent with ∠WSL.

## VII-5. Right Angles

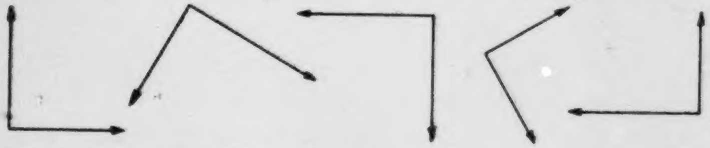
**Objective:** To introduce the idea of a right angle.

**Vocabulary:** Right angle; (Review) rectangle, square, rectangular region, square region.

**Materials:** Piece of chipboard for tracing right angles, rectangular picture frame, tracing paper and oaktag straightedge for each child.

**Suggested Procedure:**

On the chalkboard, draw right angles in several positions.

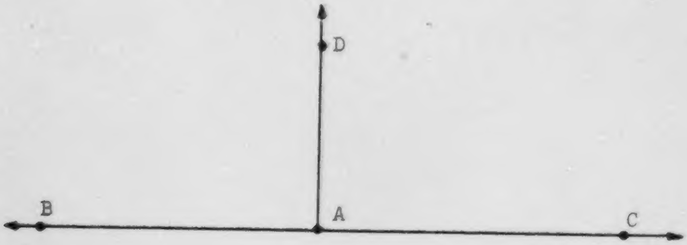


Explain that these angles are called right angles. Ask the children to describe them. ("They are square corners," etc.)

Discuss whether all right angles are congruent to each other. (Yes, they are.)

Have the children look around the room for objects suggesting right angles. There are many: corners of walls, floors, doors, windows, desks, sheets of paper, etc.

To describe a right angle exactly, draw the following figure on the chalkboard; here, points B, A, C lie on a line, and $\overline{DA}$ is perpendicular to $\overline{BC}$.



Tell the children that B, A, and C lie on the same line. Have the children tell you that $\angle DAC$ and $\angle DAB$ are right angles.

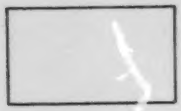
> When two right angles are placed "back to back" along a common ray ($\overrightarrow{AD}$), their other rays ($\overrightarrow{AC}$ and $\overrightarrow{AB}$) form a line ($\overleftrightarrow{BC}$).

The children can test this by lining up two desks or two sheets of paper along a common edge. Have them point out the two right angles, their common vertex, their common ray, and the line formed by their other two rays.

Draw a rectangle on the chalkboard. Remind the children of the word "rectangle". Display the picture frame. Show the four sides. Have the children find other examples in the classroom: the edges of the desk, door, window, etc. Ask for a description of a rectangle. Direct discussion to generalize that it is "a polygon with four sides (and four angles), and with all angles right angles".

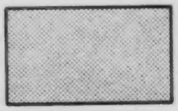
Have the children observe that opposite sides of a rectangle are congruent. Have them test this by folding a sheet of paper to match up opposite edges.

Distinguish for the children between the rectangle itself (made up of line segments) and the inside of the rectangle (the part enclosed by the rectangle). Illustrate by shading the inside of the rectangle drawn on the chalkboard. The two together--the rectangle itself plus the inside--form what we call a rectangular region.



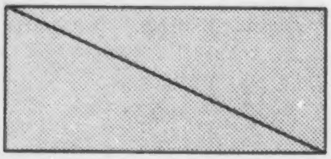
rectangle

inside



rectangular
region

Draw a diagonal of the rectangle and show how it forms two triangles with the sides. Each of these triangles has one right angle. Likewise, the diagonal cuts the rectangular region into two triangular regions (having the diagonal in common). Discuss whether these triangular regions are congruent. (They are.)



Remind the children that a square is a special kind of rectangle. A square is a rectangle whose four sides are all congruent. If helpful, refer to squares as "square rectangles".

A square plus the inside of the square forms a square region. Floor or ceiling tiles are good examples of square regions.

Pupil's book, pages 210-212

Ideas

If, when two congruent angles are placed "back to back" along a common ray and their other rays form a line, then the two angles are right angles.

Example 1

The pupils should pick out the right angles by observation.

Pupil's book, page 213

Ideas

A rectangle has two pairs of congruent sides.

A square is a rectangle with all four sides congruent.

Pupil's book, pages 214-215

Ideas

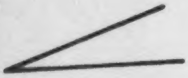
Two geometric figures are congruent if they have the same size and shape.

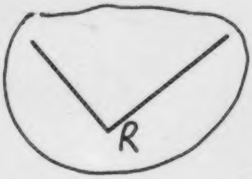
Page 214

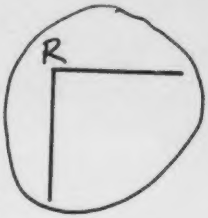
The pupils should judge congruence by observation.

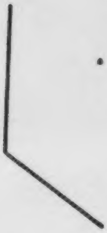
## Right Angles

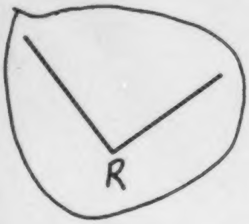
1. Mark each right angle.

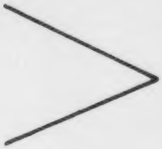


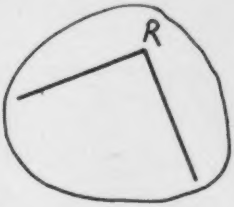


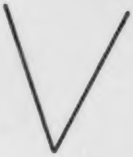




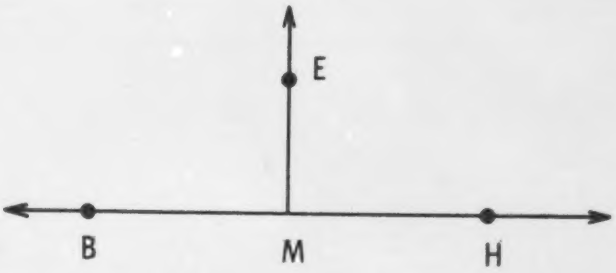








2.



Is ∠EMH congruent with ∠EMB? [✓ Yes] No

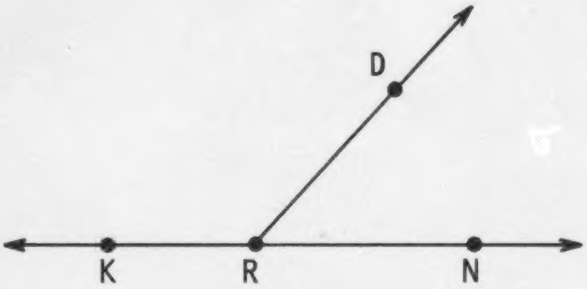
Do B, M and H lie on a line? [✓ Yes] No

Is ∠EMH a right angle? [✓ Yes] No

Is ∠EMB a right angle? [✓ Yes] No

## Right Angles

3.



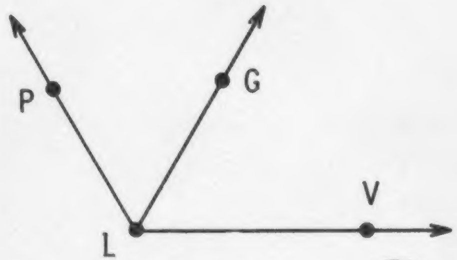
Do K, R, and N lie on a line? [✓ Yes] No

Is ∠DRN congruent with ∠DRK? Yes [✓ No]

Is ∠DRN a right angle? Yes [✓ No]

Is ∠DRK a right angle? Yes [✓ No]

4.



Is ∠GLV congruent with ∠GLP? [✓ Yes] No

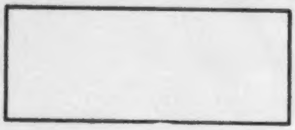
Do P, L, and V lie on a line? Yes [✓ No]

Is ∠GLV a right angle? Yes [✓ No]

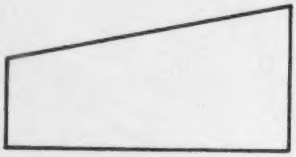
Is ∠GLP a right angle? Yes [✓ No]

## Right Angles

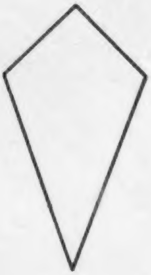
5. How many right angles does each polygon have?



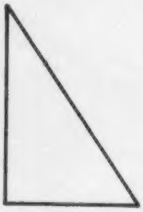
None 1 2 3 [✓ 4]



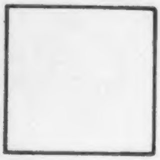
None 1 [✓ 2] 3 4



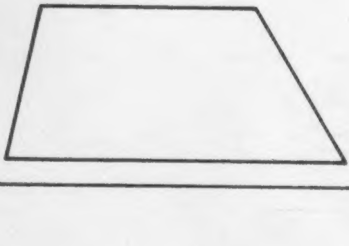
None [✓ 1] 2 3 4



None [✓ 1] 2 3 4



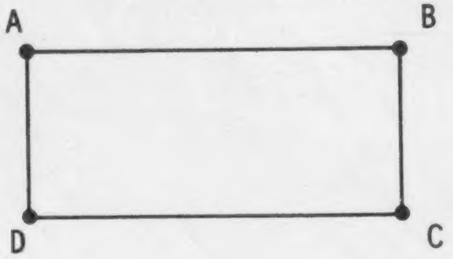
None 1 2 3 [✓ 4]



[✓ None] 1 2 3 4

## Rectangles and Squares

1. Here is a rectangle.



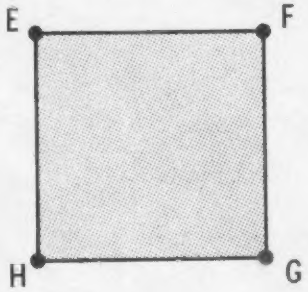
$\overline{AB}$ is congruent with $\overline{DC}$.

$\overline{AD}$ is congruent with $\overline{BC}$.

Is $\overline{AB}$ congruent with $\overline{AD}$? Yes [✓ No]

Is the rectangle a square? Yes [✓ No]

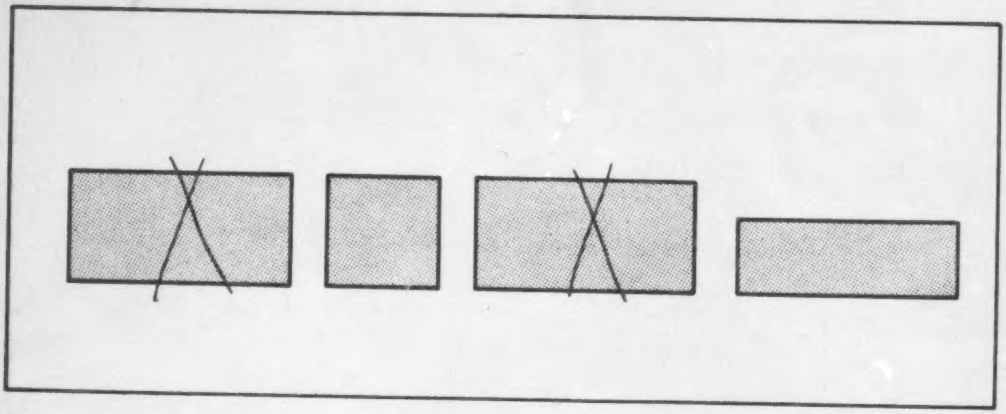
2. Here is a square region.

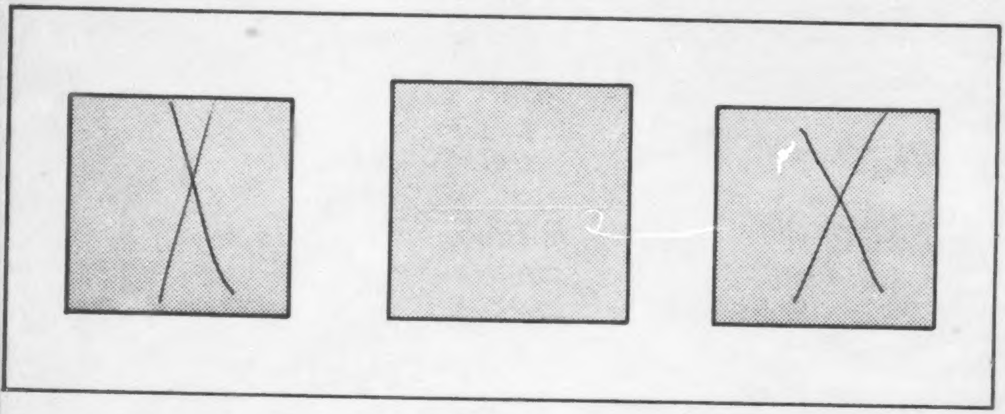


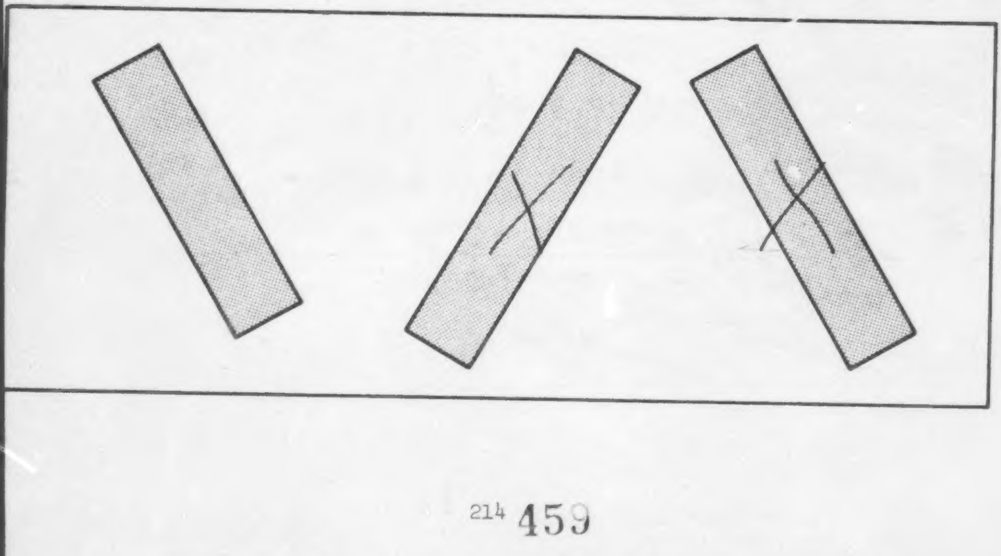
$\overline{EF}$ is congruent with $\overline{FG}$, $\overline{GH}$, and $\overline{HE}$.

## Regions

Mark the congruent rectangular regions.





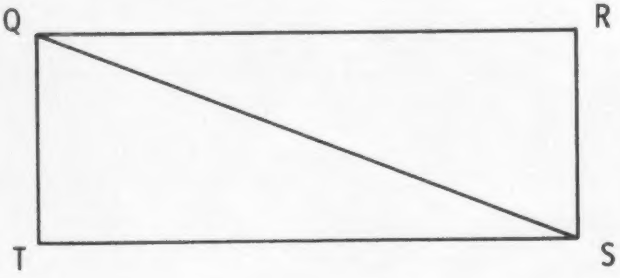


4. Here is a rectangular region.

Make a tracing of $\triangle QRS$.

Mark the points Q, R, and S on the tracing.

Fit the tracing on $\triangle STQ$.



$\overline{QR}$ is congruent with $\overline{ST}$.

$\overline{RS}$ is congruent with $\overline{TQ}$.

$\angle RSQ$ is congruent with $\angle TQS$.

$\angle SQR$ is congruent with $\angle QST$.

$\triangle QRS$ is congruent with $\triangle STQ$.

The triangular region QRS is congruent with the triangular region STQ.

# Chapter VIII

# ARRAYS AND MULTIPLICATION

## Background

Section VIII-1 reviews the idea of array (introduced in Chapter VIII of Book I). An *array* is a rectangular arrangement of objects into *rows*, each row containing the same number of objects. Shown below is an array of 3 rows, each row containing 5 x's.

x x x x x

x x x x x

x x x x x

The objects in an array are usually called elements. These objects need not be all alike. For instance, here is another array, consisting of 2 rows of 3 members each:

☆ ○ A

Z □ ▲

Arrays may also consist of rectangular arrangements of flannel board objects, or blocks on the floor, or drawers in a cabinet, or panes in a window, or compartments in a carton, etc.

An array of 3 rows of 5 elements each is referred to as a *3 by 5 array*. An array may have only 1 row, as in the case of the 1 by 3 array below.

B B B

Likewise we may have a 3 by 1 array:

Q

E

D

We study arrays because they help in understanding multiplication. This is taken up in Section VIII-2. The product $3 \times 4$ is defined as the number of members in a 3 by 4 array. Other products are defined similarly. The terminology parallels that for addition. Just as we say that the sum of the numbers 3 and 4 is 7, and write

$$3 + 4 = 7,$$

we say that the product of the numbers 3 and 4 is 12, and write

$$3 \times 4 = 12.$$

Section VIII-3 deals with several simple properties of multiplication. First, for any whole number n, a 1 by n array and an n by 1 array each have n members. From this we get the multiplication equations

$$1 \times n = n \quad \text{and} \quad n \times 1 = n.$$

An equation like

$$3 \times 4 = 4 \times 3$$

illustrates the commutative property of multiplication: either order of factors yields the same product. (Recall that we have already met, in Section II-1 of Chapter II, the commutative property of addition, in accordance with which $3 + 4 = 4 + 3$.) To see that $3 \times 4 = 4 \times 3$ amounts to observing that the number of elements in a 3 by 4 array is the same as the number of elements in a 4 by 3 array. This is evident, since turning a 3 by 4 array up on end yields a 4 by 3 array. For any whole number n, a 0 by n array has no elements because it has no rows; and an n by 0 array has no elements because each of its rows has no elements. This leads to the multiplication equations

$$0 \times n = 0 \quad \text{and} \quad n \times 0 = 0.$$

Section VIII-4 introduces the distributive property of multiplication over addition, according to which, for instance,

$$4 \times (5 + 2) = (4 \times 5) + (4 \times 2)$$

and

$$(5 + 2) \times 4 = (5 \times 4) + (2 \times 4)$$

This is used at once to generate new multiplication facts from old ones. For example, we can obtain $4 \times 7 = 28$ from $4 \times 5 = 20$ and $4 \times 2 = 8$ as follows:

$$\begin{aligned} 4 \times 7 &= 4 \times (5 + 2) \\ &= (4 \times 5) + (4 \times 2) \\ &= 20 + 8 \\ &= 28. \end{aligned}$$

## VIII-1. Arrays

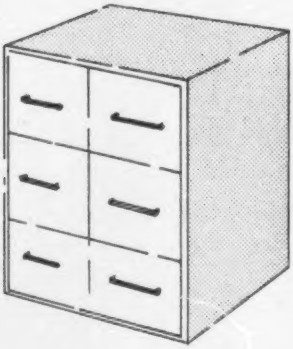
Objective: To review the idea of array.

Vocabulary: Array, row, element, member.

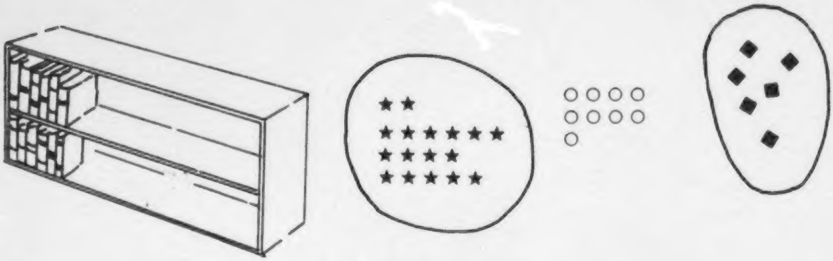
Materials: Arrays, objects for use on the flannel board, muffin tins, partitioned cartons (egg, soft drink, etc.) small manipulative objects (12 for each child), hundreds board, squared paper.

Suggested Procedure:

Look around the room for rectangular arrangements of objects: the drawers in the file cabinet, window panes, desks in the room, etc. (It may be advisable to pre-arrange objects such as pictures on a bulletin board.) Call attention to these rectangular arrangements: Look at drawers of the file cabinet. Notice that the drawers are in rows. How many rows of drawers do you see? (In the picture at the left, 3.) How many drawers are in each row? (2) How are the panes in our windows arranged? How many rows of panes are in each window? How many panes are there in each row?



Find some arrangements in the room that are not rectangular such as books on shelves, or use a flannel board to display the following:



Ask whether the objects are arranged in rows. Ask whether there is the same number of members in each row. Call attention to the fact that this is the real difference between a rectangular arrangement, such as the arrangement of window panes, and the arrangement such as that of rows of books with different numbers of members in each row. State that a rectangular arrangement is called an *array* and write the word *array* on the chalkboard. Be sure children do not confuse array with *ray*. Have the various arrays in the room mentioned again as examples of arrays. Also indicate that the objects in an array are called *elements* or *members* of the array. For example, each window pane is a member of the set of window panes in the array.

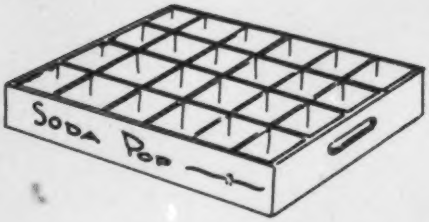
Hold up an egg carton, with two rows for six eggs each.

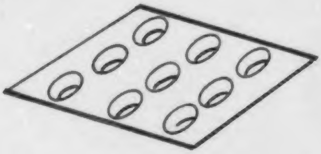
> The eggs that were in this carton were arranged in rows.
> How many rows of eggs were there? (2.)
> Did we have the same number of eggs in each row? (Yes.)
> How many eggs were in each row? (6.)

Describe the array as a "*two* *by* *six*" array, showing that the "two" refers to the number of rows and the "six" refers to the number of members in each row.

Have other arrays described: the file cabinet as a "three by two" array, for instance; the window panes, pictures, pennies arranged in many rows with two pennies in each row to be described as arrays, other objects in rows with five members, etc.

An empty soft drink case and muffin tin illustrate other arrays familiar to children.





How many rows are there?
How many members are there in each row?
How could you describe this array?

Illustrate on the flannelboard the idea that sets of objects may be arranged to form an array. For example, a set of 8 objects can be arranged to form a 2 by 4 array or a 4 by 2 array--or an 8 by 1 array or a 1 by 8 array. (Arrays having 1 row or 1 member in each row will be given special emphasis later.)

Have each child use 12 objects to try to form arrays independently. Keep a record on the chalkboard of the arrays made:

| Number of rows | Number of members in each row | Kind of array | Number of members in the array |
|---|---|---|---|
| 6 | 2 | 6 by 2 | 12 |
| 4 | 3 | 4 by 3 | 12 |
| 2 | 6 | 2 by 6 | 12 |
| 12 | 1 | 12 by 1 | 12 |
| 3 | 4 | 3 by 4 | 12 |
| 1 | 12 | 1 by 12 | 12 |

Ask children what happens when they try to make an array with 5 rows. (Can't do it; one row doesn't have five members.) Repeat for 7 rows and 9 rows.

Further Activities:

1. Have children bring to school examples of arrays. Prepare a bulletin board display using egg cartons, paper disks, or other objects showing arrays. Use descriptive labels (2 by 6 array, etc.) which can be replaced by equations as the chapter is taught.

2. Have children use squared paper to show arrays which you describe:

   "Use red crayon to show a 6 by 5 array. Use blue crayon to show a 5 by 6 array."

Pupil's book, page 216 - 217:

Children are to cross out the pictures which do not show arrays.

Pupil's book, pages 218 - 219:

This will probably have to be done as a class activity. Children are to match the pictures of arrays with the appropriate description, filling the blanks with the letter beside the picture of the array, and telling how many rows or members in each row the array has.

Pupil's book, page 220:

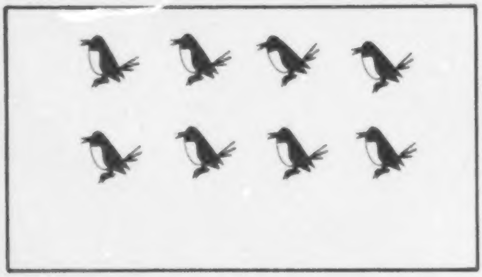
This page provides further practice in describing arrays and in finding out how many members are in the array.

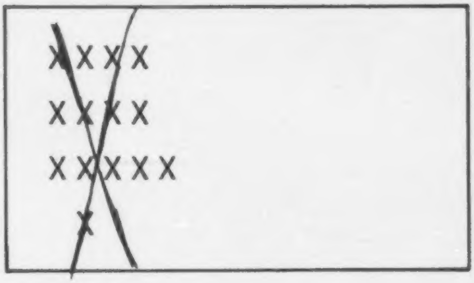
Pupil's book, page 221:

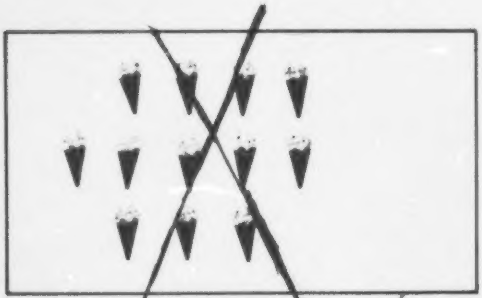
Children use X's to show arrays mentioned, and fill blanks to describe arrays.

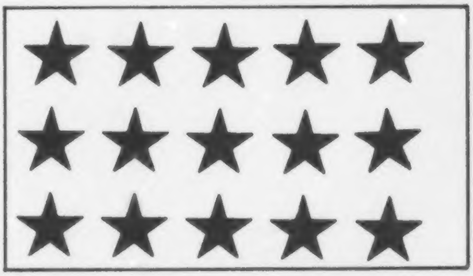
## Arrays

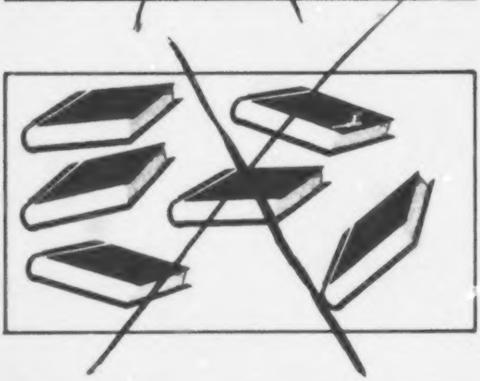
Cross out each picture below that does not show an array.

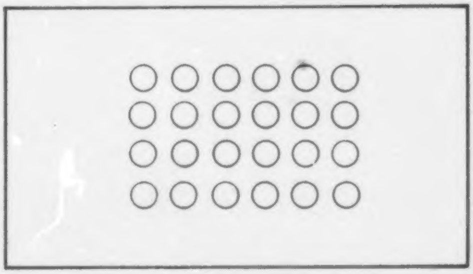


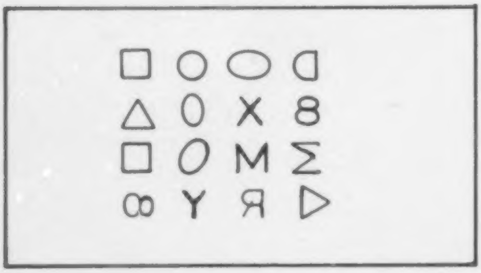


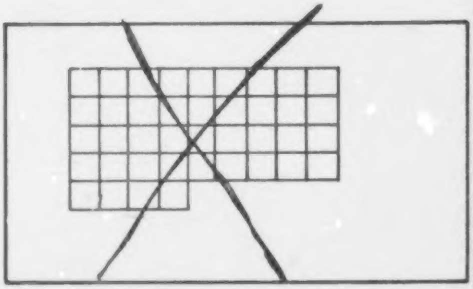






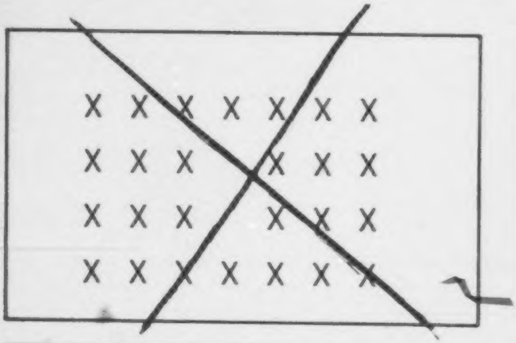


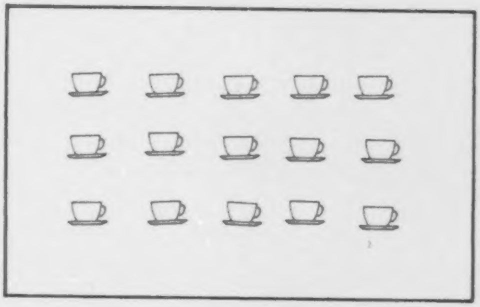


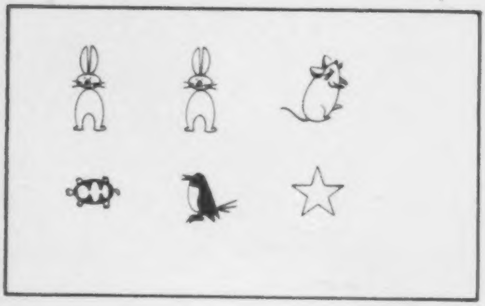


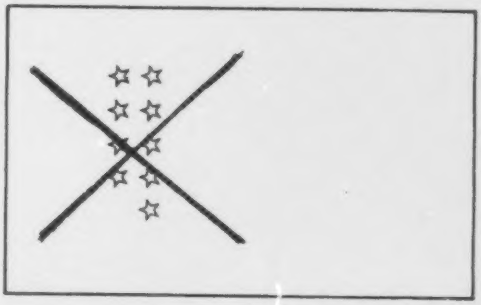
## Arrays

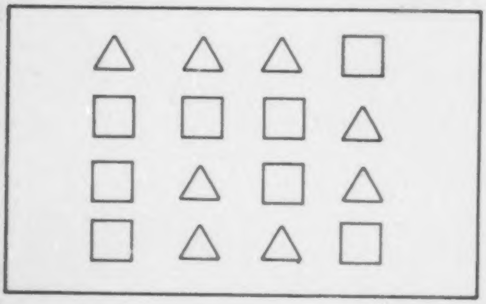
Cross out each picture below that does not show an array.

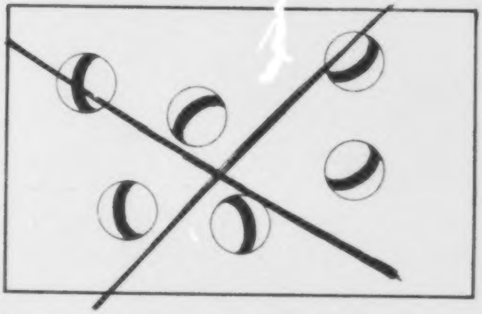


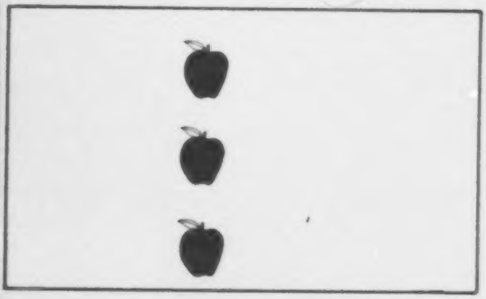


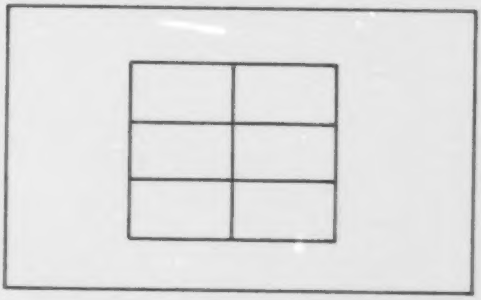






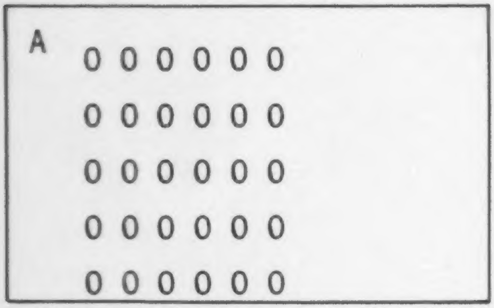




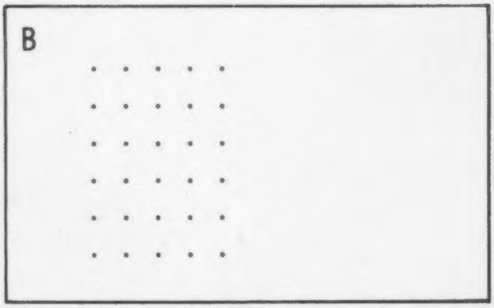


## Arrays

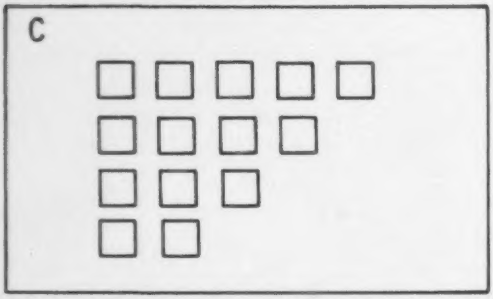
A



B



C



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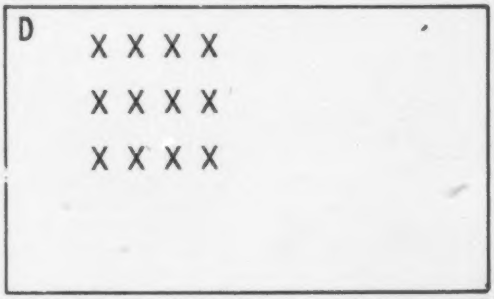
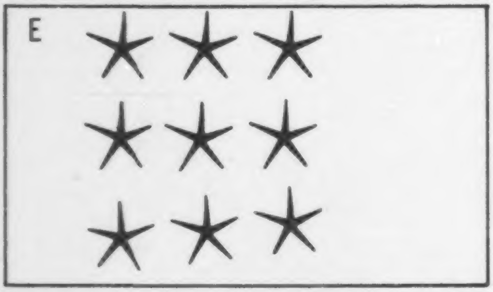
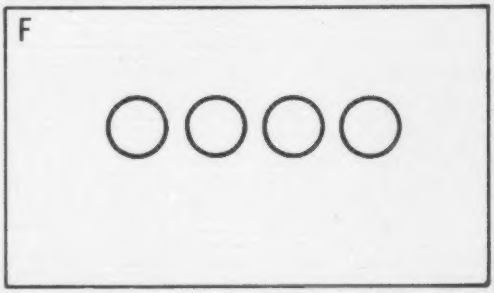


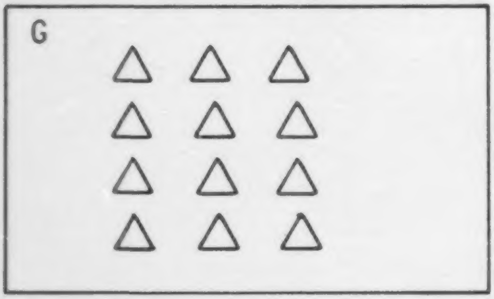
E



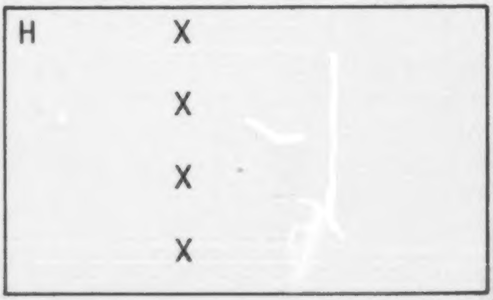
F



G



H



Match each picture on page 218 with the sentences that describe it. Fill the blanks.

Picture D has 3 rows. There are 4 members in each row.

Picture C does not show an array. It does not have the same number of things in each row.

Picture A shows a 5 by 6 array. It has 5 rows and 6 members in each row.

Picture B shows a 6 by 5 array. It has 5 members in each row and 6 rows.

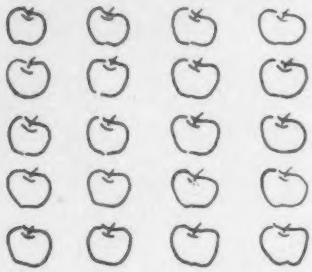
Picture G shows a 4 by 3 array. It has 4 rows and 3 members in each row.

Picture H shows a 4 by 1 array. It has 4 rows and 1 member in each row.

Picture E shows a 3 by 3 array. It has 3 members in each row and 3 rows.

Picture F shows a 1 by 4 array. It has 4 members in each row and 1 row.

## Describing Arrays



Number of rows: 5

Number of members in each row: 4

This is a 5 by 4 array.

This array has 20 members.

---

X X X

X X X

Number of rows: 2

Number of members in each row: 3

This is a 2 by 3 array.

This array has 6 members.

---

□ □ □ □ □

□ □ □ □ □

□ □ □ □ □

Number of members in each row: 5

Number of rows: 3

This is a 3 by 5 array.

This array has 15 members.

---

△ △ △ △ △ △ △

△ △ △ △ △ △ △

△ △ △ △ △ △ △

Number of members in each row: 7

Number of rows: 3

This is a 3 by 7 array.

This array has 21 members.

## Showing Arrays

Draw:

An array that has 4 rows of X's with 5 X's in each row.

X X X X X
X X X X X
X X X X X
X X X X X

We call this a 4 by 5 array.

An array that has 3 X's in each row with 5 rows.

X X X
X X X
X X X
X X X
X X X

We call this a 5 by 3 array.

An array that has 6 rows of X's with 4 X's in each row.

X X X X
X X X X
X X X X
X X X X
X X X X
X X X X

We call this a 6 by 4 array.

An array that has 6 X's in each row with 3 rows.

X X X X X X
X X X X X X
X X X X X X

We call this a 3 by 6 array.

## The Number of Members in an Array

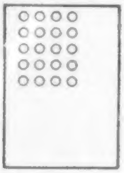
Children learn to identify the number of objects in an array by counting by rows and by addition. At this time we do not want to associate multiplication with arrays.

Using arrays described in Section 1--window panes, books, tiles, stamps, muffin tins, egg cartons, etc.--use procedures similar to the following.

Hold up the egg carton. Again have the array described (two by six). Then ask how many places there are for eggs in the carton (12). Ask children how they can find the number by counting, and by addition. Write an addition sentence for finding the number. (6 + 6 = 12.)

Likewise count by five, threes, sevens, etc., to find the number of members in other arrays. Also write addition sentences that can be used.

Use the hundreds board. Block off sections and count the number of objects exhibited. For example:

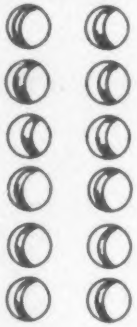


Count 4, 8, 12, 16, 20 as you move down one row at a time. Also add: 4 + 4 + 4 + 4 + 4 = 20.

Pupil's book, pages 222 - 223:

These pages may be used at this time.

## Counting Members of Arrays



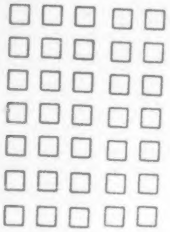
This is a 6 by 2 array.

Count by 2's.

2, 4, 6, 8, 10, 12

This array has 12 members.

2 + 2 + 2 + 2 + 2 + 2 = 12



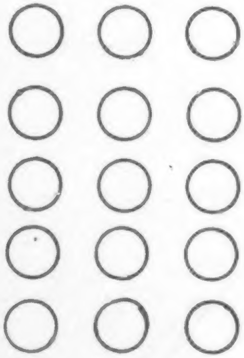
This is a 7 by 5 array.

Count by 5's.

5, 10, 15, 20, 25, 30, 35

This array has 35 members.

5 + 5 + 5 + 5 + 5 + 5 + 5 = 35



This is a 5 by 3 array.

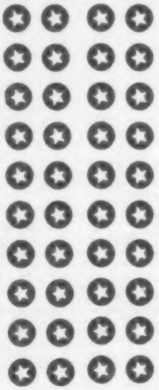
Count by 3's.

3, 6, 9, 12, 15

This array has 15 members.

3 + 3 + 3 + 3 + 3 = 15

## Counting Members of Arrays



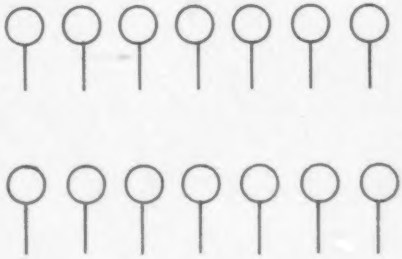
This is a 10 by 4 array.

Count by 4's.

4, 8, 12, 16, 20,

24, 28, 32, 36, 40

This array has 40 members.



This is a 2 by 7 array.

Count by 7's.

7, 14

This array has 14 members.

7 + 7 = 14

## VIII-2. Multiplication

Objective: To develop the idea of multiplication.

Vocabulary: Factor, product (Review) multiplication, times (symbol: ×).

Materials: Materials for flannel board and other objects for showing arrays.

Suggested Procedure:

Place on the flannel board a set of 3 objects and a set of 5 objects. Join the set of 5 to the set of 3. Have a child tell what you did. Ask what equation is suggested by putting these sets of objects together. (3 + 5 = 8.) Write this equation on the chalkboard.

Discuss the kind of equation. (Addition.)

Remove the materials from the flannel board and replace with a 3 by 5 array. Have the array described. (Three by five array.) Ask how many members are in the array. (15.)

> We have the number 3, for the number of rows, and the number 5, for the number of members in each row. We have 15 for the number of members in the array. We can write an equation for this idea. We say that 3 times 5 is 15. We can write an equation using a symbol instead of the word "times". We write 3 × 5 = 15. We call 3 × 5 = 15 a multiplication equation.
>
> Notice that when we add the pair of numbers 3 and 5, we get 8. When we multiply that pair of numbers, 3 and 5, we get 15.

Have some arrays displayed such as

| | | | |
|---|---|---|---|
| ○ ○ ○ | ○ ○ ○ ○ | ○ ○ ○ | ○ ○ ○ ○ ○ |
| ○ ○ ○ | ○ ○ ○ ○ | ○ ○ ○ | ○ ○ ○ ○ ○ |
| ○ ○ ○ | | | ○ ○ ○ ○ ○ |
| ○ ○ ○ | | | ○ ○ ○ ○ ○ |
| | | | ○ ○ ○ ○ ○ |

Let's write multiplication equations suggested by these arrays.

$4 \times 3 = 12$ $2 \times 4 = 8$

$2 \times 3 = 6$ $5 \times 5 = 25$

Pupil's book, pages 224 - 226:

These pages provide practice.

## Equations

▼ ▼ ▼
▼ ▼ ▼
▼ ▼ ▼
▼ ▼ ▼

4 rows
3 members in each row
12 members in the array
Equation: 4 × 3 = 12

---

X X X X X
X X X X X

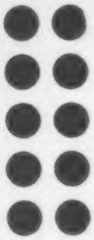
2 rows
5 members in each row
10 members in the array
Equation: 2 X 5 = 10

---

X X X
X X X
X X X
X X X
X X X
X X X

6 rows
3 members in each row
18 members in the array
Equation: 6 X 3 = 18

## Equations



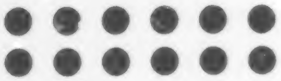
5 rows

2 members in each row

10 members in the array

Equation: 5 X 2 = 10

---



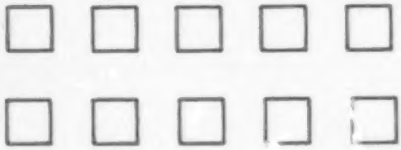
2 rows

6 members in each row

12 members in the array

Equation: 2 X 6 = 12

---



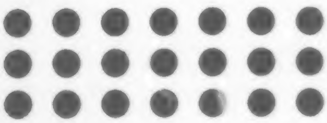
2 rows

5 members in each row

10 members in the array

Equations: 2 X 5 = 10

---



3 rows

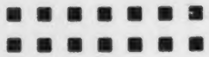
7 members in each row

21 members in the array

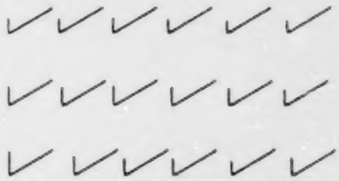
Equation: 3 X 7 = 21

**Multiplication Equations**

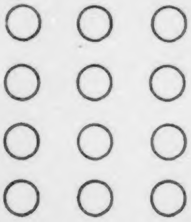
Write the equation for each array.



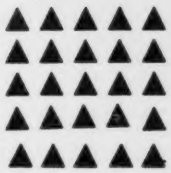
2 × 7 = 14



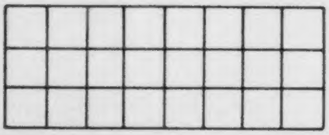
3 X 6 = 18



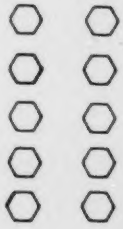
4 X 3 = 12



5 X 5 = 25



3 X 8 = 24



5 X 2 = 10

- Products and Factors

Now children learn that just as addends and sum were used to name numbers in an addition equation, we have names for the numbers in a multiplication equation. You first may wish to review the names given numbers in addition equations. For example, in the equation $5 + 3 = 8$, 5 and 3 are called addends and 8 is called the sum. Then explain that in a multiplication equation such as $5 \times 3 = 15$, 5 and 3 are called factors and 15 is called the product.

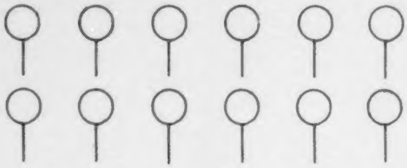
Review the equations that have been written. Ask children to name the product in each and the factors for each product.

Use other illustrations. Have children name the three numbers suggested by the arrays. Write equations. Have children name the product and its factors.

Pupil's book, pages 227 - 228:

First the equation is written. Then the product and factors are named.

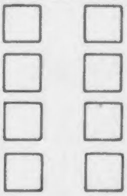
## Products and Factors



Equation: 2 X 6 = 12

Product: 12

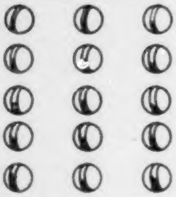
Factors: 2 and 6



Equation: 4 X 2 = 8

Product: 8

Factors: 4 and 2



Equation: 5 X 3 = 15

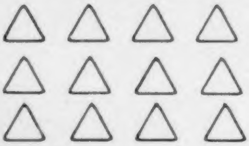
Product: 15

Factors: 5 and 3

Equation: 6 X 3 = 18

Product: 18

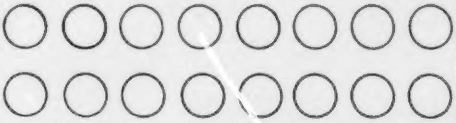
Factors: 6 and 3



Equation: 3 X 4 = 12

Product: 12

Factors: 3 and 4



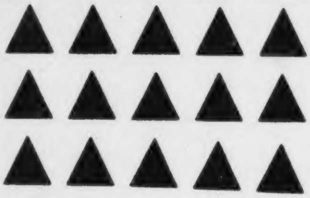
Equation: 2 X 8 = 16

Product: 16

Factors: 2 and 8

## Products and Factors

Fill the blanks.



Equation: 3 X 5 = 15

Product: 15

Factors: 3 and 5

0 0 0 0
0 0 0 0
0 0 0 0
0 0 0 0

Equation: 4 X 4 = 16

Product: 16

Factors: 4 and 4

0000000000
0000000000
0000000000
0000000000
0000000000

Equation: 5 X 9 = 45

Product: 45

Factors: 5 and 9

00000
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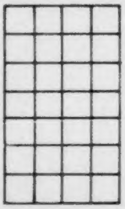
Equation: 6 X 5 = 30

Product: 30

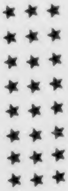
Factors: 6 and 5



Equation: 7 X 4 = 28

Product: 28

Factors: 7 and 4



Equation: 8 X 3 = 24

Product: 24

Factors: 8 and 3

## VIII-3. Some simple properties of multiplication

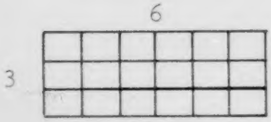
Objective: To introduce the commutative property of multiplication, and to discuss products in which a factor is 1, 2, or 0.

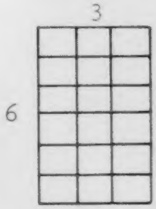
Vocabulary: Commutative property (at the teacher's discretion).

Materials: Objects for flannel board, manipulative objects for children, form for multiplication table with numerals at top only.

### The commutative property of multiplication

Show a 3 by 6 array on heavy paper. Have a child give the equation that shows the product of the numbers 3 and 6. ($3 \times 6 = 18$.) Write the equation on the chalkboard. Under it write $6 \times 3 =$ ____. Then turn the paper so as to display a $6 \times 3$ array to help children see how they can complete this equation.





Ask children whether the product of 6 and 3 is the same as the product of 3 and 6. Write $6 \times 3 = 18$.

Have children arrange arrays on the flannel board to show whether or not

$$2 \times 3 = 3 \times 2$$
$$4 \times 6 = 6 \times 4$$
$$7 \times 3 = 3 \times 7$$
$$2 \times 5 = 5 \times 2$$

Help children to generalize: Changes in the order in which two factors are used will not affect the product.

Pupil's book, page 229:

Children are to draw arrays which illustrate the commutative property.

★ Pupil's book, page 230:

(Optional) This page, for more able pupils, shows the relation between multiplication and addition, and emphasizes the idea of the commutative property.

## The Product of Two Factors

Draw a 6 by 3 array. Use X's.

X X X
X X X
X X X
X X X
X X X
X X X

6 × 3 = 18

Draw a 3 by 6 array. Use X's.

X X X X X X
X X X X X X
X X X X X X

3 × 6 = 18

Is 6 × 3 = 3 × 6? Yes

Draw a 4 by 5 array. Use X's.

X X X X X
X X X X X
X X X X X
X X X X X

4 × 5 = 20

Draw a 5 by 4 array. Use X's.

X X X X
X X X X
X X X X
X X X X
X X X X

5 × 4 = 20

Is 4 × 5 = 5 × 4? Yes

## ★ Order in Factors

Fill the blanks.

3 × 5 = 5 + 5 + 5
3 × 5 = 15

5 × 3 = 3 + 3 + 3 + 3 + 3
5 × 3 = 15

3 × 5 = 5 × 3

4 × 2 = 2 + 2 + 2 + 2
4 × 2 = 8

2 × 4 = 4 + 4
2 × 4 = 8

4 × 2 = 2 × 4

2 × 5 = 5 + 5
2 × 5 = 10

5 × 2 = 2 + 2 + 2 + 2 + 2
5 × 2 = 10

2 × 5 = 5 × 2

2 × 9 = 18
9 × 2 = 18

8 × 3 = 24
3 × 8 = 24

7 × 4 = 28
4 × 7 = 28

2 × 8 = 16
8 × 2 = 16

2 × 6 = 12
6 × 2 = 12

2 × 3 = 6
3 × 2 = 6

- One as a factor

Arrange manipulative objects on a table or desk to show arrays of 4 rows of 1 member, 1 row of 4 members; 8 rows of 1 member, 1 row of 8 members, etc. Prepare similar flannel board arrays.

Have children describe these arrays as usual and tell how many objects are in each set. Since counting by rows in the case of $n \times 1$ examples will amount to counting by ones, and counting by rows in $1 \times n$ examples will be simply saying the number once, children should readily perceive that any number times one is that number itself, and one times any number equals that number. However, it is worthwhile to verbalize this, and to emphasize by writing equations for the arrays shown:

$4 \times 1 = 4$, $1 \times 4 = 4$; $8 \times 1 = 8$, $1 \times 8 = 8$, etc.

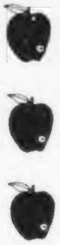
Ask a child to think of a number in the hundreds. Ask him what that number times one is. Ask what one times that number is. Use other large numbers as one of the numbers and use one as the other number. Ask for the product of each pair.

Pupil's book, pages 231 - 232:

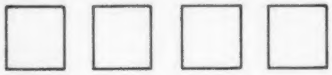
Use these pages for practice.

## One As a Factor

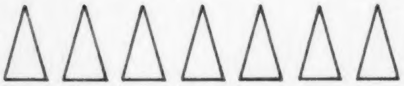
Fill the blanks.



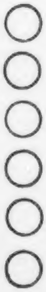
3 rows
1 member in each row
Equation: 3 X 1 = 3
Product: 3
Factors: 3 and 1



1 row
4 members in each row
Equation: 1 X 4 = 4
Product: 4
Factors: 1 and 4



1 row
7 members in each row
Equation: 1 X 7 = 7
Product: 7
Factors: 1 and 7



6 rows
1 member in each row
Equation: 6 X 1 = 6
Product: 6
Factors: 6 and 1

## One As a Factor

Fill the blanks.

---

X
X
X
X

Equation: 4 X 1 = 4
Product: 4 Factors: 4 and 1

---

X X X X

Equation: 1 X 4 = 4
Product: 4 Factors: 1 and 4

---

9 × 1 = 9

4 × 1 = 4

1 × 6 = 6

1 × 32 = 32

589 × 1 = 589

27 × 1 = 27

1 × 5 = 5

6 × 1 = 6

7 × 1 = 7

1 × 59 = 59

100 × 1 = 100

1 × 455 = 455

1 × 1 = 1

1 × 1 = 1

- Display a 2 by 4 array on the flannel board. Ask for the equation that goes with it. Leaving space above and below the equation, write:

$$2 \times 4 = 8$$

Remind children that they could have added the number of members in one row to the number of members in the other to find the number of members in both rows. Sometimes, in talking of the number of members in an array, use expressions like "Two fours are eight".

Show a 2 by 5 array. Ask for the equation and write it just below the first equation. Show a 2 by 3 array and write the equation above $2 \times 4 = 8$. Continue with $2 \times 6$, $2 \times 2$, $2 \times 7$, $2 \times 1$, $2 \times 8$, $2 \times 9$, and $2 \times 10$. Call attention to the pattern of the products; 2, 4, 6, . . . 20. Ask what kind of numbers these are. (Even numbers.) Help children to generalize: when 2 is one of the factors, the product will always be an even number.

Suggest making a table to record the products of 2 and the numbers to ten. $(2 \times x) =$ ________

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |

- Zero as a factor

Call attention to the fact that in the table on the chalkboard, each product is 2 less than the one that follows it. Ask what number is 2 less than 2 ? (0.) Enter 0 in the table. Have the children recall that $2 \times 4 = 4 + 4$. Ask what addition expression could be written to complete the equation:

$2 \times 0 =$ ____ $+$ ____. (0 + 0.)

Ask children how many members there would be in an array with 0 rows. (Zero; none at all.)

> How would we describe an array with 0 rows of 3 members each? (A 0 by 3 array.)
>
> How many members would there be in a 0 by 3 array? (0.)
> A 0 by 4 array? (0.)
> A 0 by 27 array? (0.)
> A 0 by 598 array? (0.)
> If there are 0 rows in an array, will it matter how many members each row has? (No.)

Help children to generalize: zero times any number is zero. Next, ask children how many members there would be in an array with 0 members in each row. (Zero; none.)

> How would we describe an array with 3 rows of 0 members each? (A 3 by 0 array.)
> How many members would there be in a 3 by 0 array? (0.)
> A 5 by 0 array? (0.)
> A 769 by 0 array? (0.)
> If each row in an array has 0 members, will it matter how many rows there are? (No.)

Help children to generalize: any number times zero is zero.

Pupil's book, page 233:

This page provides practice on multiplication ideas introduced up to this time.

## Multiplication

Fill the blanks.

2 × 3 = 6
2 × 8 = 16
2 × 6 = 12
5 × 1 = 5
3 × 2 = 6
7 × 1 = 7
4 × 0 = 0
0 × 9 = 0

7 × 2 = 14
8 × 0 = 0
1 × 9 = 9
2 × 2 = 4
2 × 4 = 8
0 × 8 = 0
2 × 7 = 14
1 × 8 = 8

4 × 2 = 8
2 × 5 = 10
0 × 7 = 0
9 × 1 = 9
2 × 9 = 18
1 × 5 = 5
6 × 0 = 0
2 × 1 = 2

---

0 × 6 = 0
4 × 1 = 4
1 × 39 = 39
9 × 2 = 18
6 × 2 = 12

5 × 2 = 10
92 × 0 = 0
74 × 0 = 0
2 × 0 = 0
1 × 3 = 3

1 × 7 = 7
468 × 1 = 468
1 × 92 = 92
3 × 1 = 3
8 × 2 = 16

## VIII-4. The distributive property and multiplication facts

Objective: To introduce the distributive property and use it in constructing a table of multiplication facts.

Vocabulary: Distributive property (at the teacher's discretion).

Materials: Flannel board objects, yarn, manipulative materials for children, form for multiplication table.

### Suggested Procedure:

Have children use objects to make a 2 by 5 array. Write:

$$2 \times 5 = 10.$$

Have children leave the 2 by 5 array on their desks, and a little way below it show a 1 by 5 array. Chalkboard should show:

$$2 \times 5 = 10$$
$$1 \times 5 = 5$$

Ask how many members there are in both arrays. (15.) Have children move the 1 by 5 array close to the 2 by 5 array to make a 3 by 5 array. Chalkboard should show:

$$2 \times 5 = 10$$
$$1 \times 5 = 5$$
$$3 \times 5 = 15$$

Repeat this procedure with

$$2 \times 4 = 8$$
$$2 \times 4 = 8$$
$$4 \times 4 = 16$$

and

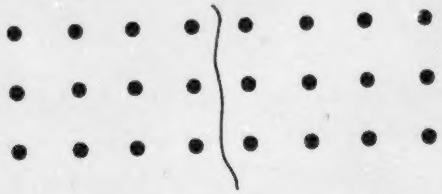
$$3 \times 5 = 15$$
$$3 \times 2 = 6$$
$$3 \times 7 = 21$$

Give many such examples. Children should begin to see how they can use what they already know to find products they do not know.

Show a 3 by 8 array on the flannel board. Write:

$$3 \times 8 = \underline{\quad}.$$

Discuss the idea of counting by eights, but suggest that children could find the number of members in the array without counting by eights. Use a piece of yarn to separate the array into two 3 by 4 arrays.



Ask children the number of members in one of the 3 by 4 arrays. Write:

$$3 \times 4 = 12$$
$$3 \times 4 = 12$$
$$3 \times 8 = \underline{\quad}$$

Children should see that the objects of the 3 by 8 array are the same objects as those in the two 3 by 4 arrays, and therefore $3 \times 8 = 24$.

Have children make arrays to show that a 4 by 9 array has the same number of members as two arrays, one 4 by 5 and the other 4 by 4, etc. Children will find it difficult to verbalize or symbolize and should not be expected to do so, but they should begin to understand that $(2 \times 5) + (1 \times 5) = 3 \times 5$, and that $(4 \times 3) + (4 \times 2) = 4 \times 5$.

By using many examples and constructing arrays, children should become aware of the ways in which they can use the multiplication facts they have learned to find the ones they do not know.

Show the chart for the multiplication table, and recall with the class the way in which they used the addition table. Explain that the first number will be shown on the side of the multiplication table and the second number across the top. Suggest that children give some of the facts they know.

> What products will be written in the zero row? (All 0's.)
> What products will be written in the zero column? (All 0's.)
> What other products can we write?

By this time, some children may be able to give all products with 1 as either factor, and most of the products in the 2, 3 and 5 rows. Remind them of the commutative property of multiplication, if necessary, so that they will also give 8 × 2, 9 × 3, etc. However, no learning of multiplication facts is expected in grade two.

Pupil's book, pages 234 - 235:

These pages should be used as a group activity.

Pupil's book, page 236:

Children may complete the page independently.

★ Pupil's book, pages 237 - 239:

(Optional) Allow more able pupils to complete as much of the work on these pages as they wish. Let them make or draw arrays, but do not expect mastery of facts in this grade. On page 239, read directions with children.

## Using Arrays

This is a 2 by 6 array.

A A A A A A

A A A A A A

2 × 6 = 12

This is a 1 by 6 array.

Q Q Q Q Q Q

1 × 6 = 6

12 + 6 = 18

This is a 3 by 6 array.

D D D D D D

D D D D D D

D D D D D D

3 × 6 = 18

---

This is a 4 by 9 array.

B B B B B B B B B

B B B B B B B B B

B B B B B B B B B

B B B B B B B B B

4 × 9 = 36

Show how it can be separated into a 3 by 9 array and a 1 by 9 array.

A 3 by 9 array has 27 members.

A 1 by 9 array has 9 members.

27 + 9 = 36

This is a 4 by 5 array.

W W W | W W
W W W | W W
W W W | W W
W W W | W W

4 × 5 = 20

Show how it can be separated into a 4 by 3 array and a 4 by 2 array.

4 × 3 = 12

4 × 2 = 8

12 + 8 = 20

---

This is a 3 by 7 array.

G G G G G | G G
G G G G G | G G
G G G G G | G G

3 × 7 = 21

Show how it can be separated into a 3 by 5 array and a 3 by 2 array.

3 × 5 = 15

3 × 2 = 6

15 + 6 = 21

Draw arrays to find the products.

Use X's.

| | |
|---|---|
| 2 × 6 = 12<br>XXXXXX<br>XXXXXX | 2 × 9 = 18<br>XXXXXXXXX<br>XXXXXXXXX |
| 1 × 6 = 6<br>XXXXXX | 1 × 9 = 9<br>XXXXXXXXX |
| 3 × 6 = 18<br>XXXXXX<br>XXXXXX<br>XXXXXX | 3 × 9 = 27<br>XXXXXXXXX<br>XXXXXXXXX<br>XXXXXXXXX |
| 2 × 7 = 14<br>XXXXXXX<br>XXXXXXX | 2 × 4 = 8<br>XXXX<br>XXXX |
| 1 × 7 = 7<br>XXXXXXX | 1 × 4 = 4<br>XXXX |
| 3 × 7 = 21<br>XXXXXXX<br>XXXXXXX<br>XXXXXXX | 3 × 4 = 12<br>XXXX<br>XXXX<br>XXXX |

★ **Multiplication. Fill the blanks.**

| | | |
|---|---|---|
| 2 × 8 = 16 | 2 × 4 = 8 | 1 × 6 = 6 |
| 1 × 8 = 8 | 1 × 4 = 4 | 2 × 6 = 12 |
| 3 × 8 = 24 | 3 × 4 = 12 | 3 × 6 = 18 |
| 1 × 5 = 5 | 2 × 7 = 14 | 2 × 9 = 18 |
| 2 × 5 = 10 | 1 × 7 = 7 | 1 × 9 = 9 |
| 3 × 5 = 15 | 3 × 7 = 21 | 3 × 9 = 27 |
| 2 × 3 = 6 | 2 × 1 = 2 | 2 × 2 = 4 |
| 1 × 3 = 3 | 1 × 1 = 1 | 1 × 2 = 2 |
| 3 × 3 = 9 | 3 × 1 = 3 | 3 × 2 = 6 |

**Fill the boxes:**

| × | 5 | 9 | 7 | 4 | 10 | 8 | 3 | 6 | 2 |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 15 | 27 | 21 | 12 | 30 | 24 | 9 | 18 | 6 |

| × | 5 | 9 | 7 | 4 | 10 | 8 | 3 | 6 | 2 |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 10 | 18 | 14 | 8 | 20 | 16 | 6 | 12 | 4 |

| × | 2 | 1 | 3 |
|---|---|---|---|
| 8 | 16 | 8 | 24 |

| × | 3 | 1 | 2 |
|---|---|---|---|
| 9 | 27 | 9 | 18 |

★ Show on the table:

products with 2 as a factor.

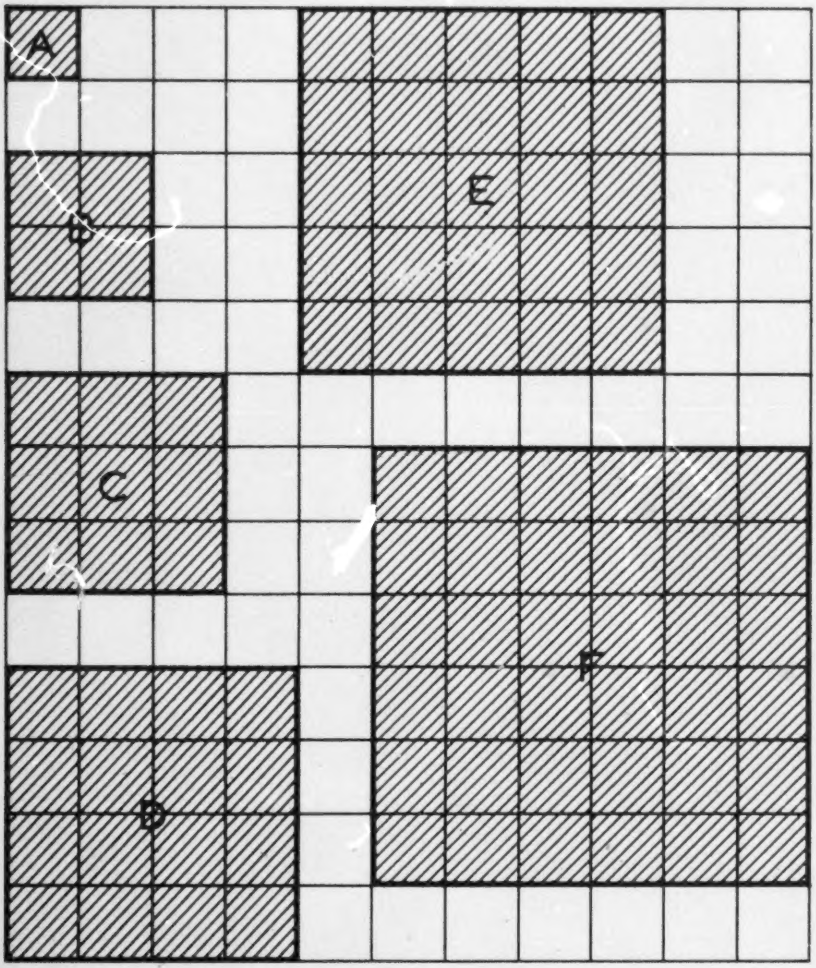
products with 0 as a factor.

products with 5 as a factor.

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 |  | 2 |  |  | 5 |  |  |  |  |
| 2 | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 3 | 0 |  | 6 |  |  | 15 |  |  |  |  |
| 4 | 0 |  | 8 |  |  | 20 |  |  |  |  |
| 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| 6 | 0 |  | 12 |  |  | 30 |  |  |  |  |
| 7 | 0 |  | 14 |  |  | 35 |  |  |  |  |
| 8 | 0 |  | 16 |  |  | 40 |  |  |  |  |
| 9 | 0 |  | 18 |  |  | 45 |  |  |  |  |

Show any other products you know. *Others will vary.*

★ A Special Kind of Array



Write equations suggested by the above arrays:

A. 1 × 1 = 1

D. 4 × 4 = 16

B. 2 × 2 = 4

E. 5 × 5 = 25

C. 3 × 3 = 9

F. 6 × 6 = 36

In each equation, the same factor is used 2 times.

Each array is a special kind of rectangle called a square.

- Using multiplication to solve problems

Although multiplication has been introduced with arrays, it is important that children realize that multiplication is also related to joining equivalent sets.

Call 5 children to the front of the room. Ask another child to give 6 small objects (buttons, beans, crayons, etc.) to each of the 5.

> If we want to find out how many buttons were given out, in all, we can join these sets. What equation would we write for this? (6 + 6 + 6 + 6 + 6 = 30.)
>
> But Jimmy counted 6 things 5 times.
>
> How many sixes did we write in the equation? (5.)
>
> When we think of 5 times 6 or 5 sixes, we are really thinking in words we use when we multiply. We can write:
>
> $$5 \times 6 = 30.$$

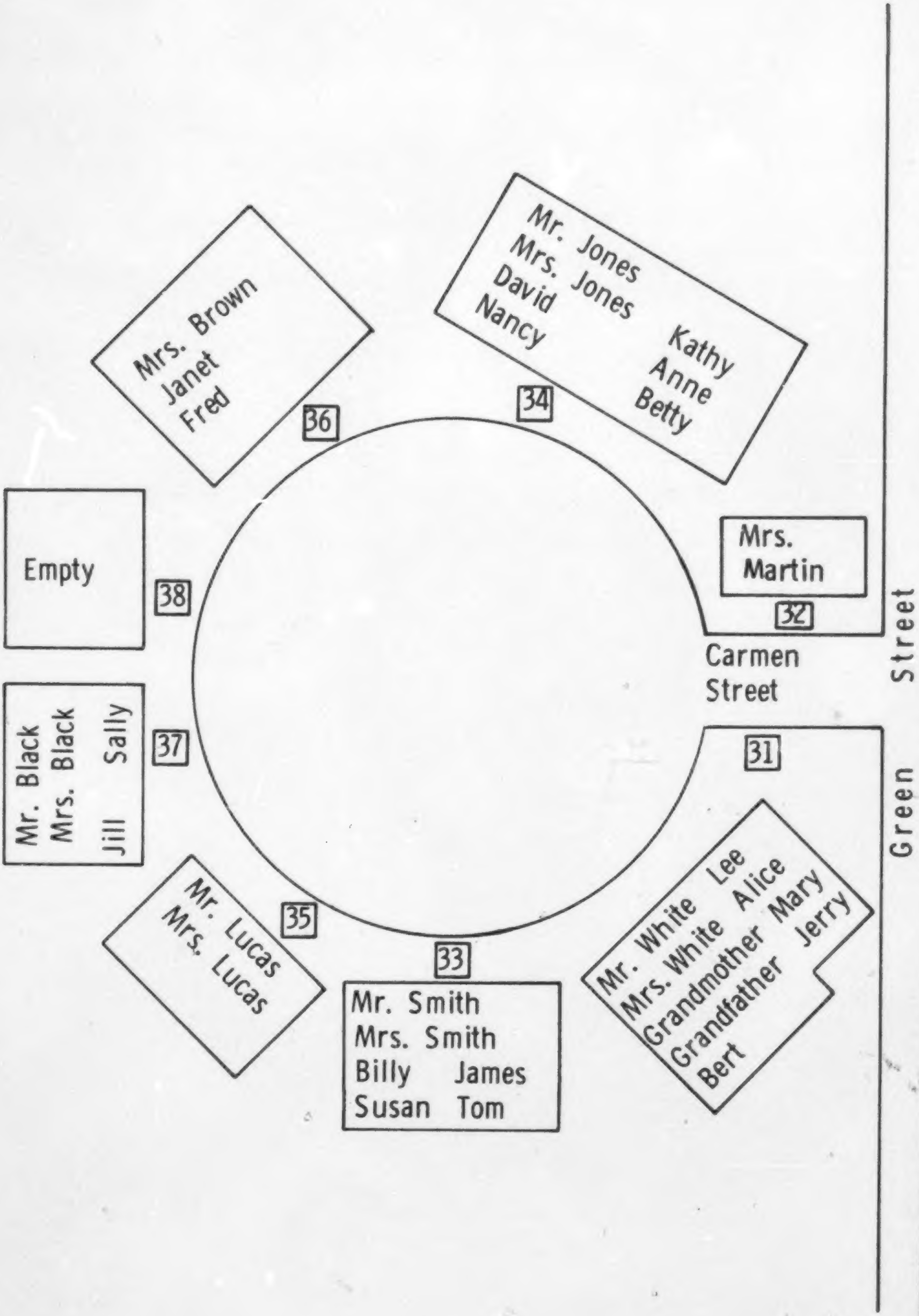
Give many examples in which multiplication may be used. Some of the following problems may be helpful:

> A room has 6 windows with 2 curtains at each window.
> How many curtains are in the room? 12
>
> A bag of candy has 7 pieces of each of 3 different kinds of candy. How many pieces of candy are in the bag? 21
>
> A man wants to buy tires for 3 trucks. Each truck needs 10 tires. How many tires must he buy? 30
>
> If you use 8 nails for each shelf of a bookcase, and it has 5 shelves, how many nails will you use? 40
>
> Milk costs 5 cents a carton. If you buy a carton of milk every school day for a week, how much will you spend for milk? 25¢

Help children to generalize: to find the total number of members in a given number of equivalent sets, you can multiply the number of members in a set by the number of sets.

Pupil's book, pages 240 - 241:

Have children look at the drawing of Carmen Street. Read the story about Mrs. Martin. Show that children must first see how many people live at number 31 Carmen Street, then multiply to find out how many cookies to send there, etc. The top of the page may be used as a class activity. After the story of Mrs. Jackson has been read aloud, children should complete the page independently.


Mr. Jones
Mrs. Jones
David
Nancy
Kathy
Anne
Betty
34
Mrs. Brown
Janet
Fred
36
Mrs. Martin
32
Empty
38
Carmen Street
Green Street
31
Mr. Black
Mrs. Black
Jill
Sally
37
Mr. White
Mrs. White
Grandmother
Grandfather
Bert
Lee
Alice
Mary
Jerry
Mr. Lucas
Mrs. Lucas
35
33
Mr. Smith
Mrs. Smith
Billy
Susan
James
Tom

## Christmas on Carmen Street

Mrs. Martin baked Christmas cookies. She wanted to send 5 cookies to every person on Carmen Street. How many cookies did she send to each house?

45 cookies to 31
10 cookies to 35
15 cookies to 36
0 cookies to 38

30 cookies to 33
20 cookies to 37
35 cookies to 34

How many cookies did she send in all? 155

Mrs. Jackson used to live at 38 Carmen St. She mailed boxes of candy to all the people who were her neighbors. She sent 3 pieces of candy for each person. How many pieces did she send to each house?

27 pieces to 31
6 pieces to 35
3 pieces to 32
9 pieces to 36

18 pieces to 33
12 pieces to 37
21 pieces to 34
0 pieces to 38

How many pieces did she send in all? 96

# CHAPTER IX

# DIVISION AND RATIONAL NUMBERS

## Background

Section IX-1 begins with problems on partitions. When we partition a set of 20 members into subsets of 5 members each, we may ask how many such subsets there will be. To answer this question using arrays, we arrange a set of 20 in rows of 5 and then count the number n of rows. This amounts to finding the factor n in the equation

$$n \times 5 = 20.$$

A different partition problem is the following. When we partition a set of 20 members into 5 equivalent subsets, we may ask how many members each of these subsets will have. To solve this problem using arrays, we distribute a set of 20 into 5 rows and then count the number, n, of members in a row. This amounts to finding the factor n in the equation.

$$5 \times n = 20.$$

Since $5 \times n = n \times 5$, this leads to the same number n as before.

Partitioning a set of 20 into sets of 5 corresponds to dividing the number 20 by the number 5. We have seen that when we partition a set of 20 into sets of 5, the number n of such sets satisfies the equation

$$n \times 5 = 20.$$

We write

$$n = \frac{20}{5}$$

or

$$n = 20 \div 5,$$

interchangeably, and say "n equals 20 divided by 5" or "n is the quotient, 20 divided by 5."

Dividing the number 20 by the number 5 also corresponds to partitioning a set of 20 into 5 equivalent subsets, since as we have seen above this results in the same number n as before.

There is a "doing and undoing" aspect of division and multiplication. Starting with 20, dividing by 5, and then multiplying the result $\frac{20}{5}$ by 5, we get back the 20 we started with. The quotient $\frac{20}{5}$ is the number n for which

$$n \times 5 = 20.$$

In finding the number of sets of 5 in a set of 20, we need not actually form arrays. Starting with the set of 20, we may successively remove sets of 5, keeping count of the number of such sets removed, until the remaining set is empty. In this way we have of course counted the number of 5's in 20. That is, we have found the quotient $\frac{20}{5}$. This could be called the "successive subtraction" aspect of division, corresponding to the "successive addition" aspect of multiplication.

Finally, we need to note that corresponding to the multiplication facts

$$1 \times n = n \quad \text{and} \quad n \times 1 = n$$

we have the division facts

$$\frac{n}{1} = n \quad \text{and} \quad \frac{n}{n} = 1.$$

Section IX-4 concerns fractions and rational numbers. We make a distinction between these. A fraction is a certain kind of written symbol; a rational number is a certain kind of number.

When we divide a number m by a number n (other than zero), the result is a number which we call their quotient. We will denote this quotient by the symbol "$\frac{m}{n}$", though the symbol "m ÷ n " is also sometimes used. Thus we have the equation

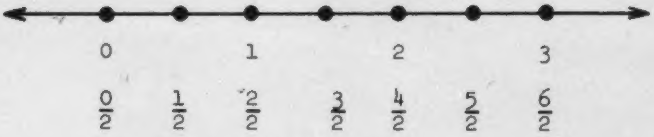
$$\frac{m}{n} = m \div n,$$

which says that the numbers $\frac{m}{n}$ and m ÷ n are the same. A specific example is $\frac{3}{2} = 3 \div 2$.

The symbol "$\frac{m}{n}$" is called a fraction. A fraction is a symbol which results from writing a numeral, drawing a bar under it, and writing a numeral beneath the bar. In a fraction the number named above the bar is called the numerator, while the number named below the bar is called the denominator. (In the symbol "m ÷ n", and sometimes in the symbol "$\frac{m}{n}$", the number m is called the dividend and n is called the divisor.)

When the numerator and denominator of a fraction are both whole numbers, then the number which this fraction designates, or names, is called rational -- because it has to do with ratios, not reason. (A number which cannot be designated by a fraction with a whole number numerator and denominator is called irrational. The numbers $\sqrt{2}$ and $\pi$ are examples of irrational numbers, it can be shown.) All whole numbers are rational numbers. The number 5, for instance, is rational because it can be designated by the fraction "$\frac{5}{1}$" (or "$\frac{10}{2}$", or "$\frac{15}{3}$", etc.) with whole number numerator and denominator.

Rational numbers can be represented by points on the number line. In Section IX-5 this is done for some of the simpler rational numbers, beginning with whole numbers like $\frac{2}{2} = 1$, $\frac{4}{2} = 2$, $\frac{6}{2} = 3$, etc. Then the point half way between $\frac{0}{2}$ and $\frac{2}{2}$ is used to represent $\frac{1}{2}$, the point half way between $\frac{2}{2}$ and $\frac{4}{2}$ is used to represent $\frac{3}{2}$ etc.



Immediately, without any attempt at a systematic treatment, a few simple sums (like $\frac{1}{2} + \frac{1}{2}$, $\frac{2}{2} + \frac{3}{2}$, etc.) are computed, using the number line visualization.

Next, pupils are led to recognize that the point half way between 0 and $\frac{1}{2}$ on the number line is the same as the point one fourth of the way from 0 to 1, and hence that the number represented by this point should be $\frac{1}{4}$. After the "fourths" have been systematically filled in on the number line, the "thirds" (and in the exercises, the "sixths") are considered. In each case a few simple sums are computed.

## IX-1. Finding factors

**Objective:** To use arrays to find unknown factors in multiplication equations.

**Vocabulary:** (No new words.)

**Materials:** Manipulative objects for children, materials for flannel board.

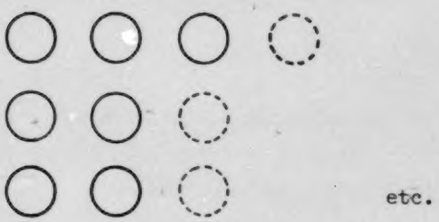
**Suggested Procedure:**

Discuss with the children the problem of finding out how many basketball teams can be formed from a group of 20 boys. Let children select 20 objects and show how they would partition the set of 20 into sets of 5 members each until they had 4 sets of 5. If no child has formed an array, show that the 5 members in each set may be put in a row. Another 5 members would be put in the next row, etc., until all 20 members had been used.

> What equation can we write about this array? $(4 \times 5 = 20)$
>
> But when we began, we knew only that we had 5 members on each team and 20 members in all. If we had used "n" in the equation for the number we didn't know, we would have written:
>
> $$n \times 5 = 20.$$
>
> Perhaps you would have thought of the number 4 at once, if you had seen the equation.

Have children select 18 objects to represent players for baseball. Since there are 9 players on a team, the number of teams may be found by forming an array with 9 members in each row. Discuss the equation to be written, and the way the array would be formed. Recall that they will partition a set of 18 members into equivalent sets of 9.

Explain that sometimes the number of rows in the array and the number of members in the array are known, but the number of members in each row must be found. If, for instance, 3 children want to share 18 cookies fairly, an array might be formed as shown:

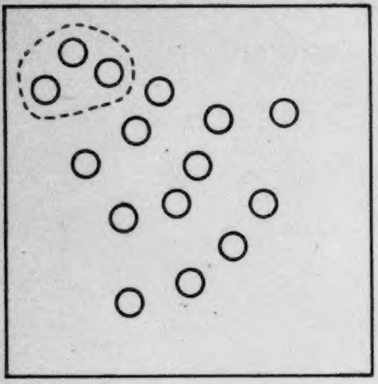


They would place an object in each row in turn until all 18 objects were distributed. The equation $3 \times n = 18$ would be written.

Pupil's book, pages 242 - 244: Children draw arrays to find the number that completes the equation. You may wish to provide additional practice by developing and using worksheets similar to these pages.

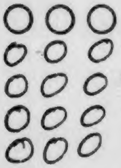
## Partitioning Sets

Find the number of rows.



This set has 15 members.

Draw an array.



There are __5__ threes in 15.

__5__ × 3 = 15

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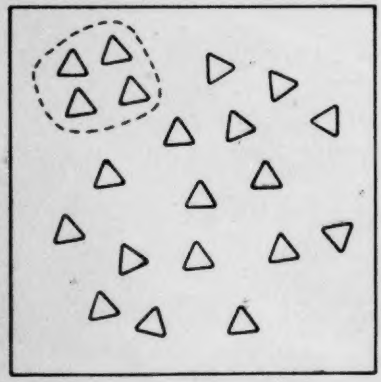
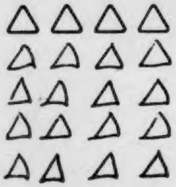


This set has 20 members.

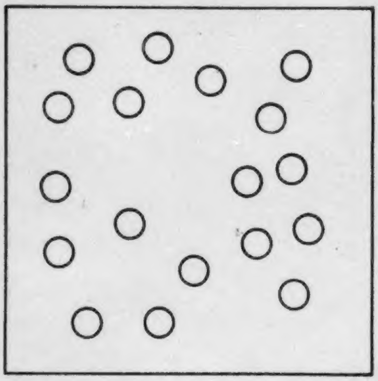
Draw an array.



There are __5__ fours in 20.

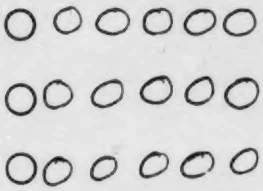
__5__ × 4 = 20

Find the number of members in each row.



This set has 18 members.

Draw an array.



3 × 6 = 18

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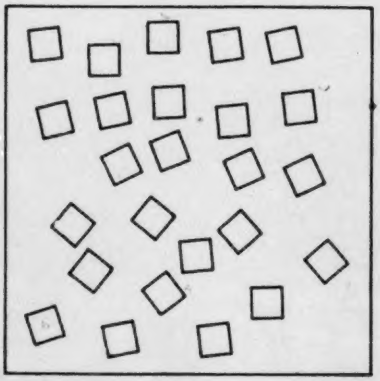
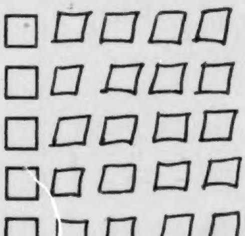


This set has 25 members.

Draw an array.

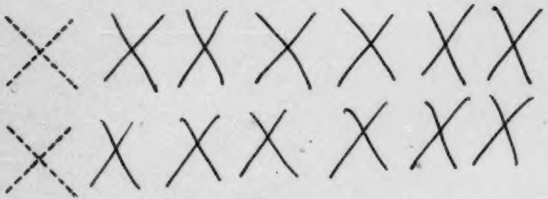


5 × 5 = 25

Use X's to draw arrays.

An array has 14 members.

It has 2 rows.



Equation: 2 × 7 = 14

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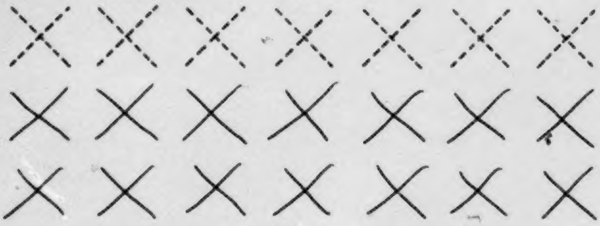
An array has 21 members.

It has 7 members in each row.

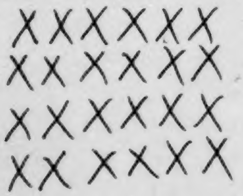


Equation: 3 × 7 = 21

---

An array has 24 members.

It has 6 members in each row.



Equation: 4 X 6 = 24

## IX-2. Division, using arrays

**Objective:** To introduce the idea of division, using arrays.

**Vocabulary:** Divide, division, quotient.

**Materials:** (No special materials.)

**Suggested Procedure:**

Recall with the class the idea of doing and undoing in addition and subtraction. To find "n" in $n + 16 = 32$, we subtract 16 from 32. That is, $n = 32 - 16$.

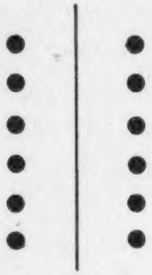
Write $n \times 2 = 12$.

Remind children that they have found what number n is by partitioning a set of 12 into subsets of 2.

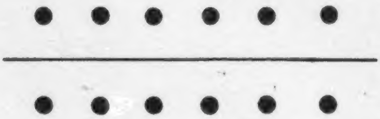
> We speak of partitioning a set into equivalent subsets.
>
> We say that we divide a number by another number.
>
> We write "Twelve divided by two" like this:
>
> $$\frac{12}{2}$$
>
> Now we can write:
>
> If $n$ is a number and $n \times 2 = 12$ then $n = \frac{12}{2}$.
>
> If we divide one number by another, the result is called the quotient.
>
> Dividing by 2 undoes multiplying by 2.

Write divide, division, and quotient, and discuss common uses of the words divide and division.

Draw a 6 by 2 array on the chalkboard. Separate it with a bar, as shown:



Draw a 2 by 6 array, and separate it, with a bar:



Explain that the bar used between the top numeral and the bottom numeral of the division expression shows that one number is divided by the other; that the top numeral names the number of the set to be partitioned and the bottom numeral shows either the number of members in each subset or the number of subsets into which the set is partitioned.

Other examples may be given. To show partitioning a given set into equivalent subsets when the number of members in each subset is known, you may use the following problem situations:

1. A boy has more marbles than he wants. He decides to sell 35 of them in bags of 5 marbles each. How many bags will he have to sell? $\frac{35}{5} = 7$

2. Some children are giving a play circus in the garage. They have 15 chairs. If they put 5 chairs in each row, how many rows of chairs can they have? $\frac{15}{5} = 3$

3. Mother wants to sew 12 buttons on a dress. There are 4 buttons on each card. How many cards of buttons will she need? $\frac{12}{4} = 3$

To show partitioning a given set into equivalent subsets when the number of subsets to be formed is known, the following may be used:

1. A boy decided to give away 20 duplicate baseball cards. He wants to give them to four of his friends. If he gives the same number of cards to each boy, how many cards will each boy get? $\frac{20}{4} = 5$

2. The 24 children in Miss White's room are going to have relay races. How many children will be on each team if they have 3 teams? $\frac{24}{3} = 8$

3. Mother put all the pennies she found into a little box. When she counted them, she found that there were 40 pennies. If she gave the same number of pennies to each of her 4 children, how many pennies did each child receive? $\frac{40}{4} = 10$

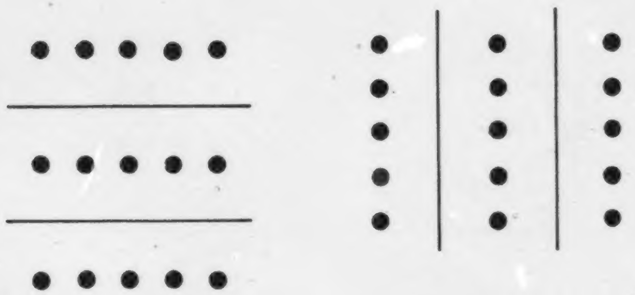
4. Miss Brown wants to have 5 rows of desks in her room. If she has 30 desks, how many will there be in each row? $\frac{30}{5} = 6$

Write several more division expressions:

$\frac{16}{8}$ $\frac{15}{3}$ $\frac{14}{7}$ $\frac{25}{5}$, etc.

Have children read them aloud and draw arrays to help them find the quotient. Call attention to the fact that the quotient times the number by which they divide gives the number named by the top numeral.

Have children observe that either of the following arrays might be made to show $\frac{15}{3}$, and that bars may be drawn to show the set partitioned into 3 equivalent subsets in either case.



Pupil's book, pages 245 - 246: Children draw arrays to help them find the quotient.

## The Quotient of Two Numbers

Draw an array. Use X's.

Fill the blanks.

2 × 6 = 12

$\frac{12}{6}$ = 2

x x x x x x
x x x x x x

---

2 × 4 = 8

$\frac{8}{2}$ = 4

x x x x
x x x x

---

2 × 9 = 18

$\frac{18}{9}$ = 2

X X X X X X X X X
X X X X X X X X X

---

3 × 7 = 21

$\frac{21}{7}$ = 3

X X X X X X X
X X X X X X X
X X X X X X X

---

5 × 3 = 15

$\frac{15}{5}$ = 3

X X X
X X X
X X X
X X X
X X X

## The Quotient of Two Numbers

Draw arrays. Use X's.

Fill the blanks.

$3 \times \underline{\ 3\ } = 9$

$\frac{9}{3} = \underline{\ 3\ }$

X X X
X X X
X X X

---

$5 \times \underline{\ 6\ } = 30$

$\frac{30}{5} = \underline{\ 6\ }$

X X X X X X
X X X X X X
X X X X X X
X X X X X X
X X X X X X

---

$\underline{\ 5\ } \times 4 = 20$

$\frac{20}{4} = \underline{\ 5\ }$

X X X X
X X X X
X X X X
X X X X
X X X X

---

$\underline{\ 7\ } \times 2 = 14$

$\frac{14}{2} = \underline{\ 7\ }$

X X
X X
X X
X X
X X
X X
X X

---

$8 \times \underline{\ 3\ } = 24$

$\frac{24}{8} = \underline{\ 3\ }$

X X X
X X X
X X X
X X X
X X X
X X X
X X X
X X X

## IX-3. Division, using subtraction

**Objectives:** To show the use of subtraction to find quotients. To learn about the division properties of 1.

**Vocabulary:** (No new terms.)

**Materials:** Materials for flannel board.

**Suggested Procedure:**

**Using Subtraction**

Give a child 12 felt cut-outs and ask him to show, on the flannel board, how to find $\frac{12}{2}$.

Have the other children observe, as he does so, that his starting set has 12 members, that he may remove a set of 2 objects and place them on the flannel board, then remove another set of 2 objects, etc. Ask what subtraction equation can be written to show what happened when the first set of 2 objects was removed from the starting set. (12 - 2 = 10.) Show the following on the chalkboard:

$$\begin{array}{r} 12 \\ -\ 2 \\ \hline 10 \end{array}$$

We have subtracted 2 from 12 one time.

$$\begin{array}{r} 12 \\ -2 \\ \hline 10 \\ -\ 2 \\ \hline 8 \end{array}$$

We have subtracted 2 two times.

Continue to subtract 2 from each successive difference, and say each time, "We have subtracted 2 _____ times" until zero is the difference.

How many 2's did we subtract? (6.)

How many 2's are there in 12? (6.)

What is $\frac{12}{2}$ ? (6.)

Write: $\frac{18}{6} =$

Ask a child to read aloud what you have written. Discuss ways to find the quotient. (Make an array. Subtract 6 from 18 as many times as possible, till you get to zero.) Have a child show the repeated subtraction. Suggest that a basic multiplication fact would tell how many times 6 can be subtracted from 18. ($3 \times 6 = 18$.)

Give other examples and have children find the quotient by repeated subtraction.

Pupil's book, page 247: Gives practice in finding the quotient by using subtraction.

* Pupil's book, page 248: (Optional) More able pupils may enjoy adding and subtracting to solve multiplication and division problems involving larger numbers.

## Using Subtraction

How many times do you subtract?

$\frac{27}{9}$ = 3

$$\begin{array}{r} 27 \\ -\ 9 \\ \hline 18 \\ -\ 9 \\ \hdashline 9 \\ -\ 9 \\ \hdashline 0 \end{array}$$

$\frac{32}{8}$ = 4

$$\begin{array}{r} 32 \\ -\ 8 \\ \hline 24 \\ -8 \\ \hline 16 \\ -8 \\ \hline 8 \\ -8 \\ \hline 0 \end{array}$$

$\frac{15}{5}$ = 3

$$\begin{array}{r} 15 \\ -\ 5 \\ \hline 10 \\ -5 \\ \hline 5 \\ -5 \\ \hline 0 \end{array}$$

$\frac{14}{7}$ = 2

$$\begin{array}{r} 14 \\ -\ 7 \\ \hline 7 \\ -7 \\ \hline 0 \end{array}$$

$\frac{30}{5}$ = 6

$$\begin{array}{r} 30 \\ -\ 5 \\ \hline 25 \\ -5 \\ \hline 20 \\ -\ 5 \\ \hline 15 \\ -5 \\ \hline 10 \\ -5 \\ \hline 5 \\ -5 \\ \hline 0 \end{array}$$

$\frac{28}{4}$ = 7

$$\begin{array}{r} 28 \\ -\ 4 \\ \hline 24 \\ -4 \\ \hline 20 \\ -4 \\ \hline 16 \\ -4 \\ \hline 12 \\ -4 \\ \hline 8 \\ -4 \\ \hline 4 \\ -4 \\ \hline 0 \end{array}$$

## ★ Using Addition or Subtraction

Add to find the answer.

| | |
|---|---|
| 3 × 32 = 96<br>32<br>+32<br>64<br>+32<br>96 | 4 × 21 = 84<br>21<br>+21<br>42<br>+21<br>+21<br>84 |
| 2 × 65 = 130<br>65<br>+65<br>130 | 3 × 34 = 102<br>34<br>+34<br>+34<br>102 |

Subtract to find the answer.

| | |
|---|---|
| $\frac{98}{49}$ = 2<br>98<br>- 49<br>- 49<br>0 | $\frac{84}{12}$ = 7<br>84<br>- 12<br>72<br>-12<br>60<br>-12<br>48<br>-12<br>36<br>-12<br>24<br>-12<br>12<br>-12<br>0 |

One in division

Give a child 9 felt cut-outs and ask him to show, in an array, how many sets he can make with 1 member in each set. (9.) Does this show how many 1's there are in 9? (Yes: 9.)

Write:

$$\frac{9}{1} = 9 \qquad 9 \times 1 = 9$$

Give another child the 9 cut-outs and ask him to show how many sets he can make with 9 members in each set. (1.) Does this show how many 9's there are in 9? (Yes: 1.)

Write:

$$\frac{9}{9} = 1 \qquad 1 \times 9 = 9$$

Give several examples, including some with numbers like

$$\frac{65}{1} = 65 \qquad \text{and} \qquad \frac{98}{98} = 1.$$

Help children to generalize: If any number is divided by 1, the quotient is that number; if any number is divided by itself, the quotient is 1.

Pupil's book, pages 249 - 250: Provide practice on the ideas presented in section IX-3.

## Finding Quotients

Find n. Use multiplication facts, arrays, or subtraction.

$n \times 6 = 18$

$n = \frac{18}{6}$

$n =$ 3

$n \times 1 = 14$

$n = \frac{14}{1}$

$n =$ 14

$8 \times n = 24$

$n = \frac{24}{8}$

$n =$ 3

$n \times 62 = 62$

$n = \frac{62}{62}$

$n =$ 1

$n \times 5 = 45$

$n = \frac{45}{5}$

$n =$ 9

$1 \times n = 97$

$n = \frac{97}{1}$

$n =$ 97

$2 \times n = 18$

$n = \frac{18}{2}$

$n =$ 9

$75 \times n = 75$

$n = \frac{75}{75}$

$n =$ 1

## Finding Quotients
Fill the blanks.

$\frac{27}{9} =$ 3

$\frac{50}{10} =$ 5

$\frac{20}{2} =$ 10

$\frac{12}{4} =$ 3

$\frac{16}{4} =$ 4

$\frac{43}{43} =$ 1

$\frac{70}{10} =$ 7

$\frac{595}{1} =$ 595

## IX-4. Rational Numbers

**Objective**: To build the concept of rational number.

**Vocabulary**: Rational number, fraction, one half, one third, one fourth, etc.

**Materials**: Construction paper with heavy markings to show congruent regions as indicated in text. Commercial or teacher-made fraction kit.

**Suggested Procedure**:

Write: $\frac{8}{2}$

> What number is this? (4.) When we write a numeral, a bar under it, and another numeral below the bar, we call this expression a fraction.

As you proceed, illustrate on the chalkboard.

> The fraction $\frac{8}{2}$, (read eight-halves) names the number which is the result of dividing 8 by 2. What is another name for this number? (4).
>
> The fraction $\frac{4}{2}$ (read four-halves) is another name for what number? (2).
>
> The fraction $\frac{2}{2}$ (read two-halves) is another name for what number? (1).

Write the fraction $\frac{1}{2}$ but do not read it aloud.

What would this mean to you? (One divided by two.) If we want to show what $\frac{8}{2}$ is, we can partition a set of 8 members into 2 equivalent subsets and find that each subset has 4 members. To find out what $\frac{4}{2}$ is, we do the same thing, only we start with a set of 4 members. For $\frac{2}{2}$ we start with a set of 2 members. What set will we start with to find out what 1 divided by 2 is? (A set of 1 member.)

Show a piece of paper and ask how it could be shared by 2 people. (Fold it in the middle and give each child 1 piece.)

We'll fold it, so you can see what each person would get if we cut it in two. When you name one of these pieces, what do you usually call it? (One half.)

Write $\frac{1}{2}$.

How many halves are there in 1 piece of paper? (2.)

Write $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$. (Show the folded paper and point to first one half and then the other.)

What is another name for this number? (Point to $\frac{2}{2}$. Answer: 1.)

And if we have 2 halves (Unfold paper and display it.), do we have 1 sheet of paper? (Yes.) If these pieces were not the same size, could we call them halves? (No.)

If we split one thing into more than 2 pieces, could we call each piece one half of it? (No.)

Write: $\frac{1}{3}$

What would this mean to you? (One divided by three.) What would this fraction describe? (One thing split into 3 congruent pieces.) The fraction "$\frac{1}{3}$" names a number. What did we just call this number? (One divided by three.)

We can also call this number one third. How many thirds are there in 1? (3.)

What is 3 divided by 3? (Write $\frac{3}{3}$. Answer: 1.) When we say "three thirds" or "3 divided by 3," what number are we talking about? (The number 1.)

Continue with the fractions $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$, each time asking how many of them it would take to make 1, etc.

Suppose, instead of thinking about splitting just 1 thing into 2 congruent parts, we think about splitting one set of things into 2 equivalent subsets. Think of a set of 4 members. What is $\frac{1}{2}$ of 4? (2.)

Write $\frac{4}{2}$ on the board.

We can write: $\frac{1}{2}$ of $4 = \frac{4}{2} = 2$.

Now think of a set of 16 numbers.

What is $\frac{1}{2}$ of 16? (8.)

What is 16 divided by 2? (8.)

We write: $\frac{1}{2}$ of $16 = \frac{16}{2} = 8$.

If we want to take one third instead of one half, into how many equivalent subsets would we partition the set? (3.)

Think of a set of 12 members.

What is $\frac{1}{3}$ of 12? (4.)

What is 12 divided by 3? (4.)

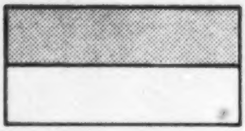
We would write: $\frac{1}{3}$ of $12 = \frac{12}{3} = 4$.

Give more examples, as needed.

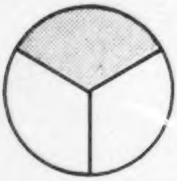
Pupil's book, page 251: reinforces these ideas.

## Rational Numbers and Fractions

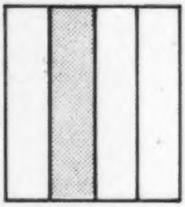
Ring the fraction that shows what part is shaded.



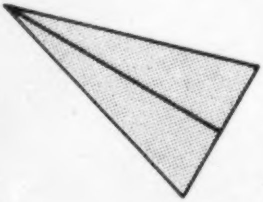
$\frac{2}{2}$ [✓ $\frac{1}{2}$] $\frac{1}{3}$ $\frac{1}{4}$



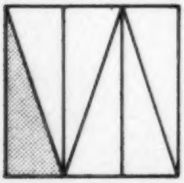
$\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{3}$ [✓ $\frac{1}{3}$]



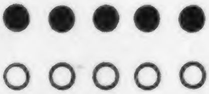
$\frac{1}{2}$ $\frac{1}{3}$ [✓ $\frac{1}{4}$] $\frac{1}{6}$



[✓ $\frac{2}{2}$] $\frac{1}{3}$ $\frac{1}{2}$ $\frac{2}{3}$



$\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{2}$ [✓ $\frac{1}{6}$]



$\frac{1}{4}$ [✓ $\frac{1}{2}$] $\frac{1}{3}$ $\frac{1}{6}$

- Show a sheet of paper folded into fourths. Ask into how many congruent parts it has been divided. (4.) Ask what fraction describes each of these parts. ($\frac{1}{4}$.)

  > If we use a fraction to talk about some of the parts, the numeral below the bar of the fraction tells us how many congruent parts the region has been divided into. If the numeral below the bar is a 6, we think of a region that has been divided into 6 congruent parts, or of a set that has been partitioned into 6 equivalent subsets, etc. If we want to think about 1 of the congruent parts of a region, we write a 1 above the bar. If we want to think about 2 of the parts, we write a 2 above the bar.
  >
  > Suppose we want to talk about this much of this sheet of paper. (Write $\frac{3}{4}$.)

  Have a child point to 3 of the sections of paper. Display other regions, divided into thirds, fourths, sixths, and eights, and have children point out given fractional parts: $\frac{2}{3}$, $\frac{5}{8}$, $\frac{5}{6}$, etc. Mark parts of some of the regions and have children write the fraction shown.

  Use two sheets of paper, each divided into three congruent parts.

  > Suppose we think of trying to divide these two pieces of paper so that 3 children can share them. What could we do?

If children are hesitant, suggest letting each child take 1 of the congruent parts of each piece.

| | | |
|---|---|---|
| A's part | B's part | C's part |

| | | |
|---|---|---|
| A's part | B's part | C's part |

Cut the paper into thirds, and show that each child's share will be two thirds of one sheet of paper. Write $\frac{2}{3}$ = two thirds, and read this "Two divided by three is two thirds."

Write on the board a variety of fractions of the sort considered so far, such as

$\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{8}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{5}{6}$, etc.

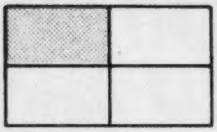
Does each of these fractions name a number? (Yes.) In each of these fractions do the numerals above and below the bar name whole numbers? (Yes.) Numbers which can be named by such fractions are called rational numbers. Can you think of a simpler name for the rational number $\frac{2}{2}$ ? (Yes: "1".) So 1 is a rational number. Can you think of a simpler name for the rational number $\frac{4}{2}$ ? (Yes: "2".) For the rational number $\frac{8}{2}$ ? (Yes: "4".) Are 2 and 4 both rational numbers? (Yes.) Are all whole numbers rational numbers? (Yes.)

Pupil's Book, page 252: Remind children to record the number of congruent parts in the unit region below the bar, and above the bar how many of the parts are shaded.

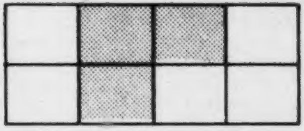
Pupil's Book, page 253 - 254: Children are to ring fractional parts of sets, and shade fractional parts of regions.

## Fractions

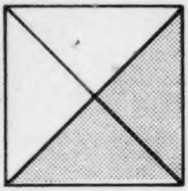
Write the fraction that shows what part is shaded.



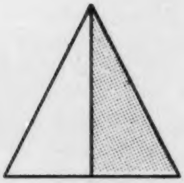
$\frac{1}{4}$



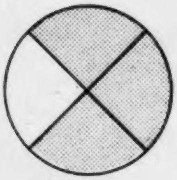
$\frac{3}{8}$



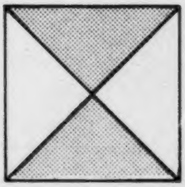
$\frac{2}{4}$ or $\frac{1}{2}$



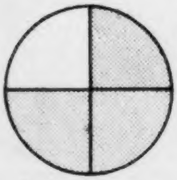
$\frac{1}{2}$



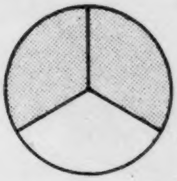
$\frac{3}{4}$



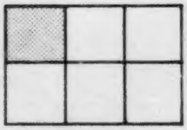
$\frac{2}{4}$ or $\frac{1}{2}$



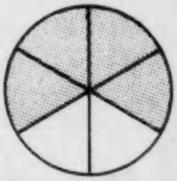
$\frac{3}{4}$



$\frac{2}{3}$



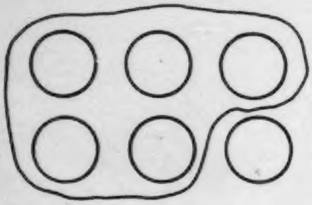
$\frac{1}{6}$



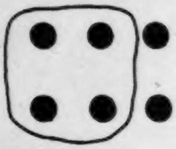
$\frac{4}{6}$ or $\frac{2}{3}$

## Fractions

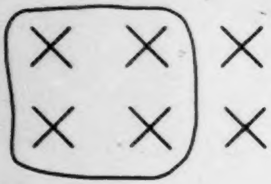
Ring the part of the set shown by the fraction:



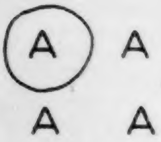
$\frac{5}{6}$



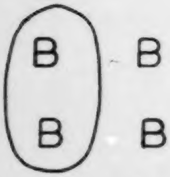
$\frac{2}{3}$



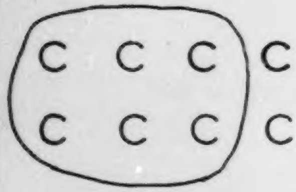
$\frac{4}{6}$



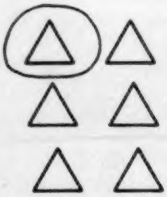
$\frac{1}{4}$



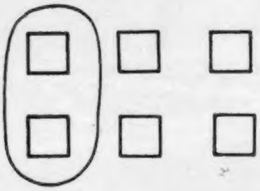
$\frac{1}{2}$



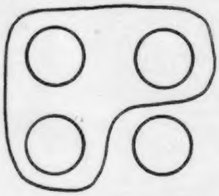
$\frac{3}{4}$



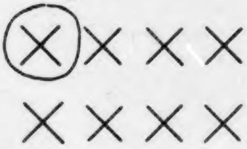
$\frac{1}{6}$



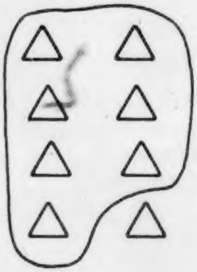
$\frac{1}{3}$



$\frac{3}{4}$



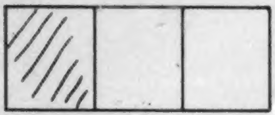
$\frac{1}{8}$



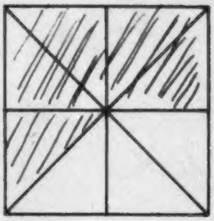
$\frac{7}{8}$

## Fractions

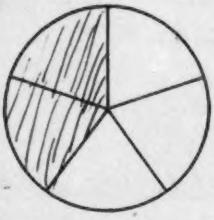
Shade the part shown by the fraction.



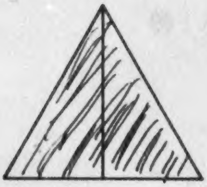
$\frac{1}{3}$



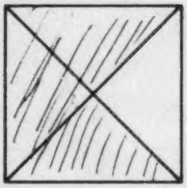
$\frac{5}{8}$



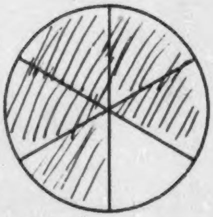
$\frac{2}{5}$



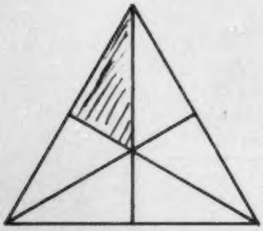
$\frac{2}{2}$



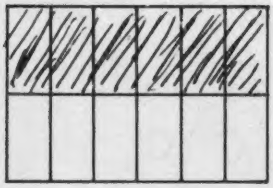
$\frac{3}{4}$



$\frac{5}{6}$



$\frac{1}{6}$



$\frac{6}{12}$

## IX-5. Rational numbers and the number line

Objective: To use the number line to visualize some of the simplest rational numbers and how they are added.

Vocabulary: (No new words.)

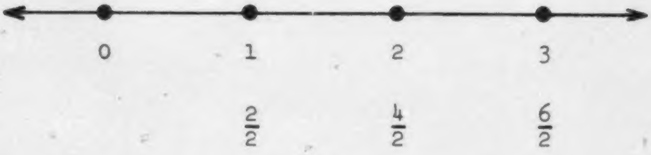
Materials: Number line on chalkboard, with 1 foot segments between points marked.

Suggested Procedure:

Review ideas about the number line: it goes on and on in both directions; every whole number has a place on the number line; it is the same distance from 2 to 3 as from 0 to 1 or from 98 to 99, etc. After the first few numerals, 0, 1, 2, 3, etc., have been written below their points on the number line, explain that underneath these you are going to show some of the other names for these same numbers.

> Let's write another name for the number 1. We know that the number $\frac{2}{2}$ is the same as the number 1. We also know that $\frac{4}{2}$ is the same number as 2 and that $\frac{6}{2}$ is the same as 3.

Label points as shown below:



We see where to write $\frac{2}{2}$ because $\frac{2}{2}$ is another name for 1. Where would $\frac{1}{2}$ go? (Halfway between 0 and 1.)

Make a paper model of the unit segment, fold it in half, and mark the point between 0 and 1, labeling it $\frac{1}{2}$.

We have $\frac{1}{2}$, $\frac{2}{2}$ and then a big jump to $\frac{4}{2}$. What should go between $\frac{2}{2}$ and $\frac{4}{2}$? ($\frac{3}{2}$) (Mark it.)

What goes between $\frac{4}{2}$ and $\frac{6}{2}$? ($\frac{5}{2}$. Mark it.) So we can take half jumps from $\frac{1}{2}$ to $\frac{2}{2}$ to $\frac{3}{2}$ to $\frac{4}{2}$ to $\frac{5}{2}$ to $\frac{6}{2}$. Could we go on taking half jumps? (Yes.)

Is there a point we could label $\frac{20}{2}$? (Yes. 10.)

Is there a point we could label $\frac{100}{2}$? (Yes.)

Lead children to see that for each half jump from right to left along the number line, the number of halves becomes one less, and that it fits in with this if 0 is named "$\frac{0}{2}$".

We have used the number line to help us add whole numbers. Can we use it to help us add halves?

Write: $\frac{1}{2} + \frac{1}{2} =$ _____

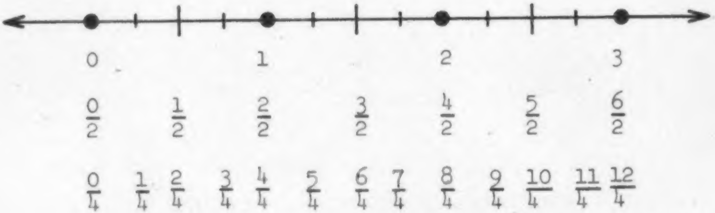
Have the child draw arrows to show half jumps on the number line. Observe that 2 half jumps means that he stops at $\frac{2}{2}$ or 1. Remind the children that they have already seen the same result when they looked at congruent regions, as illustrated with a piece of paper in the previous lesson. Write other examples, like: $\frac{3}{2} + \frac{2}{2} =$ ______, $1 + \frac{1}{2} =$ _____, etc. When children seem to understand that halves have a place on the number line and can be added, go on to fourths.

Since we could mark a point half-way between 0 and 1, do you think we could mark a point half-way between 0 and $\frac{1}{2}$? (Yes.)

Ask for suggestions about how to find the half-way mark. Use the model of the segment used to find $\frac{1}{2}$, and fold the folded paper again. Mark the point midway between 0 and $\frac{1}{2}$. Ask what the number of that point would be. Unfold the paper model to show that you have used 1 of the 4 congruent parts of the model, or $\frac{1}{4}$ of it.

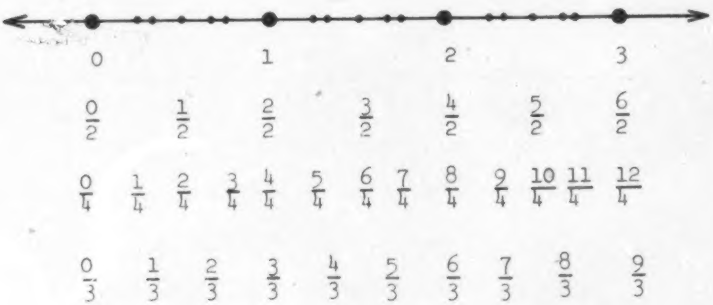
Develop the idea that the model $(\frac{1}{4})$ can be laid off to the right on the number line as often as we please. Mark points and discuss the fact that each successive point is one more fourth of a unit segment to the right.

Label points:


0
1
2
3
$\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$
$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$ $\frac{9}{4}$ $\frac{10}{4}$ $\frac{11}{4}$ $\frac{12}{4}$

Children should observe that the number $\frac{4}{4}$ is indeed found to be the number 1, that $\frac{8}{4}$ is 2, and that $\frac{12}{4}$ is 3. Ask what $\frac{2}{4}$, $\frac{6}{4}$, and $\frac{10}{5}$ are. ($\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, . . .)

Have the children add, using fourths, as you did with halves. Lead to discussion of thirds on the number line by having 1 renamed as $\frac{3}{3}$. Fold a model segment in thirds, mark points, and label them as before.

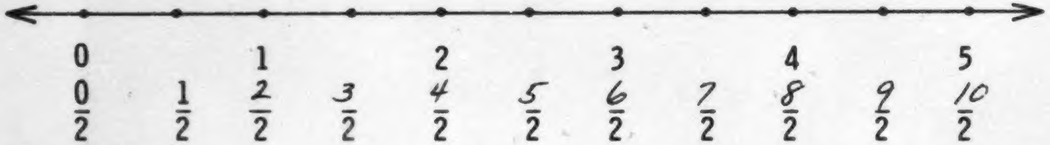

0
1
2
3
$\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$
$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$ $\frac{9}{4}$ $\frac{10}{4}$ $\frac{11}{4}$ $\frac{12}{4}$
$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$ $\frac{7}{3}$ $\frac{8}{3}$ $\frac{9}{3}$

Help children to generalize: A rational number has a place on the number line, and rational numbers have many names.

Pupil's book, pages 255 - 256: These pages will probably require considerable supervision. Children are first to write numerals above the bar, for each rational number, and then to draw arrows to show the sums indicated.

Pupil's book, page 256: Extends the idea of rational numbers on the number line to sixths.

The Number Line


0 1 2 3 4 5
0/2 1/2 2/2 3/2 4/2 5/2 6/2 7/2 8/2 9/2 10/2

Name points to show halves. Show the sum of $\frac{3}{2}$ and $\frac{3}{2}$. $\frac{3}{2} + \frac{3}{2} =$ $\frac{6}{2}$ or 3

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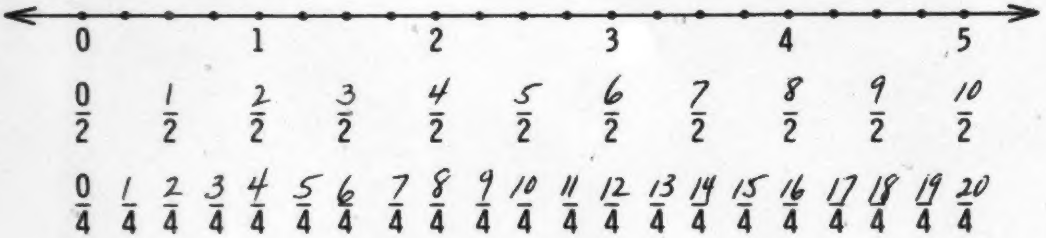
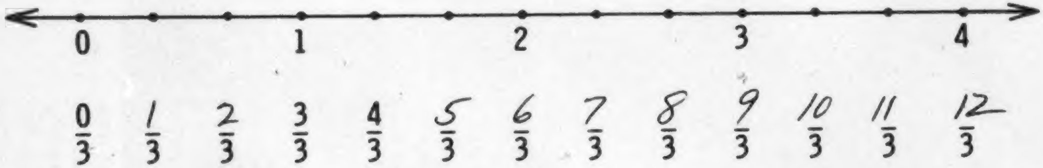
255

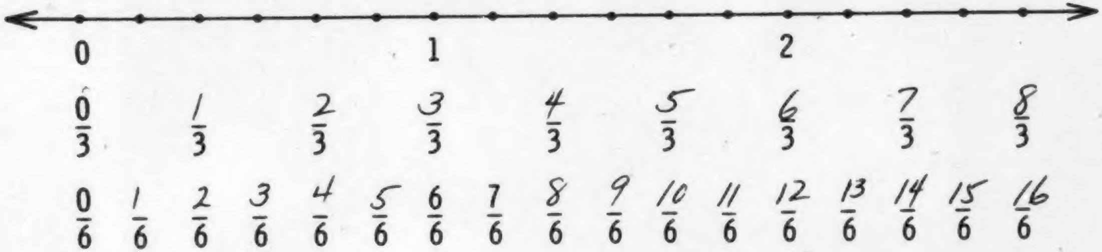

0 1 2 3 4 5
0/2 1/2 2/2 3/2 4/2 5/2 6/2 7/2 8/2 9/2 10/2
0/4 1/4 2/4 3/4 4/4 5/4 6/4 7/4 8/4 9/4 10/4 11/4 12/4 13/4 14/4 15/4 16/4 17/4 18/4 19/4 20/4

Name points to show halves and fourths. $\frac{2}{4} + \frac{3}{4} = \frac{5}{4}$

The Number Line


0
1
2
3
4
$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$ $\frac{7}{3}$ $\frac{8}{3}$ $\frac{9}{3}$ $\frac{10}{3}$ $\frac{11}{3}$ $\frac{12}{3}$

Name points to show thirds. $\frac{2}{3} + \frac{3}{3} = \frac{5}{3}$


0
1
2
$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{5}{3}$ $\frac{6}{3}$ $\frac{7}{3}$ $\frac{8}{3}$
$\frac{0}{6}$ $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$ $\frac{7}{6}$ $\frac{8}{6}$ $\frac{9}{6}$ $\frac{10}{6}$ $\frac{11}{6}$ $\frac{12}{6}$ $\frac{13}{6}$ $\frac{14}{6}$ $\frac{15}{6}$ $\frac{16}{6}$

Name points to show thirds and sixths. $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

The following is a list of all those who participated in the preparation of this volume:

Leslie Beatty, Chula Vista City School District, Chula Vista, California
Truman Botts, University of Virginia
Leon W. Cohen, University of Maryland
Zigmund Drapalski, WTVS, Detroit Public Schools, Detroit, Michigan
Jean Dunn, Palo Alto Unified School District, Palo Alto, California
Wade Ellis, Oberlin College, Oberlin, Ohio
Mary O. Folsom, University of Miami, Florida
Mary E. Giamperoli, Edith C. Baker School, Chestnut Hill, Massachusetts
Leonard Gillman, University of Rochester, Rochester, New York
E. Glenadine Gibb, State College of Iowa
Muriel Greig, McColl School, Detroit, Michigan
Adrien L. Hess, Montana State College
Stanley B. Jackson, University of Maryland
John L. Kelley, University of California, Berkeley
Sharon Logan, Oak Ridge Elementary School, Arlington, Virginia
William F. McClintock, Stanislaus State College, Turlock, California
Mary McCulloch, University School, Northern Illinois University, DeKalb, Illinois
Patricia Michels, Joaquin Miller School, Oakland, California
Rose Mijanovich, Joaquin Miller School, Oakland, California
Mildred Pierce, Humbert School, Cedar Falls, Iowa
Frank W. Sinden, Bell Telephone Laboratories, Murray Hill, New Jersey
Jane Stenzel, Cambrian Elementary School District, San Jose, California
J. Fred Weaver, Boston University